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$$= \frac{1}{\sin^2(2x+3)} \cdot \frac{d}{dx}\left[\sin(2x+3)\right]^2$$ *(do in your head)*

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$$= \frac{4\cos(2x+3)}{\sin(2x+3)}$$

Remark

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&= 2\ln\Big[\sin(2x+3)\Big]
\end{aligned}
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$$\begin{aligned} \ln\left[\sin^2(2x+3)\right] &= \ln\left[\sin(2x+3)\right]^2 \\ &= 2\ln\left[\sin(2x+3)\right] \end{aligned}$$

Apply chain rule twice.

Example Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the following:

(6) $y = \frac{1}{(x^2 + 3)^{40}} - 3\mathrm{e}^{4x+1}$

Solution

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Solution

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{(x^2+3)^{40}} - \frac{\mathrm{d}}{\mathrm{d}x}3\mathrm{e}^{4x+1}$$

Example Find $\frac{dy}{dx}$ for the following:

(6) $y = \frac{1}{(x^2+3)^{40}} - 3e^{4x+1}$

Solution

$$\frac{dy}{dx} = \frac{d}{dx}\frac{1}{(x^2+3)^{40}} - \frac{d}{dx}3e^{4x+1}$$

$$= \frac{d}{dx}(x^2+3)^{-40} -$$

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$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{(x^2+3)^{40}} - \frac{\mathrm{d}}{\mathrm{d}x}3e^{4x+1}$$

$$= \frac{\mathrm{d}}{\mathrm{d}x}(x^2+3)^{-40} - 3\frac{\mathrm{d}}{\mathrm{d}x}e^{4x+1}$$

$$= (-40)(x^2+3)^{-41} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(x^2+3) -$$

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&= \frac{\mathrm{d}}{\mathrm{d}x}(x^2+3)^{-40} - 3\frac{\mathrm{d}}{\mathrm{d}x}e^{4x+1} \\
&= (-40)(x^2+3)^{-41}\cdot\frac{\mathrm{d}}{\mathrm{d}x}(x^2+3) - 3e^{4x+1}\cdot\frac{\mathrm{d}}{\mathrm{d}x}(4x+1) \\
&= \cdots
\end{aligned}$$

Remark $$\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{(x^2+3)^{40}} = \frac{(x^2+3)^{40}\cdot\frac{\mathrm{d}}{\mathrm{d}x}1 - 1\cdot\frac{\mathrm{d}}{\mathrm{d}x}(x^2+3)^{40}}{\left[(x^2+3)^{40}\right]^2}$$ Quotient Rule

Example Find $\dfrac{dy}{dx}$ for the following:

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Solution

$$\frac{dy}{dx} = \frac{d}{dx}\frac{1}{(x^2+3)^{40}} - \frac{d}{dx}3e^{4x+1}$$

$$= \frac{d}{dx}(x^2+3)^{-40} - 3\frac{d}{dx}e^{4x+1}$$

$$= (-40)(x^2+3)^{-41}\cdot\frac{d}{dx}(x^2+3) - 3e^{4x+1}\cdot\frac{d}{dx}(4x+1)$$

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Remark

$$\frac{d}{dx}\frac{1}{(x^2+3)^{40}} = \frac{(x^2+3)^{40}\cdot\frac{d}{dx}1 - 1\cdot\frac{d}{dx}(x^2+3)^{40}}{\left[(x^2+3)^{40}\right]^2}$$

Quotient Rule

$$= \frac{0 - \text{need chain rule}}{(x^2+3)^{80}}$$

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(7) $y = \mathrm{e}^{x+1} \ln(x^2 + 1)$

Solution

Example Find $\frac{dy}{dx}$ for the following:

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$$= \mathrm{e}^{x+1} \cdot \frac{1}{x^2 + 1} \cdot \frac{\mathrm{d}}{\mathrm{d}x}(x^2 + 1) +$$

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$$= \mathrm{e}^{x+1}\cdot\frac{1}{x^2+1}\cdot\frac{\mathrm{d}}{\mathrm{d}x}(x^2+1) + \ln(x^2+1)\cdot\mathrm{e}^{x+1}\cdot\frac{\mathrm{d}}{\mathrm{d}x}(x+1)$$

$$= \cdots$$

Last Operation Rule (last come, first serve)

