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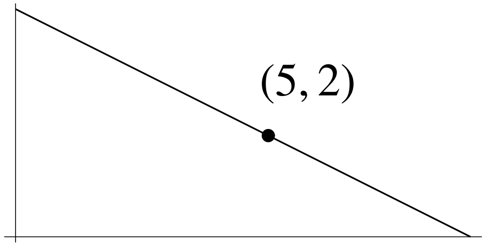

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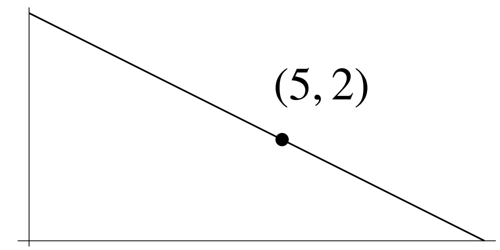

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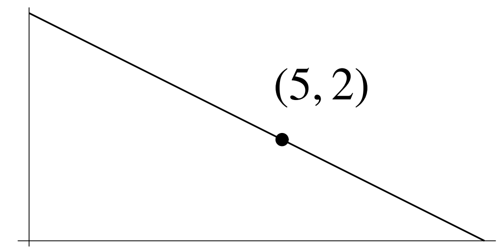

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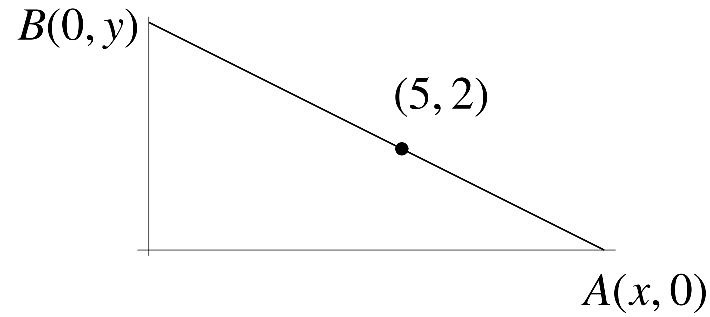

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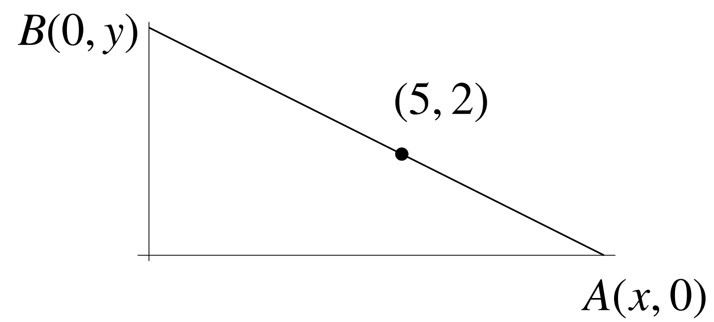

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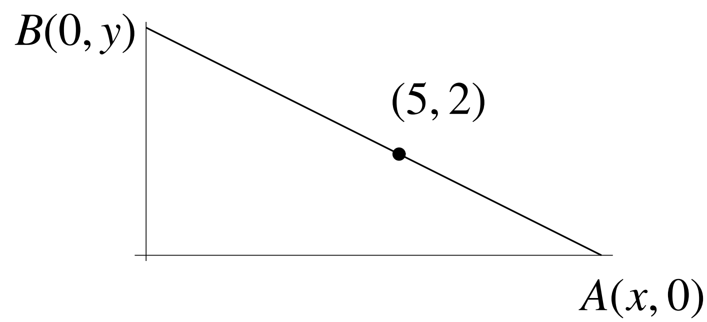

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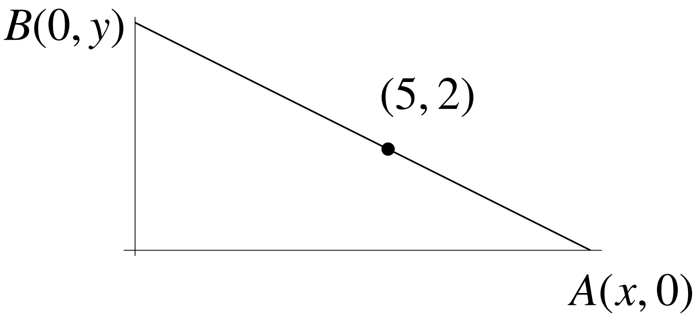

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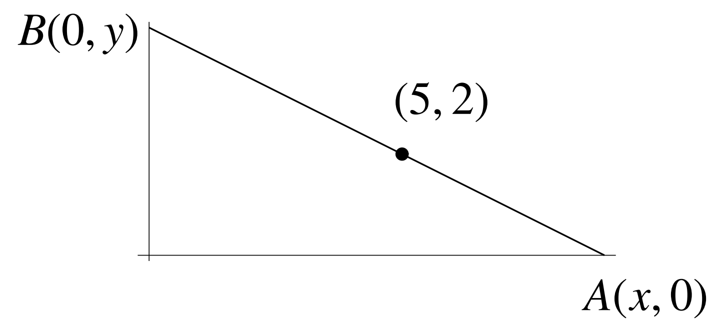

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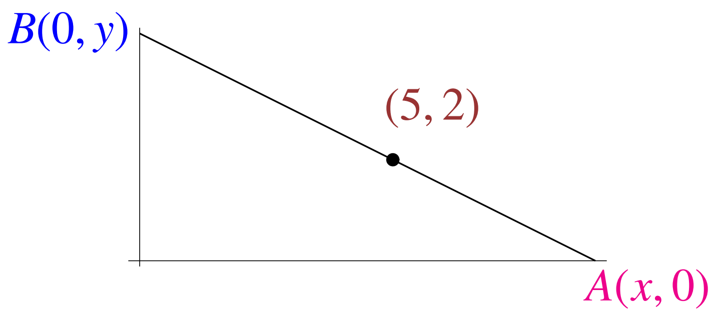

