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$$x \sin y + \cos y^2 = 1$$

at the point $(1, 0)$.

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$$1\cdot\cos 0\cdot\left.\frac{dy}{dx}\right|_{(1,0)} + \sin 0 - 2\cdot 0\cdot\sin 0^2\cdot\left.\frac{dy}{dx}\right|_{(1,0)} = 0$$

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$$\begin{aligned}\sin y &= (2y\sin y^2 - x\cos y)\frac{dy}{dx}\\ \frac{dy}{dx} &= \frac{\sin y}{2y\sin y^2 - x\cos y}\end{aligned}$$

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$$\begin{aligned}1\cdot\cos 0\cdot\left.\frac{dy}{dx}\right|_{(1,0)} + \sin 0 - 2\cdot 0\cdot\sin 0^2\cdot\left.\frac{dy}{dx}\right|_{(1,0)} &= 0\\ \left.\frac{dy}{dx}\right|_{(1,0)} + 0 - 0 &= 0\end{aligned}$$

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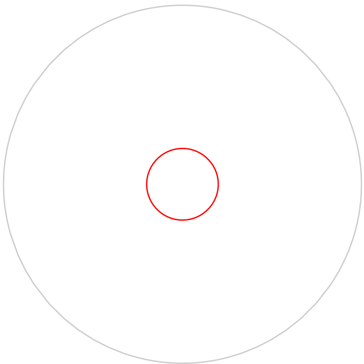

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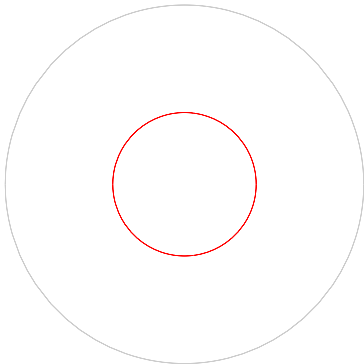

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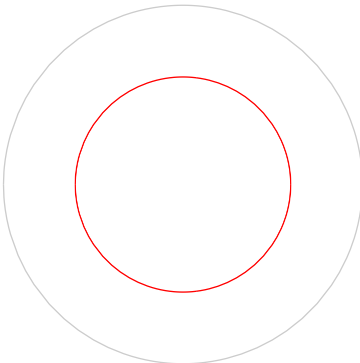

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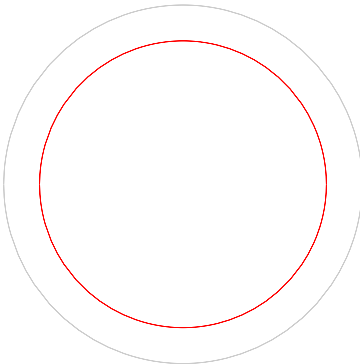

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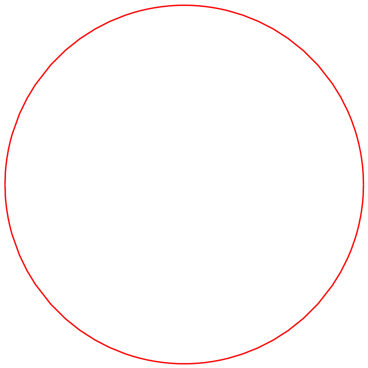

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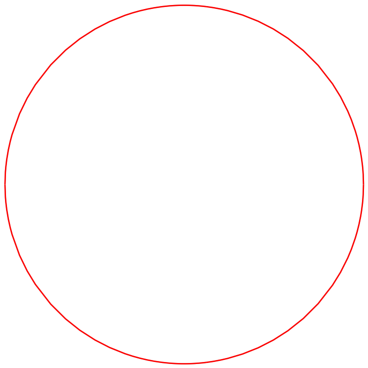

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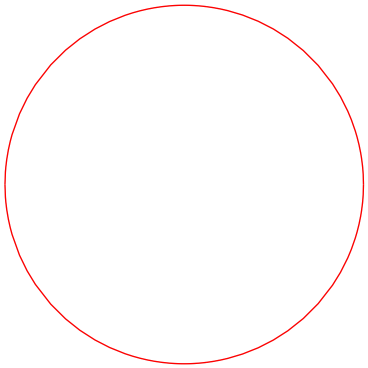