In differentiation, to determine which rule to apply,

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- if the last operation is $+$, apply $\frac{\mathrm{d}}{\mathrm{d}x}\big(f(x)+g(x)\big)=\frac{\mathrm{d}}{\mathrm{d}x}f(x)+\frac{\mathrm{d}}{\mathrm{d}x}g(x)$

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- if the last operation is $\div$, apply $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{f(x)}{g(x)}=\dfrac{g(x)\cdot\frac{\mathrm{d}}{\mathrm{d}x}f(x)-f(x)\cdot\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{g(x)^2}$

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- if the last operation is $\mathrm{e}^{\text{something}}$, apply

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- if the last operation is $\div$, apply $\frac{\mathrm{d}}{\mathrm{d}x}\,\frac{f(x)}{g(x)} = \frac{g(x)\cdot\frac{\mathrm{d}}{\mathrm{d}x}f(x) - f(x)\cdot\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{g(x)^2}$

- if the last operation is $\mathrm{e}^{\text{something}}$, apply $\frac{\mathrm{d}}{\mathrm{d}x}\,\mathrm{e}^{f(x)} = \mathrm{e}^{f(x)}\cdot\frac{\mathrm{d}}{\mathrm{d}x}f(x)$

- if the last operation is $\sin$ (something), apply

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In differentiation, to determine which rule to apply,

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- if the last operation is $\div$, apply $$\frac{\mathrm{d}}{\mathrm{d}x}\frac{f(x)}{g(x)}=\frac{g(x)\cdot\frac{\mathrm{d}}{\mathrm{d}x}f(x)-f(x)\cdot\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{g(x)^2}$$
- if the last operation is $\mathrm{e}^{\text{something}}$, apply $$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{f(x)}=\mathrm{e}^{f(x)}\cdot\frac{\mathrm{d}}{\mathrm{d}x}f(x)$$
- if the last operation is $\sin(\text{something})$, apply $$\frac{\mathrm{d}}{\mathrm{d}x}\sin f(x)=\cos f(x)\cdot\frac{\mathrm{d}}{\mathrm{d}x}f(x)$$

Derivative of exponential functions *with base different from* e

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Method 1 express in base e

Method 2 logarithmic differentiation

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Example Find $\dfrac{\mathrm{d}}{\mathrm{d}x} 5^{x^2+\cos x}$.

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Derivative of exponential functions *with base different from* e

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Example Find $\frac{\mathrm{d}}{\mathrm{d}x} 5^{x^2+\cos x}$.

Method 1 $5^{x^2+\cos x} = \mathrm{e}^{\ln 5^{x^2+\cos x}}$ $\qquad b^{\log_b n} = n$

Derivative of exponential functions *with base different from* e

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Example Find $\dfrac{\mathrm{d}}{\mathrm{d}x} 5^{x^2+\cos x}$.

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$$5^{x^2+\cos x} = \mathrm{e}^{\ln 5^{x^2+\cos x}} \qquad b^{\log_b n} = n$$

$$= \mathrm{e}^{(x^2+\cos x)\ln 5}$$

Derivative of exponential functions *with base different from* e

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$$= \mathrm{e}^{(x^2+\cos x)\ln 5} \cdot (\ln 5) \cdot (2x - \sin x)$$

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$$= \mathrm{e}^{(x^2+\cos x)\ln 5} \cdot \frac{\mathrm{d}}{\mathrm{d}x}[(x^2 + \cos x)\ln 5]$$

$$= \mathrm{e}^{(x^2+\cos x)\ln 5} \cdot (\ln 5) \cdot (2x - \sin x) = (\ln 5)(2x - \sin x)5^{x^2+\cos x}$$

Method 2 Put $y = 5^{x^2+\cos x}$

- *Take natural logarithm* on both sides

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$$\begin{aligned} \ln y &= \ln 5^{x^2+\cos x} \\ \ln y &= (x^2 + \cos x)\ln 5 \end{aligned}$$

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$$\frac{\mathrm{d}}{\mathrm{d}x} \ln y = \frac{\mathrm{d}}{\mathrm{d}x}(x^2 + \cos x) \ln 5$$

$$\frac{1}{y} \cdot \frac{\mathrm{d}y}{\mathrm{d}x}$$

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Method 2 Put $y = 5^{x^2+\cos x}$

- *Take natural logarithm* on both sides

$$\begin{aligned} \ln y &= \ln 5^{x^2+\cos x} \\ \ln y &= (x^2+\cos x)\ln 5 \end{aligned}$$

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Technique for differentiating functions that are *not* given *in the usual form* $y = f(x)$.