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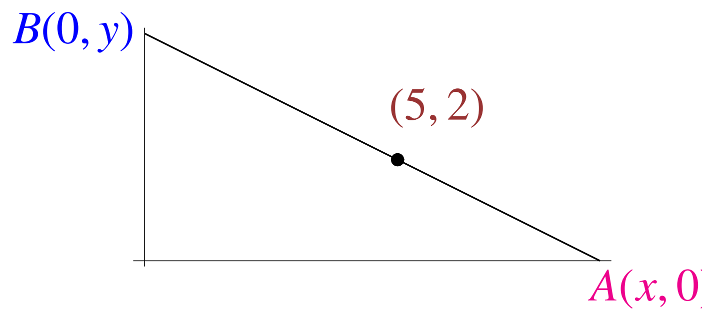

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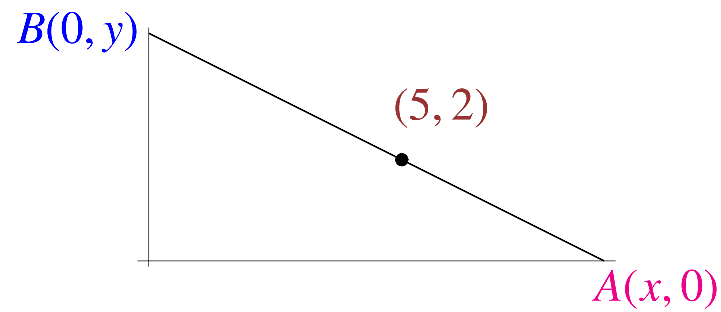

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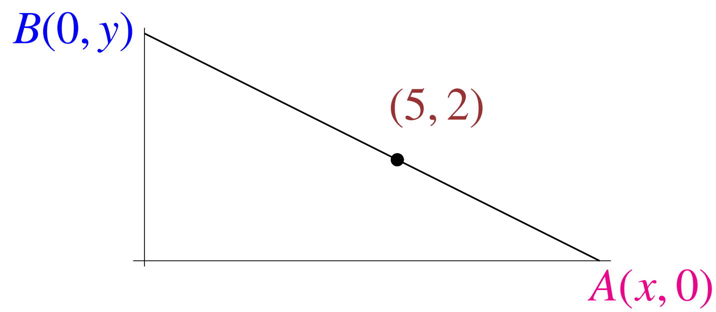

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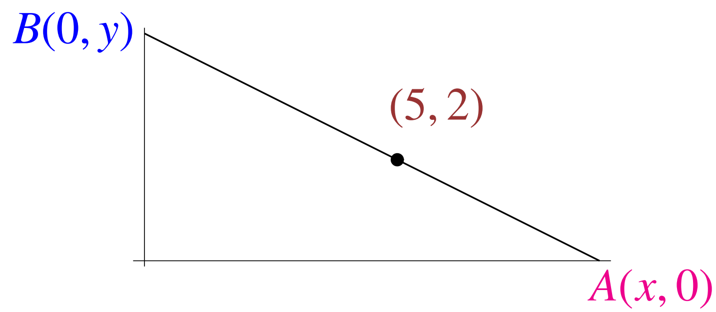

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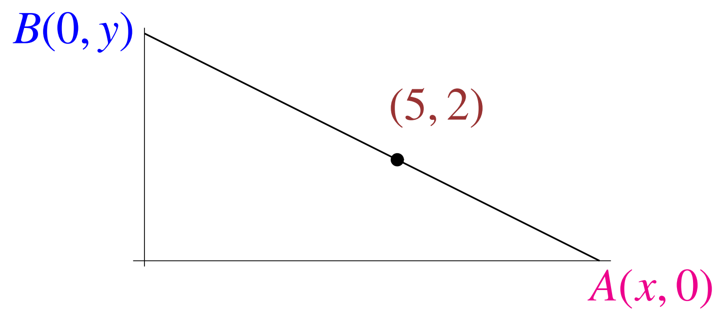

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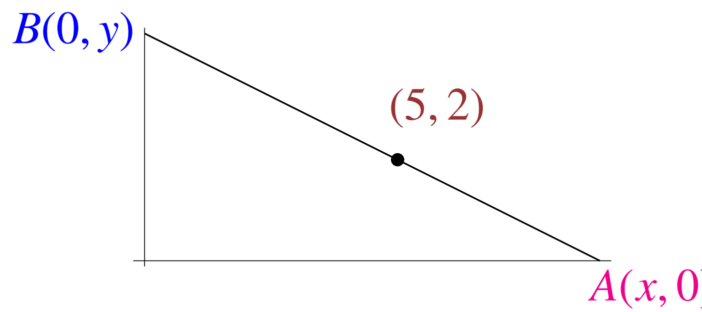

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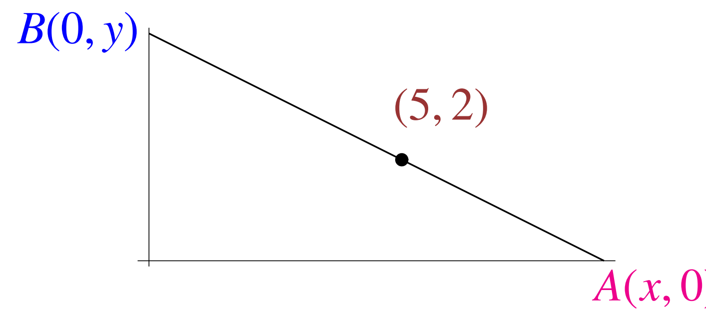

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$$= 2x + \frac{(x-5)^2 \cdot 8x - 4x^2 \cdot 2(x-5)}{(x-5)^4}$$

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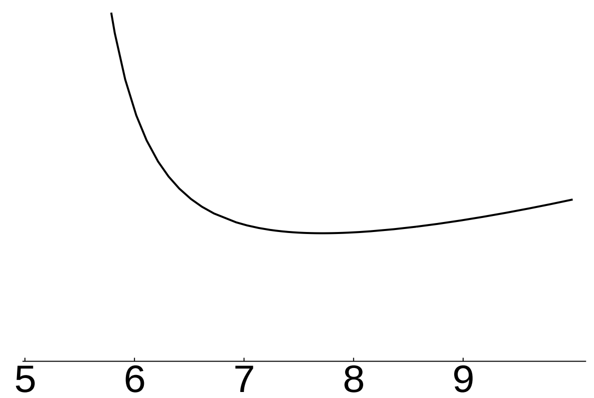