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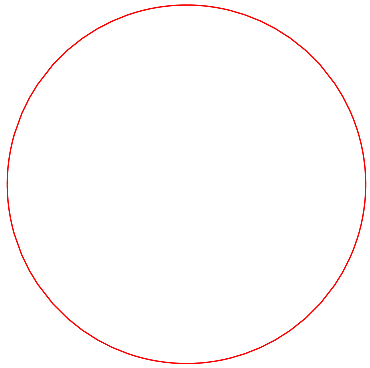

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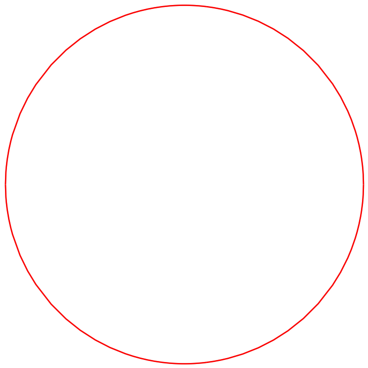

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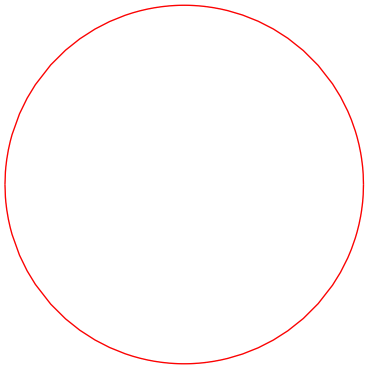

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Solution (cont.)

Differentiate (1) *with respect to time* t:

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The area is increasing at the rate of 30π cm^2 per second.

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- Plugin given data.

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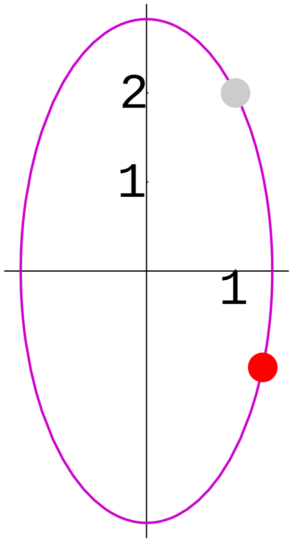

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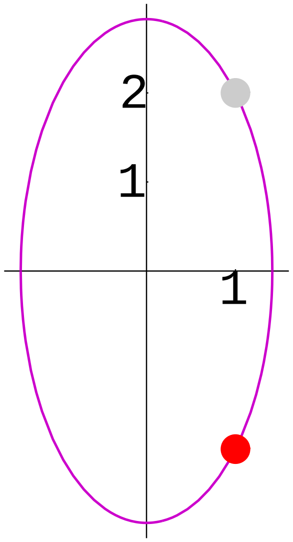

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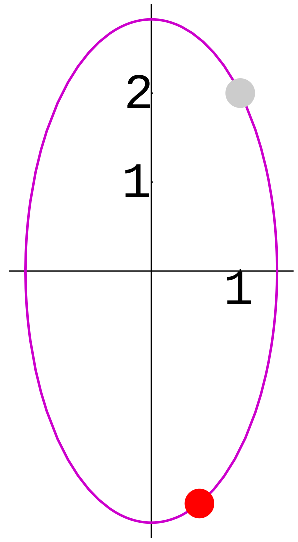

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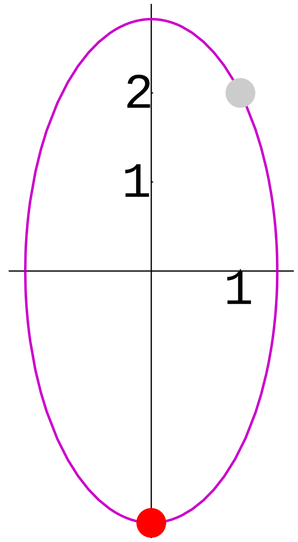

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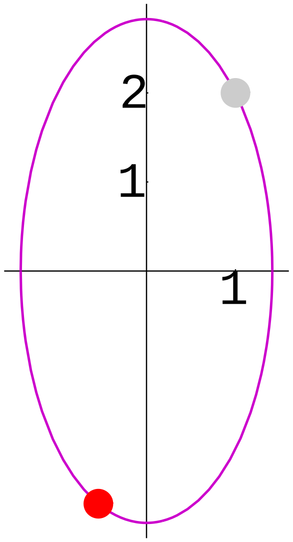