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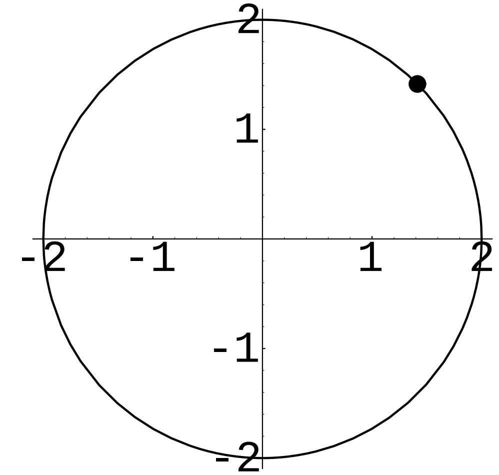

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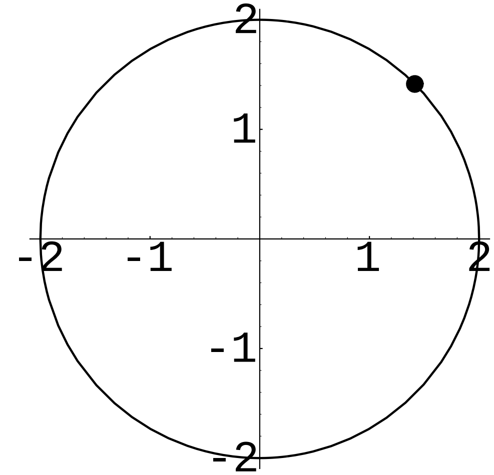

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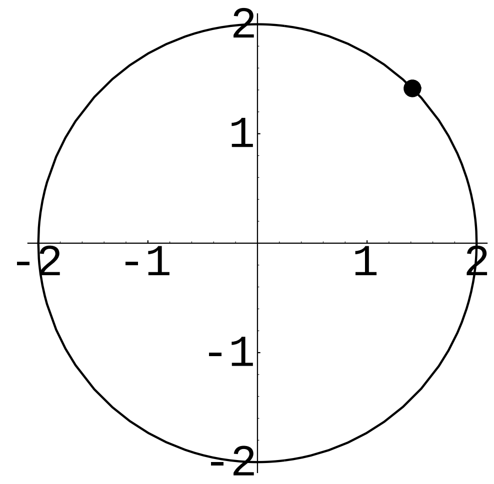

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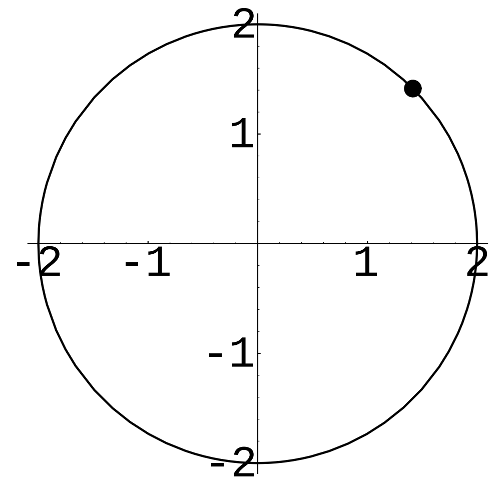

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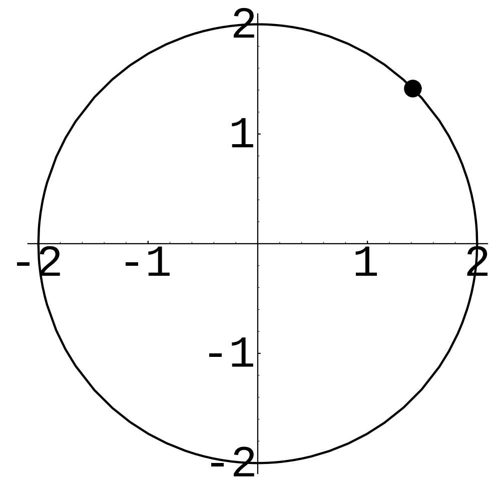

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- $(\sqrt{2}, \sqrt{2})$ lies on the upper semi-circle.

