L'Hôpital's Rule

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Example Find $\lim_{x \to 1} \frac{x^2 - 1}{x^2 + 2x - 3}$

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Short Form

$$\lim_{x \to 1} \frac{x^2 - 1}{x^2 + 2x - 3} \qquad \frac{0}{0}$$

$$= \lim_{x \to 1} \frac{2x}{2x + 2}$$

$$= \frac{2}{2 + 2} = \frac{1}{2}$$

Example Find $\lim_{x \to 0} \dfrac{e^x - x - 1}{x^2}$

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$$= 0$$

Example Find $\displaystyle\int \frac{(x+1)\mathrm{e}^{x+\ln x}}{x}\,\mathrm{d}x$

Method 2

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Consider $\displaystyle\frac{\mathrm{d}}{\mathrm{d}x}\Big((x+1)\cdot \mathrm{e}^{x}\Big)$

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Consider $\displaystyle\frac{d}{dx}\Big((x+1)\cdot e^{x}\Big) = (x+1)\cdot\frac{d}{dx}e^{x} + e^{x}\cdot\frac{d}{dx}(x+1)$

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Therefore,

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$$= (x+1)e^{x} - e^{x} + C$$

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$$= (x+1)e^{x} - e^{x} + C$$
$$= x\,e^{x} + C$$

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- *surprising* that $\int_{-\infty}^{\infty} \mathrm{e}^{-x^2}\,\mathrm{d}x$ can be evaluated

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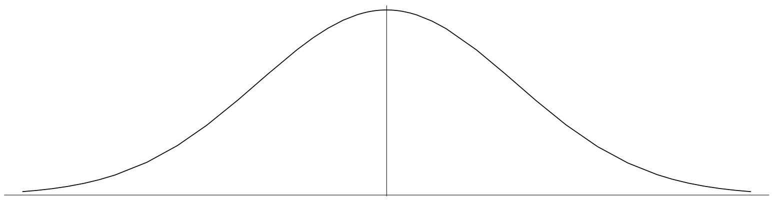

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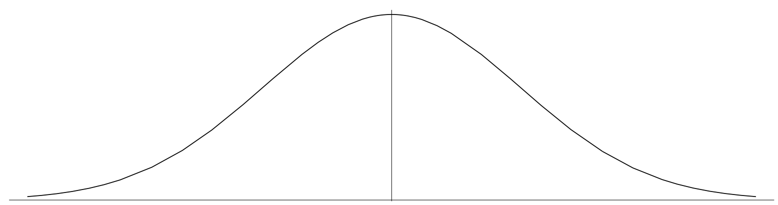

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}$$ (use double integral)