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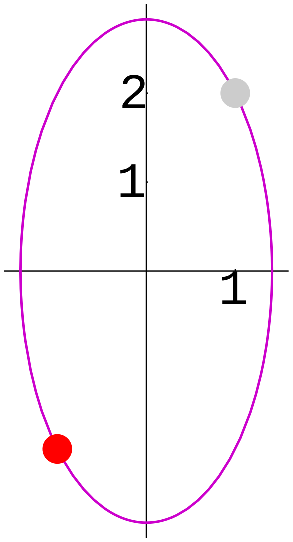

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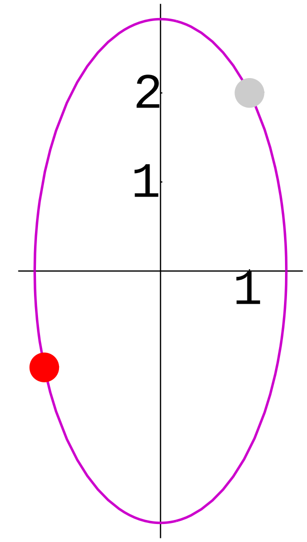

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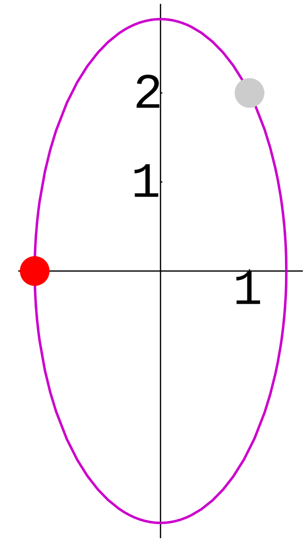

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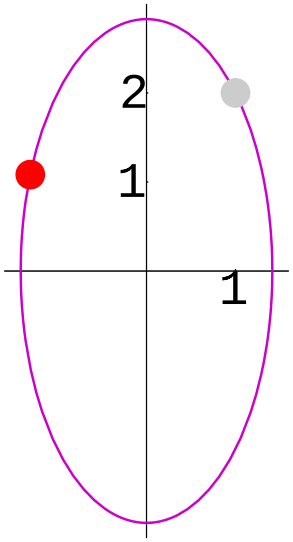

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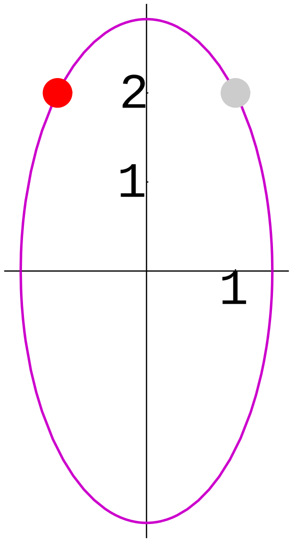

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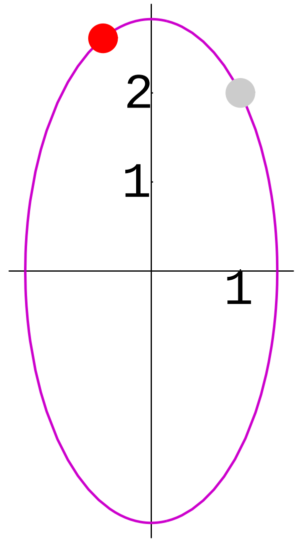

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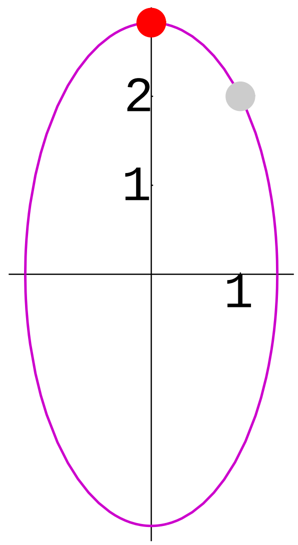

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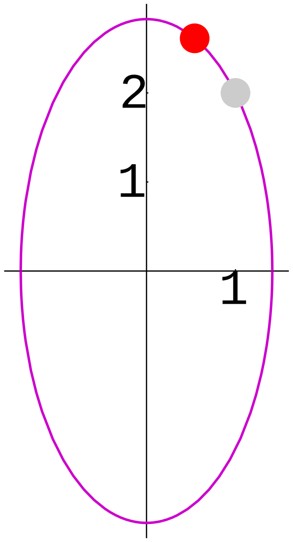

