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- For convenience, we write $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = x^2$ to represent the function $f = (\mathbb{R}, \mathbb{R}, G)$ where $G = \{(x, y) \in \mathbb{R}^2 : y = x^2\}$.

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  - ◇ Sometimes, instead of $f\big|_{[0,\infty)}$, we simply write f.

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- a real number $2 \in \mathbb{R}$
- a constant function $2 : \mathbb{R} \longrightarrow \mathbb{R},\; 2(x) = 2$
- a multiplication operator $2 : C(-\infty, \infty) \longrightarrow C(-\infty, \infty),\; 2[f] = 2 \cdot f$