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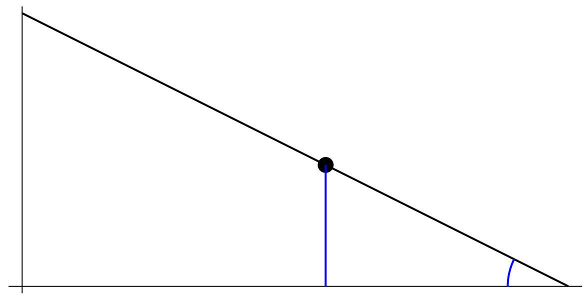
- Let θ be the angle between the line segment and the x-axis.



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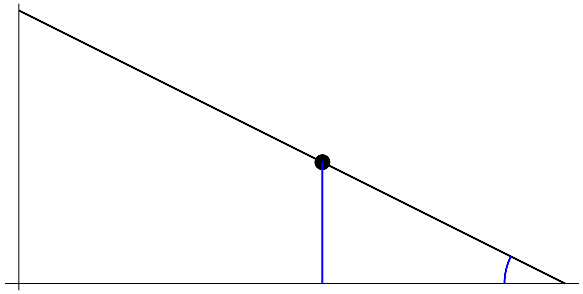

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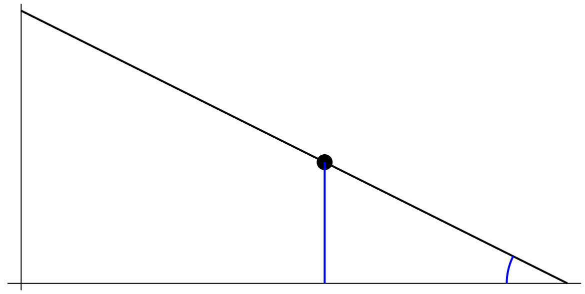

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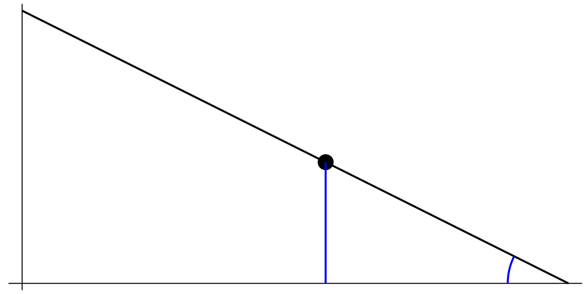

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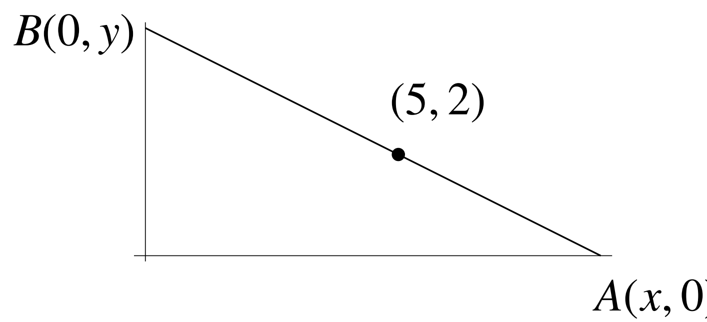

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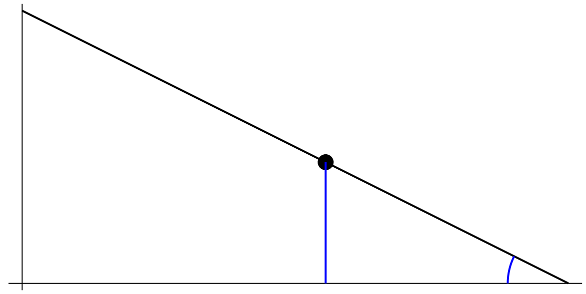

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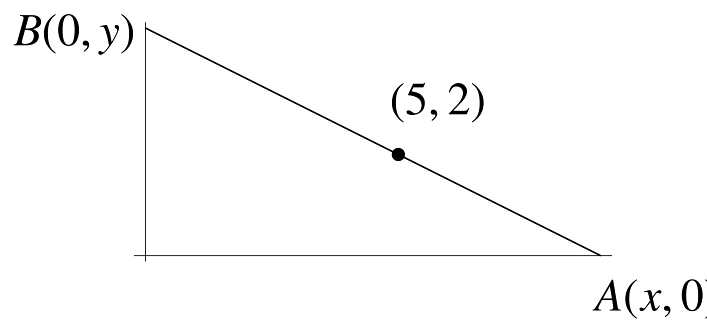

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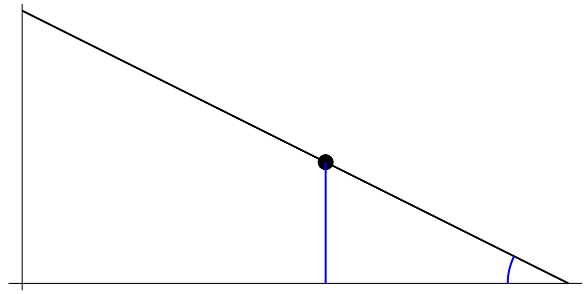

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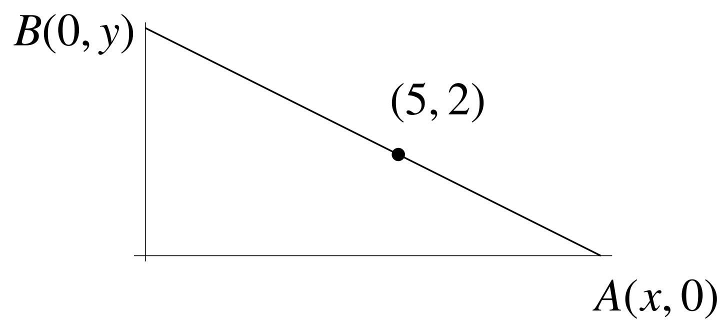