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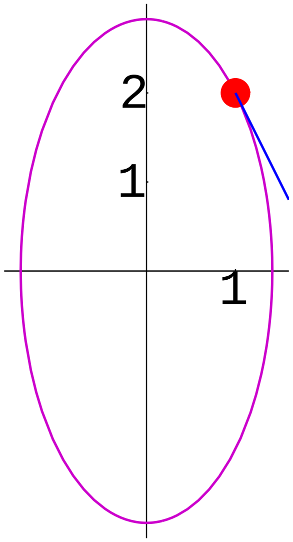

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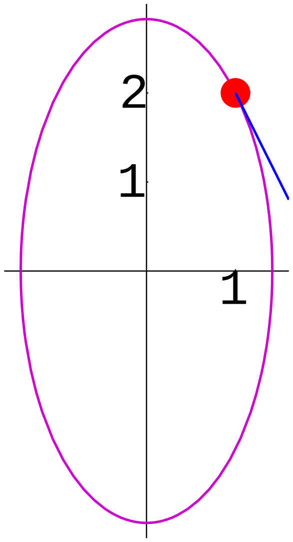

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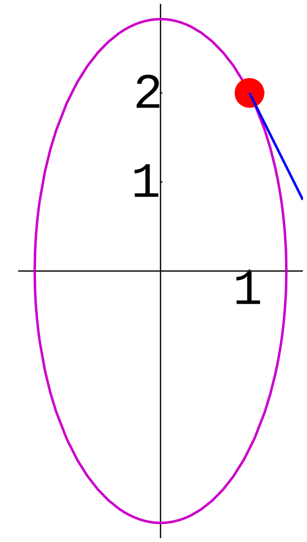

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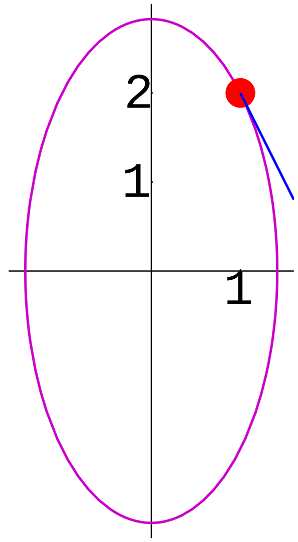

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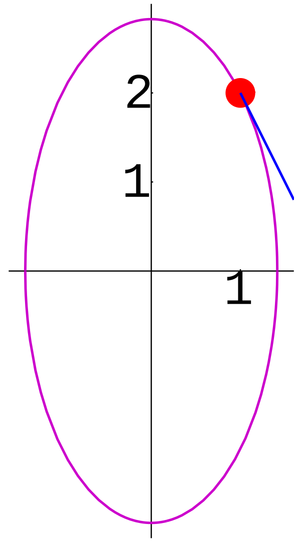

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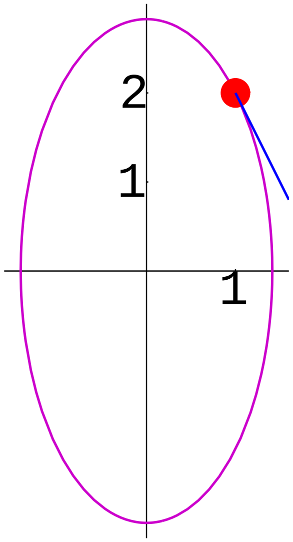

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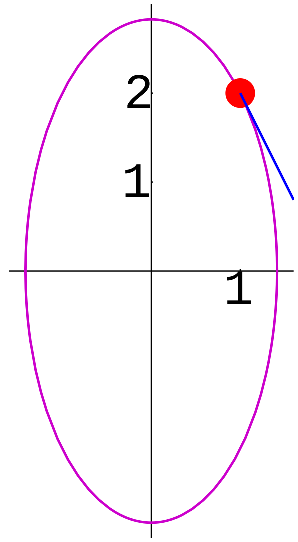

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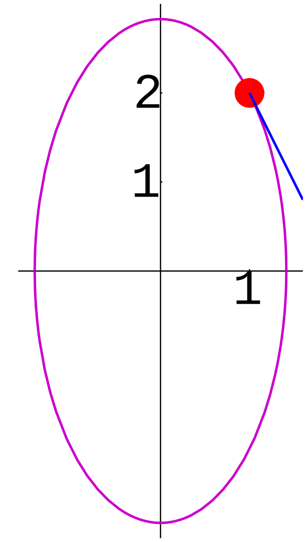

