Strategy for u-substitution

- Consider the integrand as product of two functions $\int \alpha(x) \cdot \beta(x) \, \mathrm{d}x$
- Choose $u =$ *an expression appearing in one function* such that

  $\frac{\mathrm{d}u}{\mathrm{d}x}$ is the other function, OR

  $\frac{\mathrm{d}u}{\mathrm{d}x}$ is a *multiple of the other function*

Example Find $\int \sin(2x - 3)\,\mathrm{d}x$.

Example Find $\int \sin(2x-3)\,\mathrm{d}x$.

consider $\int \sin(2x-3)\cdot 1\,\mathrm{d}x$

Solution

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$$\int \sin(2x-3)\,\mathrm{d}x \quad = \quad \int \frac{1}{2}\sin(2x-3)\cdot 2\,\mathrm{d}x$$

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$$\begin{aligned}\int \sin(2x-3)\,\mathrm{d}x &= \int \frac{1}{2}\sin(2x-3)\cdot 2\,\mathrm{d}x \\ &= \int \frac{1}{2}\sin u\,\mathrm{d}u \qquad \textit{substitution} \\ &= \frac{1}{2}(-\cos u) + C\end{aligned}$$

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Solution

Let $u = 2x - 3$

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$$= \frac{1}{2}(-\cos u) + C$$

$$= -\frac{1}{2}\cos(2x-3) + C$$

Substitution Method for Definite Integrals

Example Evaluate $\int_0^4 x\sqrt{x^2+9}\,\mathrm{d}x$.

Substitution Method for Definite Integrals

Example Evaluate $\int_0^4 x\sqrt{x^2+9}\,\mathrm{d}x$. *consider* $\int x \cdot \sqrt{x^2+9}\,\mathrm{d}x$ *or*

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Substitution Method for Definite Integrals

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Step 1 Find indefinite integral by substitution.

Substitution Method for Definite Integrals

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Let $u = x^2 + 9$

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Step 2 Apply Fund Thm

Substitution Method for Definite Integrals

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Substitution formula for definite integrals

$$\int_a^b f(g(x))\, g'(x)\, \mathrm{d}x = \int_{g(a)}^{g(b)} f(u)\, \mathrm{d}u$$

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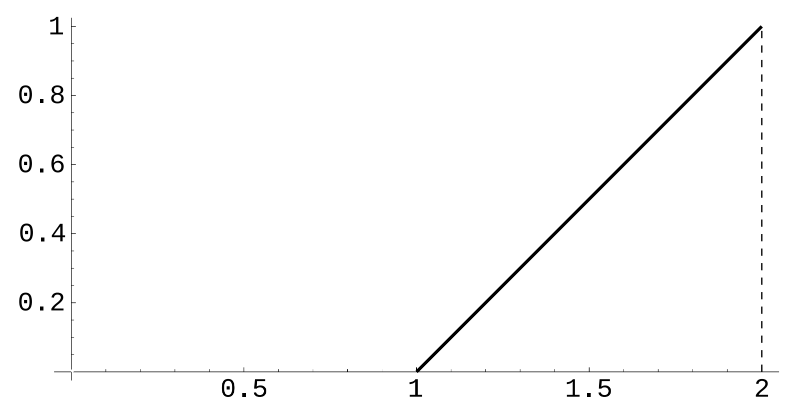

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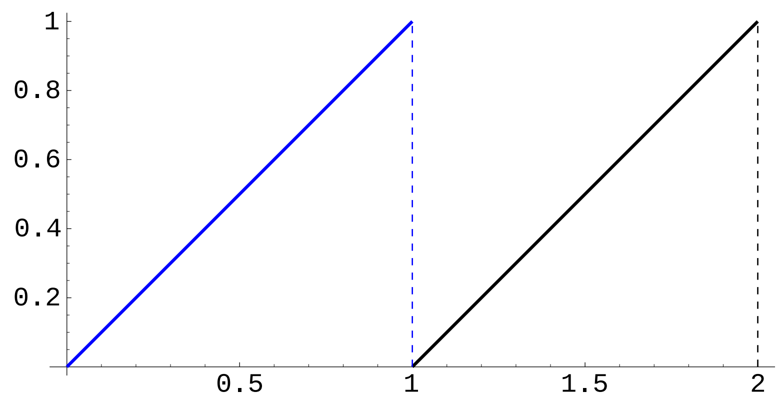

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Let $u = x^2 + 9$

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When $x = 4$, $u = 25$;

Example Evaluate $\int_0^4 x\sqrt{x^2+9}\,\mathrm{d}x$.

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$$= \int_9^{25} \frac{1}{2}u^{\frac{1}{2}}\,\mathrm{d}u$$ *substitution*

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Solution

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Solution

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Then $du = (2x+2)\,dx$

Example Evaluate $\int_0^1 (x+1)e^{x^2+2x}\,dx$. *consider* $\int (x+1)\cdot e^{x^2+2x}\,dx$ *or*

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Solution

Let $u = x^2 + 2x$

Then $du = (2x+2)\,dx$

When $x = 1$, $u = 3$;

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$$\int_0^1 (x+1)e^{x^2+2x}\,dx = \int_0^1 \frac{1}{2}e^{x^2+2x}\cdot(2x+2)\,dx$$

Example Evaluate $\int_0^1 (x+1)e^{x^2+2x}\,dx$.

consider $\int (x+1)\cdot e^{x^2+2x}\,dx$ *or* $\int e^{x^2+2x}\cdot(x+1)\,dx$

Solution

Let $u = x^2 + 2x$

Then $du = (2x+2)\,dx$

When $x = 1$, $u = 3$;

$x = 0$, $u = 0$.

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$$= \int_0^3 \frac{1}{2}e^{u}\,du$$ *substitution*

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