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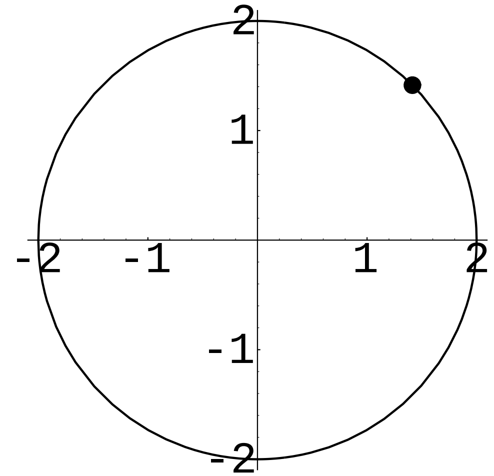
- Differentiating $$\frac{dy}{dx} = \frac{d}{dx}(4 - x^2)^{\frac{1}{2}}$$



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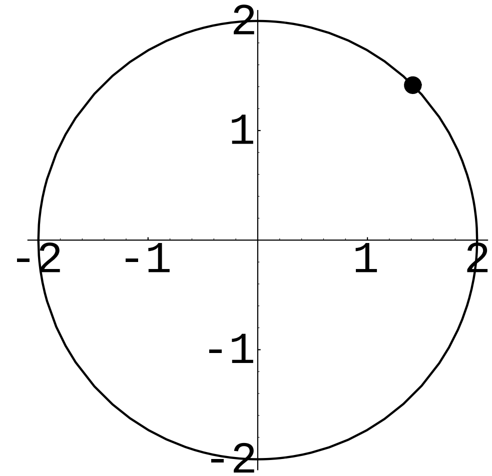

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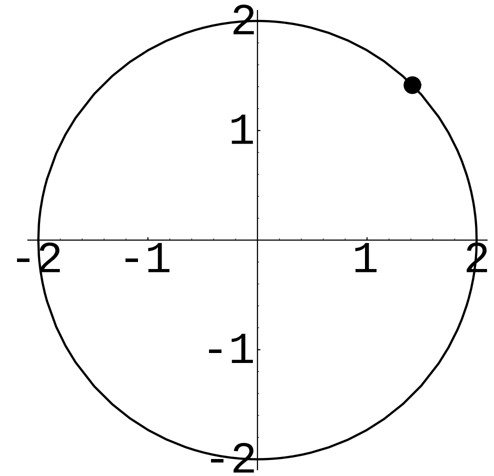

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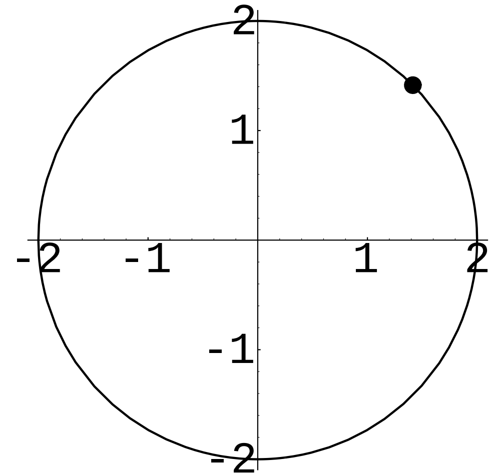

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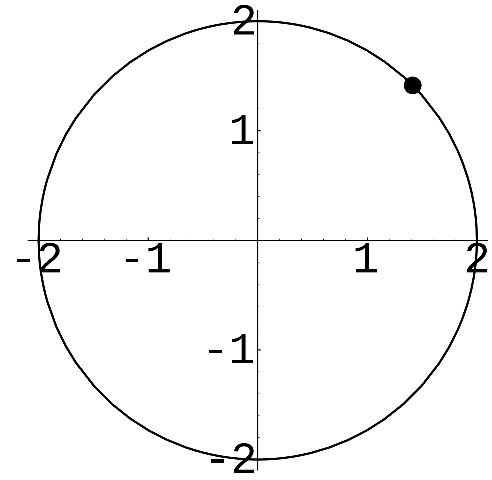