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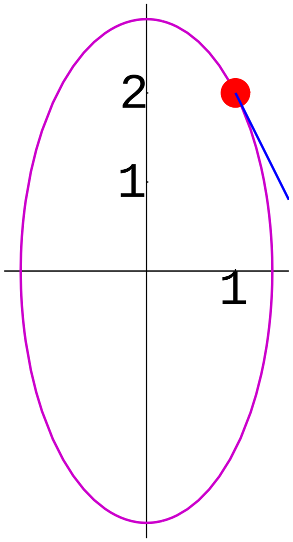
$$4 \cdot 1 \cdot 3 +$$



Example A point is moving along the graph of $4x^2 + y^2 = 8$. When the point is at $(1, 2)$, its x-coordinate is increasing at the rate of 3 units per second. How fast is the y-coordinate changing at that moment?

Solution Differentiate equation of ellipse w.r.t. time t

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}(4x^2 + y^2) &= \frac{\mathrm{d}}{\mathrm{d}t} 8 \\ 4 \cdot 2x \cdot \frac{\mathrm{d}x}{\mathrm{d}t} + 2y \cdot \frac{\mathrm{d}y}{\mathrm{d}t} &= 0 \\ 4x \frac{\mathrm{d}x}{\mathrm{d}t} + y \frac{\mathrm{d}y}{\mathrm{d}t} &= 0 \end{aligned}$$

At that moment $x = 1$, $y = 2$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 3$.

$$4 \cdot 1 \cdot 3 + 2 \frac{\mathrm{d}y}{\mathrm{d}t} = 0$$

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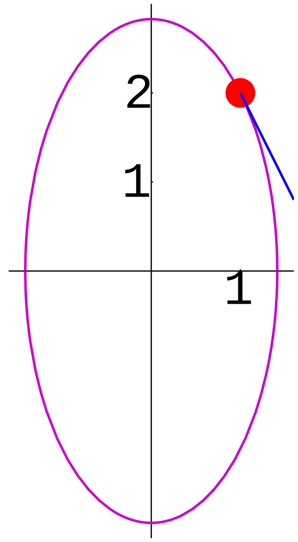
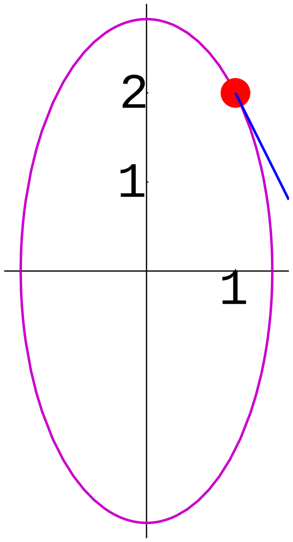


Solution Differentiate equation of ellipse w.r.t. time t

$$\begin{aligned}
\frac{d}{dt}(4x^2 + y^2) &= \frac{d}{dt}8 \\
4 \cdot 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} &= 0 \\
4x\frac{dx}{dt} + y\frac{dy}{dt} &= 0
\end{aligned}$$

At that moment $x = 1$, $y = 2$ and $\frac{dx}{dt} = 3$.

$$\begin{aligned}
4 \cdot 1 \cdot 3 + 2\frac{dy}{dt} &= 0 \\
\frac{dy}{dt} &= -6
\end{aligned}$$



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$$4 \cdot 1 \cdot 3 + 2\frac{dy}{dt} = 0$$

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The y-coordinate is decreasing at the rate of 6 units per second.

More graph sketching

Example Let $f(x) = x \ln x$.

(1) Find and classify the critical point(s) of f.

(2) Find the interval(s) on which f is increasing/decreasing, convex/concave.

(3) Sketch the graph of f.