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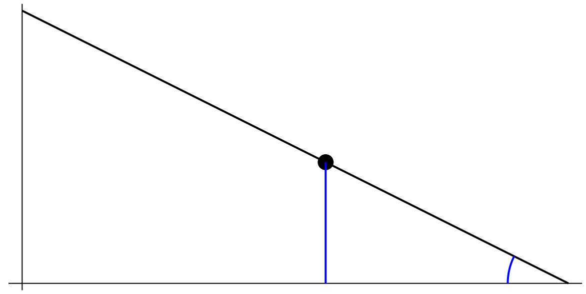

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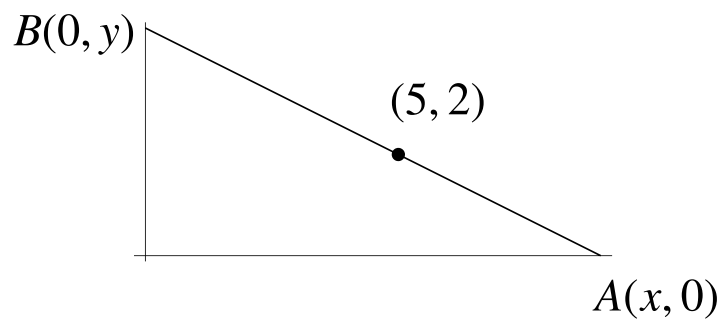

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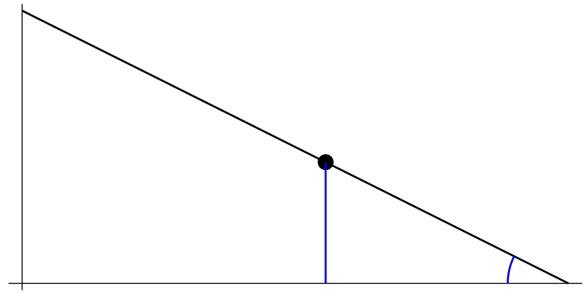
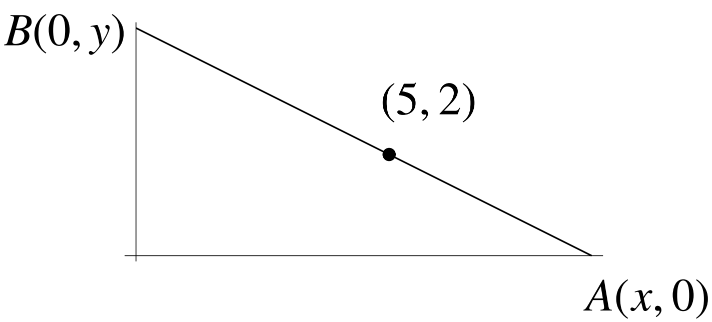


- Express L in terms of θ, where $0 < \theta < \dfrac{\pi}{2}$.
- Use differentiation to minimize L, *see lecture notes*.

Alternative Method 2

- Let m be the slope of the line segment.
- Express x and y in terms of m.
- Hence express L in terms of m
- Remaining steps, exercise



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(3) What percentage of the target group will have purchased the CD when the maximum profit is reached?

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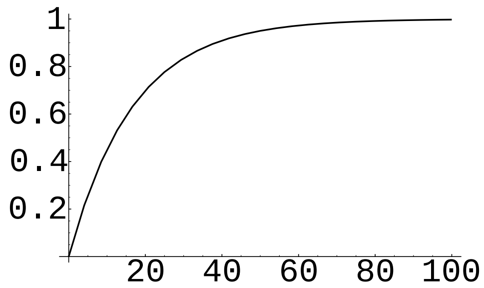

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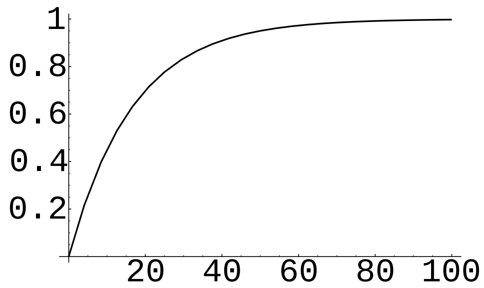

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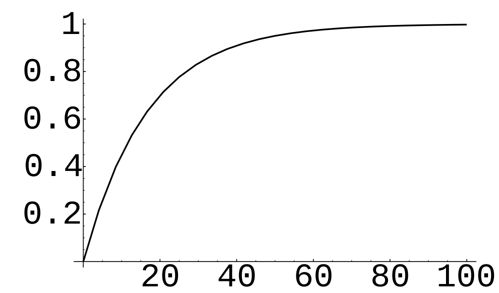

Meaning of $1 - e^{-0.06t}$

- When $t = 0$, it is 0.
- It increases with t *(more and more people know)*
- When t is large, it is close to 1 *(almost everyone knows)*

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- $$P'(t) \quad = \quad 1000000 \cdot -\mathrm{e}^{-0.06t} \cdot \frac{\mathrm{d}}{\mathrm{d}t}(-0.06t) - 12000$$

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	$t = 26$	$t = 27$
$P(t)$	277864	278101

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(1) 27 days of TV promotion should be used to maximize the profit.

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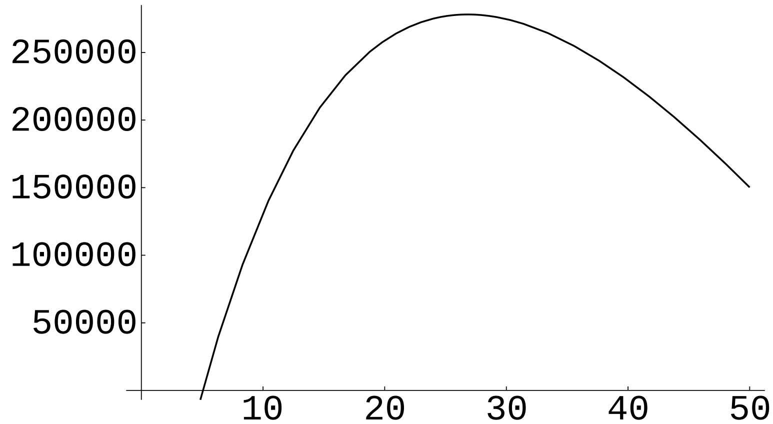
(1) 27 days of TV promotion should be used to maximize the profit.