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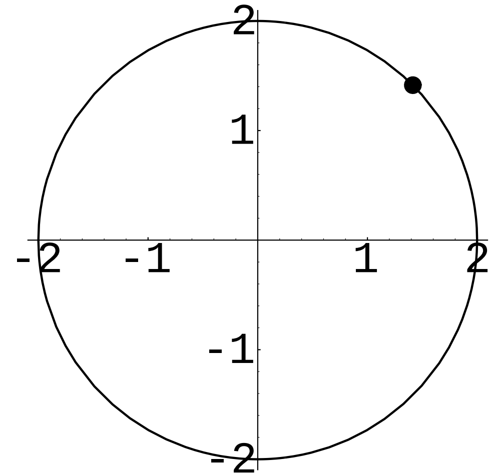

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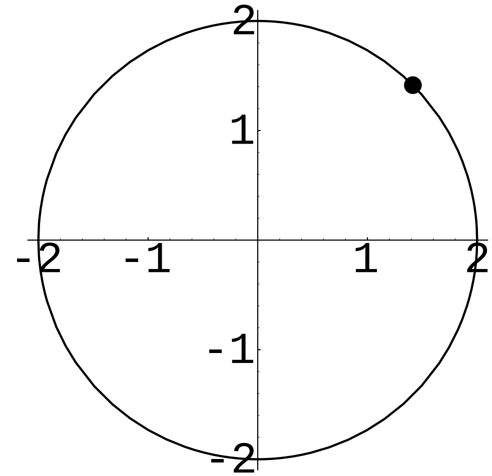

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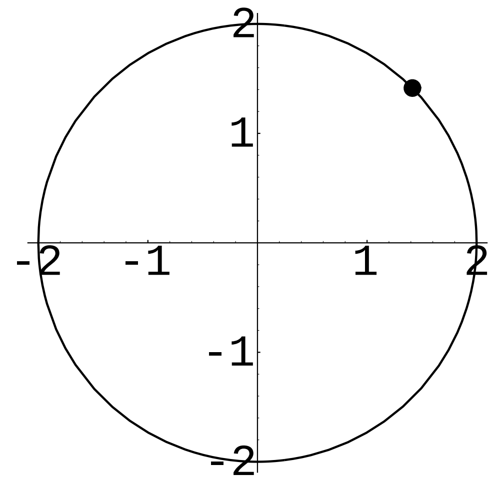

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This method is called *implicit differentiation*.

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$r = \frac{1}{2}$ $\quad \frac{\mathrm{d}}{\mathrm{d}x} f(x)^{\frac{1}{2}} = \frac{1}{2} f(x)^{-\frac{1}{2}} \frac{\mathrm{d}}{\mathrm{d}x} f(x)$ $\quad \frac{\mathrm{d}}{\mathrm{d}x} y^{\frac{1}{2}} = \frac{1}{2} y^{-\frac{1}{2}} \frac{\mathrm{d}y}{\mathrm{d}x}$

Power Rule + Chain Rule $\frac{\mathrm{d}}{\mathrm{d}x}f(x)^r = r f(x)^{r-1}\frac{\mathrm{d}}{\mathrm{d}x}f(x)$

$r = 2$ $\quad \frac{\mathrm{d}}{\mathrm{d}x}f(x)^2 = 2f(x)\,\frac{\mathrm{d}}{\mathrm{d}x}f(x)$ $\quad$ put $y = f(x)$ $\quad \frac{\mathrm{d}}{\mathrm{d}x}y^2 = 2y\,\frac{\mathrm{d}y}{\mathrm{d}x}$

$r = 3$ $\quad \frac{\mathrm{d}}{\mathrm{d}x}f(x)^3 = 3f(x)^2\,\frac{\mathrm{d}}{\mathrm{d}x}f(x)$ $\quad \frac{\mathrm{d}}{\mathrm{d}x}y^3 = 3y^2\,\frac{\mathrm{d}y}{\mathrm{d}x}$

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In $\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{f(x)} = \mathrm{e}^{f(x)}\frac{\mathrm{d}}{\mathrm{d}x}f(x)$, put $y = f(x)$:

Power Rule + Chain Rule $\frac{\mathrm{d}}{\mathrm{d}x} f(x)^r = r f(x)^{r-1} \frac{\mathrm{d}}{\mathrm{d}x} f(x)$

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$r = \frac{1}{2}$ $\frac{\mathrm{d}}{\mathrm{d}x} f(x)^{\frac{1}{2}} = \frac{1}{2} f(x)^{-\frac{1}{2}} \frac{\mathrm{d}}{\mathrm{d}x} f(x)$ $\frac{\mathrm{d}}{\mathrm{d}x} y^{\frac{1}{2}} = \frac{1}{2} y^{-\frac{1}{2}} \frac{\mathrm{d}y}{\mathrm{d}x}$

