MATH 2405 Differential Equations

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- Learn some methods for solving DE
- Understand why the methods work
- Understand the main theorems

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A joke ?

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> To which the physicist responds, *"No, no! Some Scottish sheep are black !"*
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> The mathematician gazes heavenward in supplication, and then intones, *"In Scotland there exists a field, containing a sheep, at least one side of which is black."*

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A ridicule or a compliment?

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- physical sciences,
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- biological sciences,
- social sciences . . .

A vector-host epidemic model for BSE

$$\begin{cases} S'(t) = -\Big(aM(t) + b(1 - M(t))\Big)S(t) \\ D'(t) = aM(t)S(t) \\ M'(t) = cD(t)\Big(1 - M(t)\Big) \\ S(0) = 1, \quad D(0) = 0, \quad M(0) = m \end{cases}$$

Data

Year	1986	1987	1988	$\cdots$	$\cdots$	$\cdots$	2001
Cases	0	442	2469	$\cdots$	$\cdots$	$\cdots$	499

SIR model for SARS

$$
\begin{cases}
S'(t) = -aS(t)I(t) \\
I'(t) = aS(t)I(t) - bI(t) \\
R'(t) = bI(t) \\
S(0) = p_0 - 1, \quad I(0) = 1, \quad R(0) = 0
\end{cases}
$$

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Chapter 0: Introduction

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$$\frac{\mathrm{d}A}{\mathrm{d}t} = -kA \tag{1}$$

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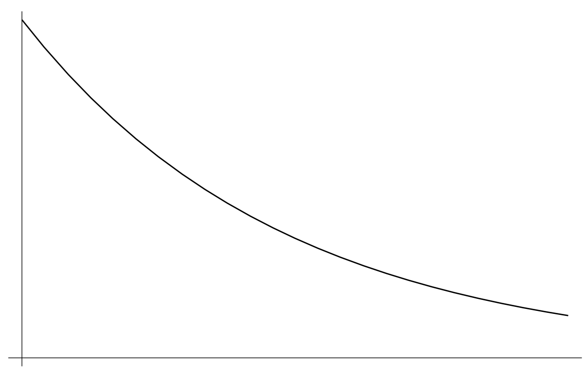
show decay.nb



(2) Newton's Law of Cooling:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = -k(T - T^*) \tag{2}$$

- $T = T(t)$ is temperature of object at time t
- T^* is the temperature of the surrounding
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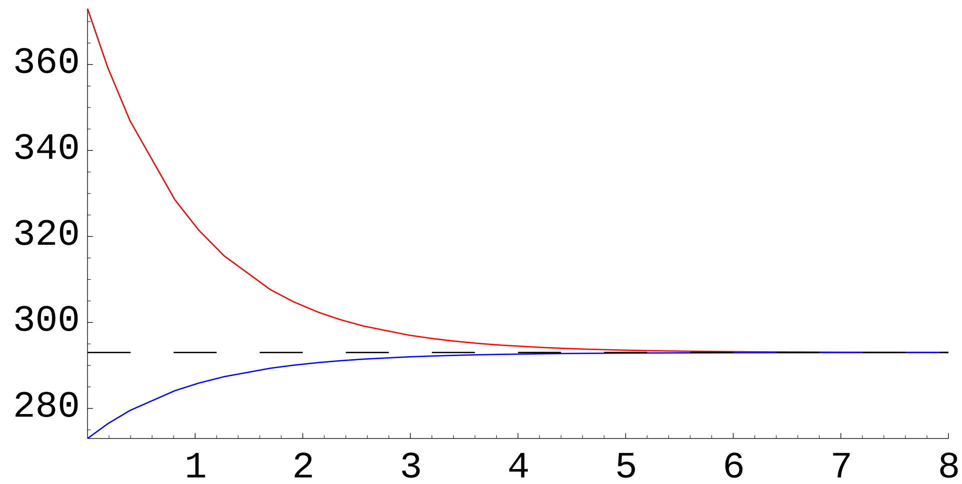

(3) Logistic equation:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP(P^* - P) \tag{3}$$