(2) The maximum profit is 278101 dollars.

(3) $1 - e^{-0.06 \times 27} = 0.80 = 80\%$ of target group will have purchased the CD.

- Graph of the profit function



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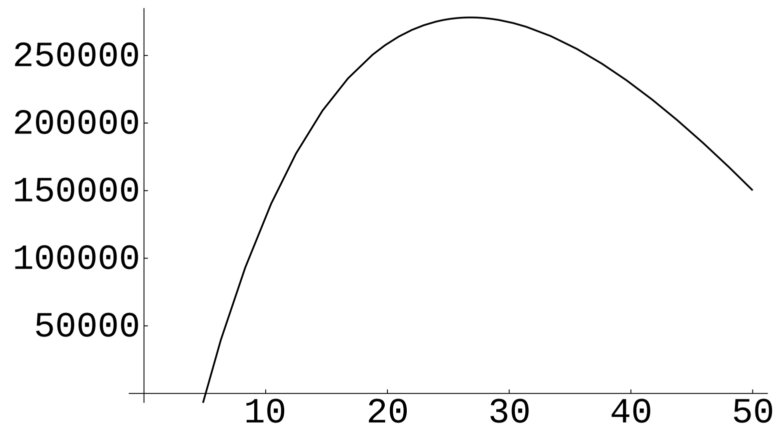
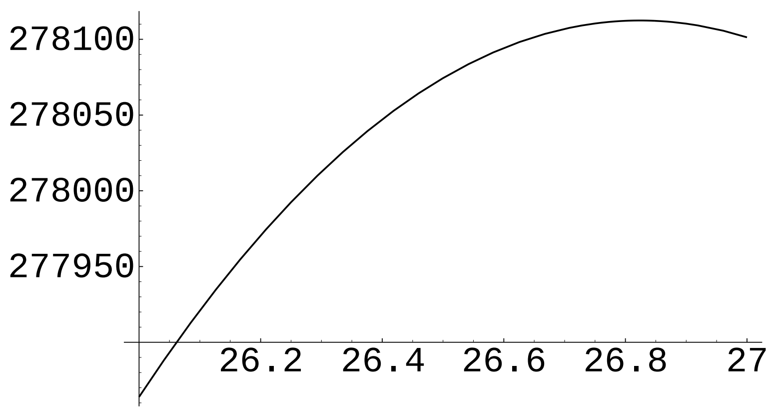


- Closer look around the critical point



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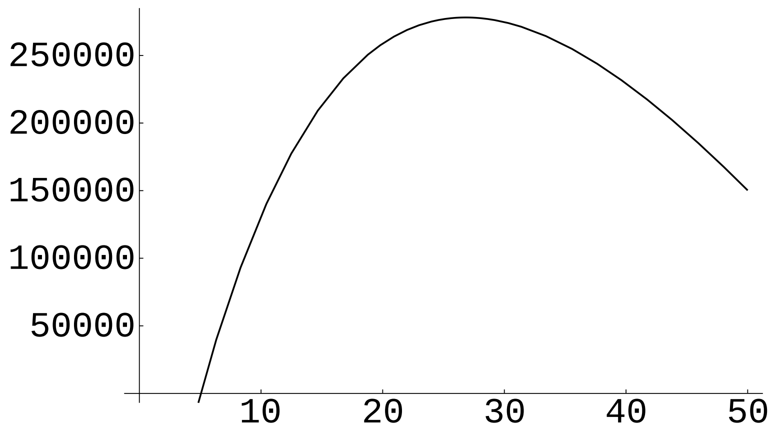

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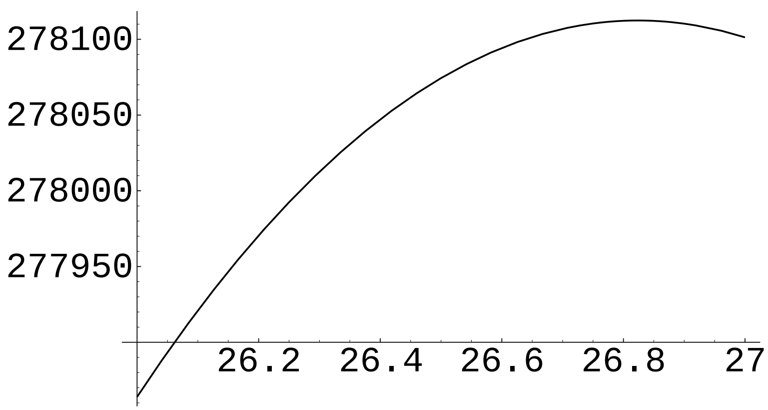
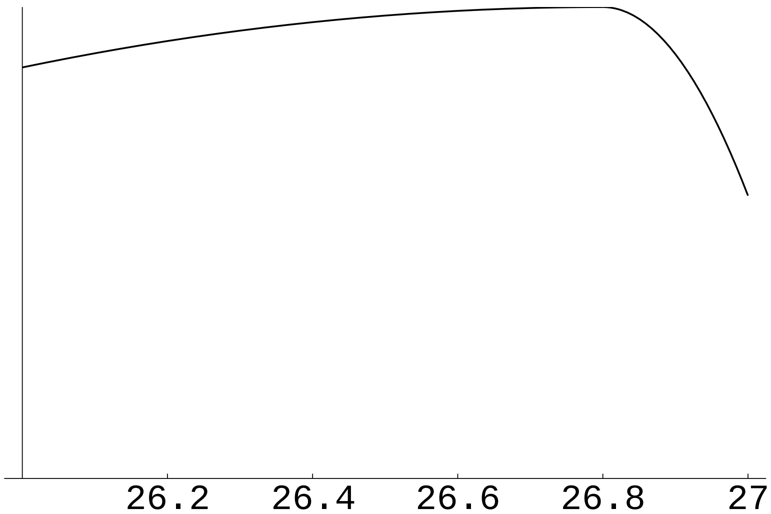
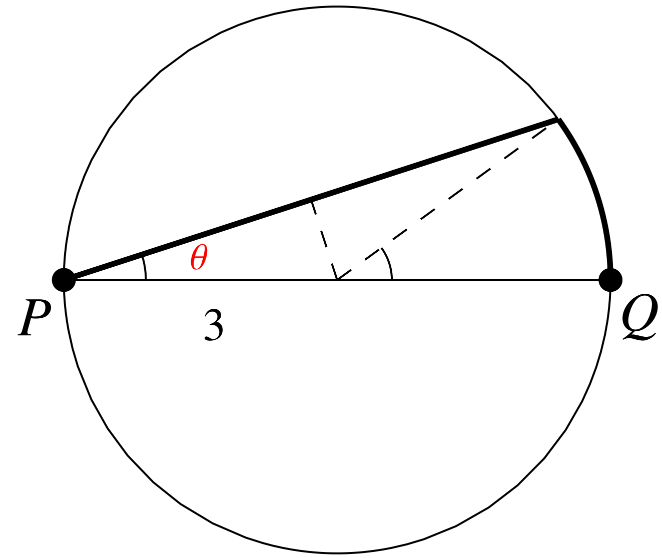