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- *Solve* $f'(x) = 0$ $$\begin{aligned} \ln x &= -1 \\ x &= \mathrm{e}^{-1} \end{aligned}$$ critical point of f

	$0 < x < e^{-1}$	$x = e^{-1}$	$x > e^{-1}$
$f'(x) = 1 + \ln x$		0	
f			

	$0 < x < e^{-1}$	$x = e^{-1}$	$x > e^{-1}$
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f	$\searrow$		$\nearrow$

- f is decreasing on $(0, \mathrm{e}^{-1})$
  increasing on $(\mathrm{e}^{-1}, \infty)$

	$0 < x < \mathrm{e}^{-1}$	$x = \mathrm{e}^{-1}$	$x > \mathrm{e}^{-1}$
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f	↘		↗

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	$0 < x < 1$	$x = 1$	$x > 1$
$f(x)$	$-$	0	$+$

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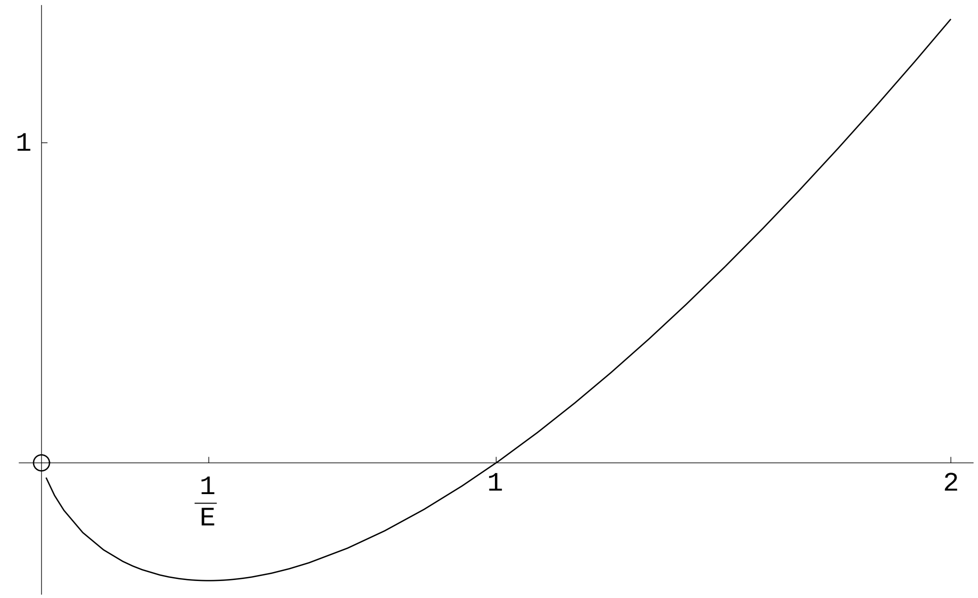

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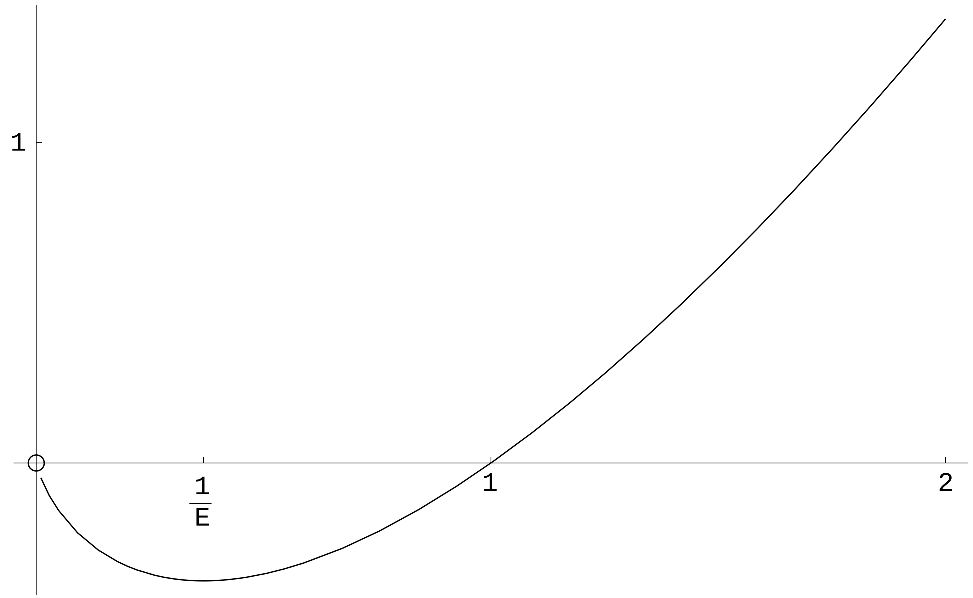

Remark $\lim_{x \to 0+} x \ln x = 0$