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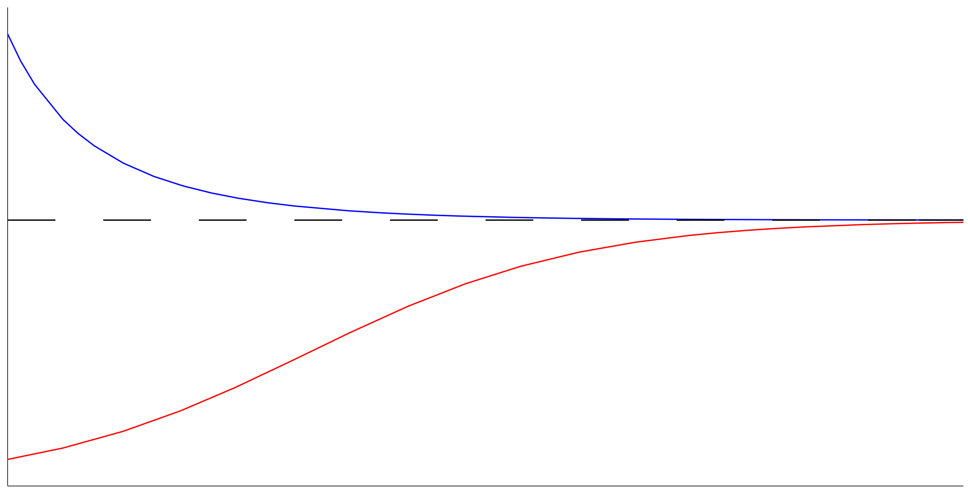
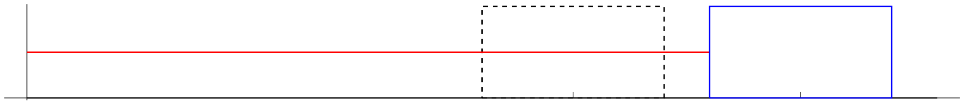


(4) Damped unforced vibration:

$$M\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0 \tag{4}$$

- $x = x(t)$ is displacement at time t
- M = mass
- k = spring constant
- c = resistance constant

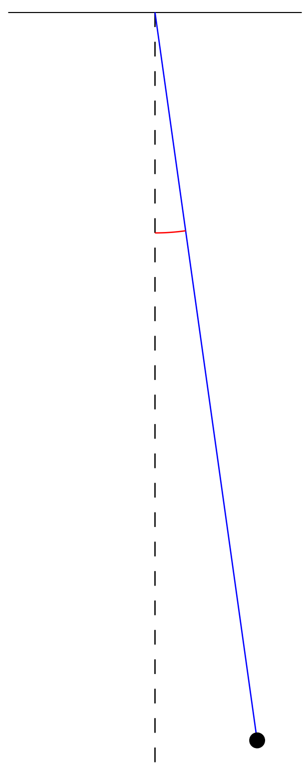
show damped.nb



(5) Simple pendulum:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0 \tag{5}$$

- $\theta = \theta(t)$ is angle that arm makes with vertical
- L is length of pendulum
- g is gravitation constant



show pendulum.nb

(6) Airy's equation:

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + xy = 0 \tag{6}$$

related to theory of diffraction.

Remark $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - xy = 0$ is also called Airy's equation.

show airy.nb

(7) Heat equation:

$$a^2\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)=\frac{\partial w}{\partial t} \tag{7}$$

- $w = w(x, y, z, t)$ is temperature of a point (x, y, z) at time t
- a is a constant

show heat.nb

(8) Volterra's prey-predator equation:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = x\,(a - by) \\[2ex] \dfrac{\mathrm{d}y}{\mathrm{d}t} = -y\,(c - dx) \end{cases} \tag{8}$$

- $x = x(t)$ is prey population
- $y = y(t)$ predator population at time t.
- a, b, c, d are positive constants

show prey-predator.nb

Terminology If unknown function is

- *function of a single variable*, the differential equation is called an *ordinary differential equation* (ODE)
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$$\boxed{\text{DE=ODE}}$$

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- (4), (5) and (6) order 2

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- (1), (2) and (3) order 1
- (4), (5) and (6) order 2
- (8) is a first order system.

Terminology

- DE of order n can be written as (called *implicit* form)

$$F(t, y, y', y'', \ldots, y^{(n)}) = 0$$

where y' denotes $\dfrac{\mathrm{d}y}{\mathrm{d}t}$ etc.

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(2) 2nd order $\quad F(t, y, y', y'') = 0$ implicit form

$y'' = f(t, y, y')$ explicit form

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$$a_n(t)y^{(n)} + a_{n-1}(t)y^{(n-1)} + \cdots + a_0(t)y = g(t)$$

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$$F(t, y, y', y'', \ldots, y^{(n)}) = 0$$

in (a, b) is a function $\varphi : (a, b) \longrightarrow \mathbb{R}$ such that

$$F(t, \varphi(t), \varphi'(t), \varphi''(t), \ldots, \varphi^{(n)}(t)) = 0 \qquad \forall\, t \in (a, b)$$

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$$y' + 2ty^2 = 0$$

Show that $\varphi(t) = \dfrac{1}{t^2}$ is a solution.

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Remark The interval $(0, \infty)$ is maximal:

> *If φ is a solution to the DE in an interval (a, b) and $(a, b) \supset (0, \infty)$, then $(a, b) = (0, \infty)$.*

Definition Let φ be a solution to a DE. An *interval of validity* of φ is an open interval (α, β) such that

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To solve a DE means

- to find all solutions to the DE;
- if possible, for each solution, determine its interval(s) of validity.

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- The following are solutions:
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    - ○ if $c = -p < 0$, φ_c solution on $(-\infty, -\sqrt{p})$, $(-\sqrt{p}, \sqrt{p})$ and $(\sqrt{p}, \infty)$.

      $\varphi_c(t) = \dfrac{1}{t^2 - p}$ is undefined at $\pm\sqrt{p}$