nearer may not give larger value

Example A man at a point P on the shore of a circular lake of radius 3 kilometers wants to reach the point Q on the shore diametrically opposite P (see figure). He can row 1 kilometer per hour and walk 3 kilometers per hour.



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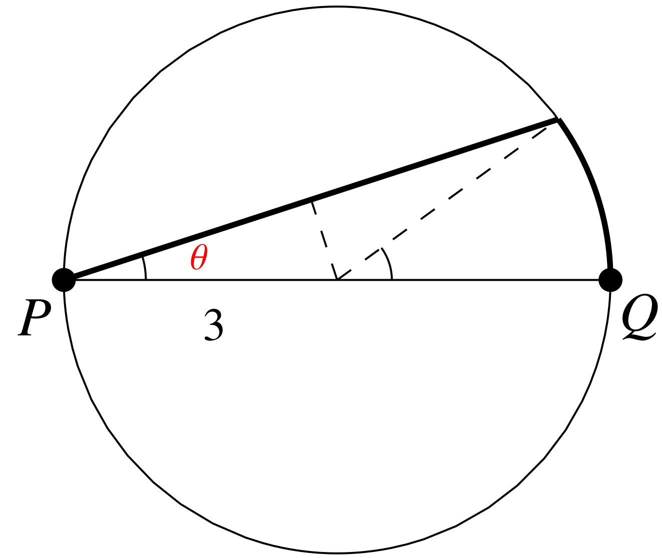

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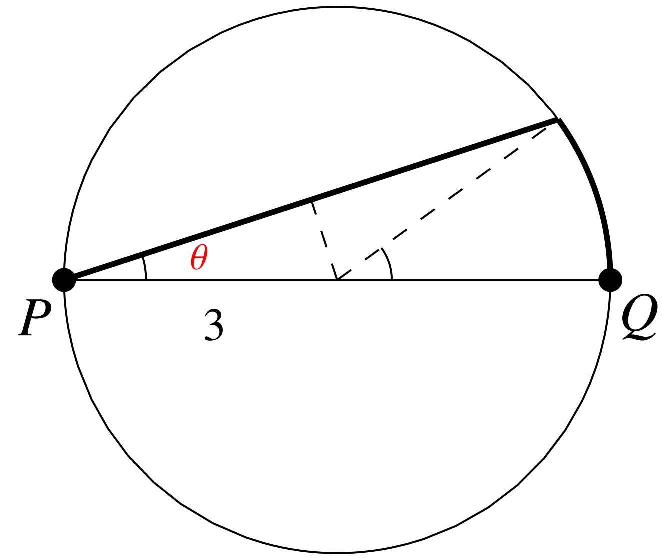

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(a) Show that the time T (in hours) required to reach Q is $T(\theta) = 6\cos\theta + 2\theta$.

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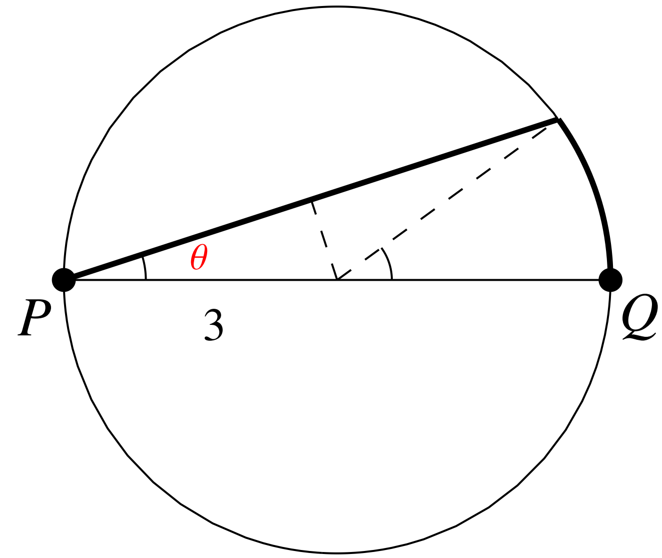

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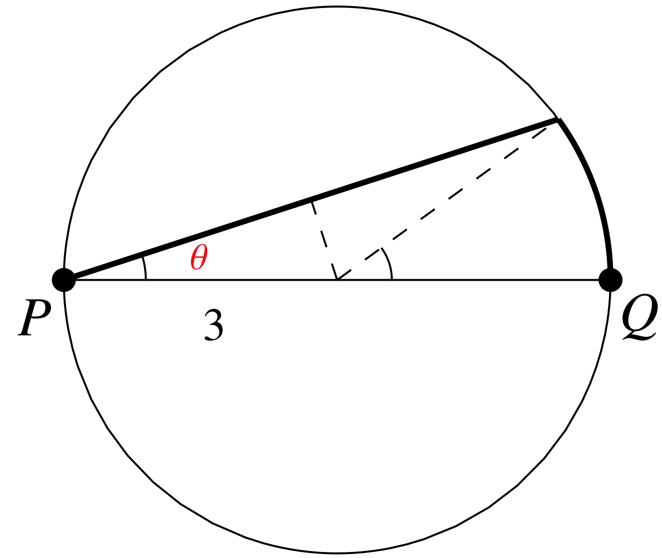

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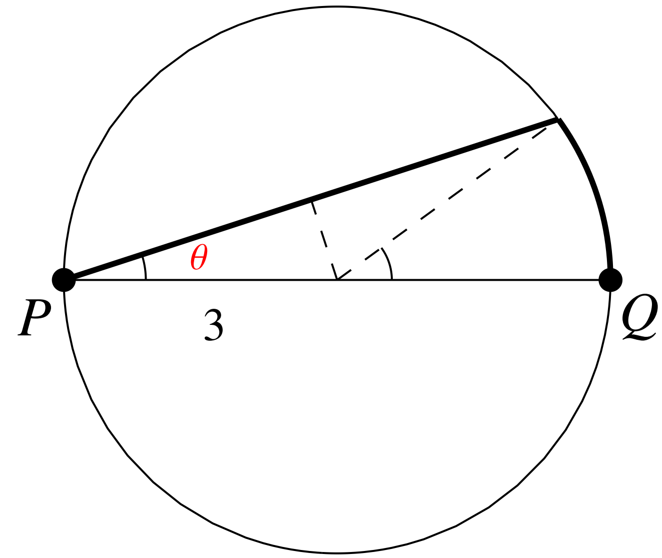

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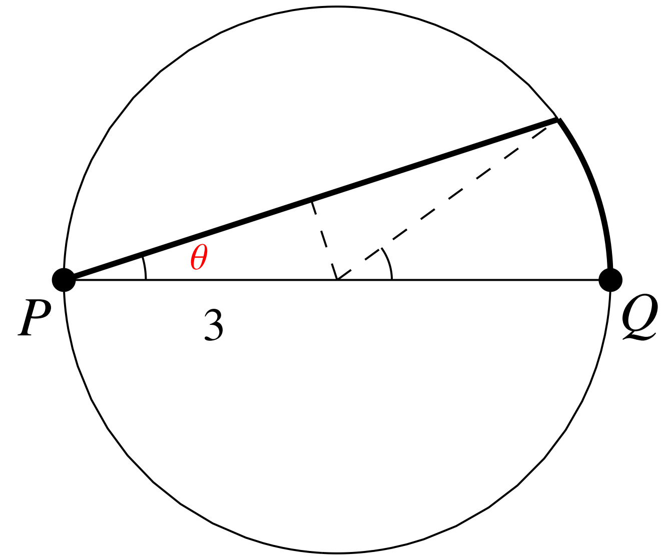

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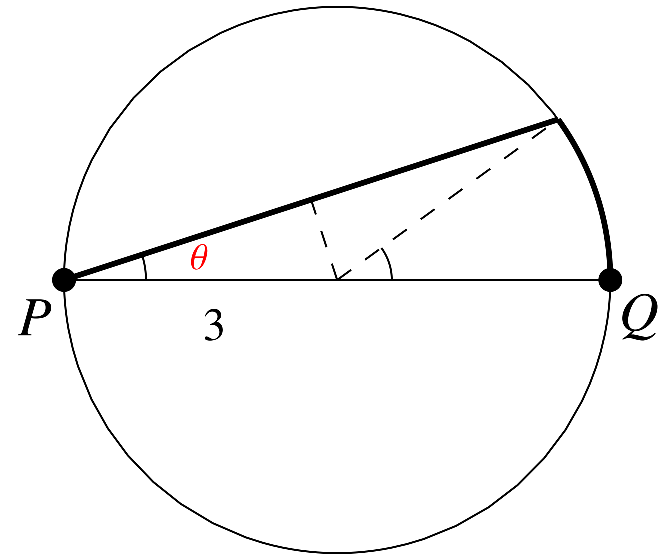

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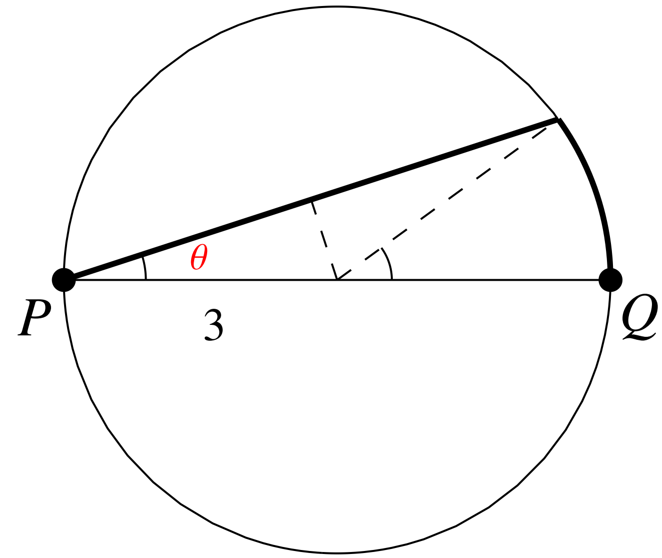

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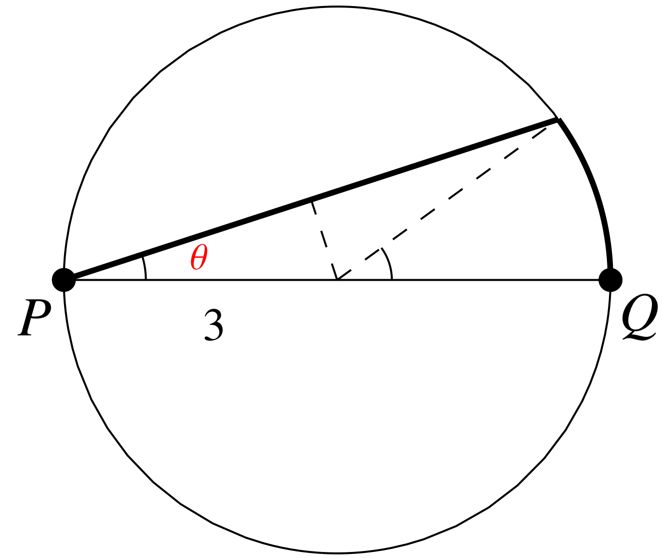

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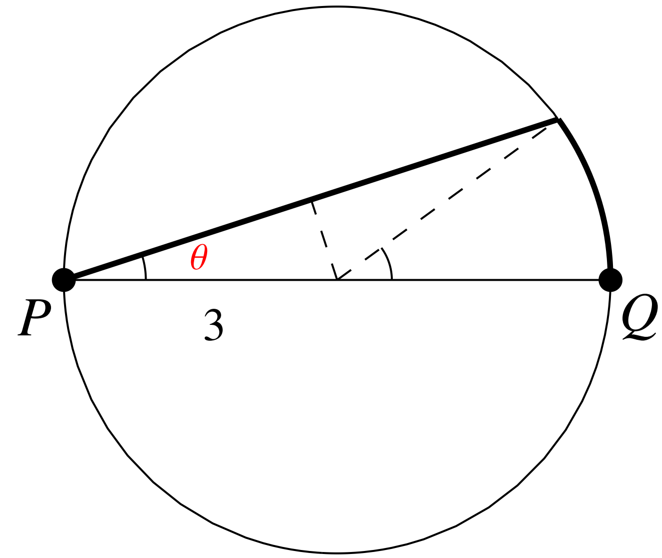

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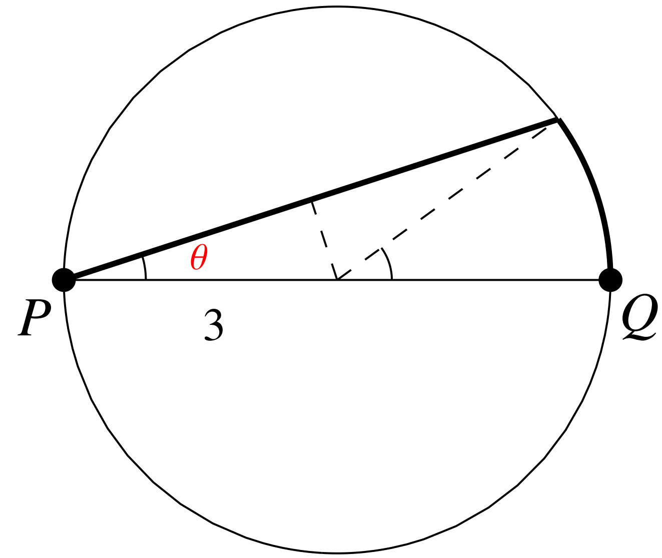

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T			

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$T(\theta)$		

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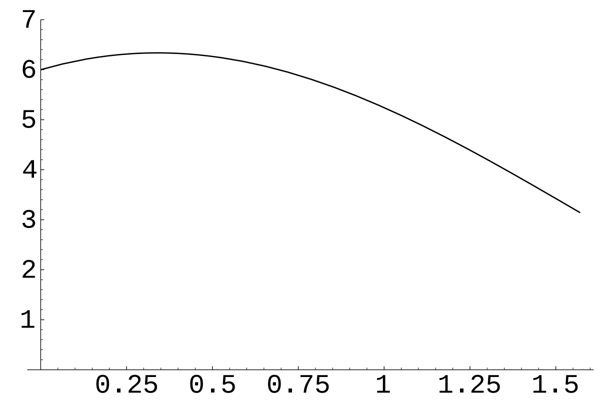

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	$\theta = 0$	$\theta = \frac{\pi}{2}$
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More Integration

- More Formulas
- Substitution Method
- More Applications of Definite Integrals

Objectives

- To use **formulas** and the **substitution method** to do integration.

More Formulas

(0) $\int x^r \, \mathrm{d}x = \frac{1}{r+1} x^{r+1} + C$ where $r \neq -1$

(1) $\int \sin x \, \mathrm{d}x = -\cos x + C$

(2) $\int \cos x \, \mathrm{d}x = \sin x + C$

(3) $\int \sec^2 x \, \mathrm{d}x = \tan x + C$

(4) $\int \mathrm{e}^x \, \mathrm{d}x = \mathrm{e}^x + C$

(5) $\int \frac{1}{x} \, \mathrm{d}x = \ln |x| + C$

Proof for (1)

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$$\frac{\mathrm{d}}{\mathrm{d}x}(-\cos x) \quad = \quad -\frac{\mathrm{d}}{\mathrm{d}x}\cos x$$

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\frac{\mathrm{d}}{\mathrm{d}x}(-\cos x) &= -\frac{\mathrm{d}}{\mathrm{d}x}\cos x \\
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&= \sin x
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$$\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}$$

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Note Domain of $\ln x$ is $(0, \infty)$

Domain of $\dfrac{1}{x}$ is $\mathbb{R} - \{0\} = (-\infty, 0) \cup (0, \infty)$

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Proof for (5)

$$\boxed{\int \frac{1}{x}\,\mathrm{d}x = \ln|x| + C}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \ln x = \frac{1}{x}$$

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$$\boxed{\int \frac{1}{x}\,\mathrm{d}x = \ln|x| + C}$$

Case 1 For $x > 0$, we have $|x| = x$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}$$

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Example *(Indefinite Integral)*

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(1) $$\int (x^2 + \sin x)\,\mathrm{d}x = \int x^2\,\mathrm{d}x + \int \sin x\,\mathrm{d}x$$

Example *(Indefinite Integral)*

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$$\int (x^2 + \sin x)\,\mathrm{d}x = \int x^2\,\mathrm{d}x + \int \sin x\,\mathrm{d}x$$
$$= \frac{x^3}{3} - \cos x + C$$

Example *(Indefinite Integral)*

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Example *(Definite Integral)*

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