In $\frac{\mathrm{d}}{\mathrm{d}x} \mathrm{e}^{f(x)} = \mathrm{e}^{f(x)} \frac{\mathrm{d}}{\mathrm{d}x} f(x)$, put $y = f(x)$: $\frac{\mathrm{d}}{\mathrm{d}x} \mathrm{e}^y = \mathrm{e}^y \frac{\mathrm{d}y}{\mathrm{d}x}$

General Form	Alternative Form
$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)]^r = r[f(x)]^{r-1} \cdot \frac{\mathrm{d}}{\mathrm{d}x}f(x)$	$\frac{\mathrm{d}}{\mathrm{d}x}u^r = ru^{r-1}\frac{\mathrm{d}u}{\mathrm{d}x}$

General Form	Alternative Form
$\frac{d}{dx}[f(x)]^r = r[f(x)]^{r-1} \cdot \frac{d}{dx}f(x)$	$\frac{d}{dx}u^r = ru^{r-1}\frac{du}{dx}$
$\frac{d}{dx}\sin[f(x)] = \cos[f(x)] \cdot \frac{d}{dx}f(x)$	$\frac{d}{dx}\sin u = \cos u\,\frac{du}{dx}$

General Form	Alternative Form
$\frac{d}{dx}[f(x)]^r = r[f(x)]^{r-1} \cdot \frac{d}{dx}f(x)$	$\frac{d}{dx}u^r = ru^{r-1}\frac{du}{dx}$
$\frac{d}{dx}\sin[f(x)] = \cos[f(x)] \cdot \frac{d}{dx}f(x)$	$\frac{d}{dx}\sin u = \cos u \frac{du}{dx}$
$\frac{d}{dx}\cos[f(x)] = -\sin[f(x)] \cdot \frac{d}{dx}f(x)$	$\frac{d}{dx}\cos u = -\sin u \frac{du}{dx}$

General Form	Alternative Form
$\dfrac{\mathrm{d}}{\mathrm{d}x}[f(x)]^r = r[f(x)]^{r-1} \cdot \dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$	$\dfrac{\mathrm{d}}{\mathrm{d}x}u^r = ru^{r-1}\dfrac{\mathrm{d}u}{\mathrm{d}x}$
$\dfrac{\mathrm{d}}{\mathrm{d}x}\sin[f(x)] = \cos[f(x)] \cdot \dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$	$\dfrac{\mathrm{d}}{\mathrm{d}x}\sin u = \cos u\,\dfrac{\mathrm{d}u}{\mathrm{d}x}$
$\dfrac{\mathrm{d}}{\mathrm{d}x}\cos[f(x)] = -\sin[f(x)] \cdot \dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$	$\dfrac{\mathrm{d}}{\mathrm{d}x}\cos u = -\sin u\,\dfrac{\mathrm{d}u}{\mathrm{d}x}$
$\dfrac{\mathrm{d}}{\mathrm{d}x}\tan[f(x)] = \sec^2[f(x)] \cdot \dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$	$\dfrac{\mathrm{d}}{\mathrm{d}x}\tan u = \sec^2 u\,\dfrac{\mathrm{d}u}{\mathrm{d}x}$

General Form	Alternative Form
$\dfrac{d}{dx}[f(x)]^r = r[f(x)]^{r-1} \cdot \dfrac{d}{dx}f(x)$	$\dfrac{d}{dx}u^r = ru^{r-1}\,\dfrac{du}{dx}$
$\dfrac{d}{dx}\sin[f(x)] = \cos[f(x)] \cdot \dfrac{d}{dx}f(x)$	$\dfrac{d}{dx}\sin u = \cos u\,\dfrac{du}{dx}$
$\dfrac{d}{dx}\cos[f(x)] = -\sin[f(x)] \cdot \dfrac{d}{dx}f(x)$	$\dfrac{d}{dx}\cos u = -\sin u\,\dfrac{du}{dx}$
$\dfrac{d}{dx}\tan[f(x)] = \sec^2[f(x)] \cdot \dfrac{d}{dx}f(x)$	$\dfrac{d}{dx}\tan u = \sec^2 u\,\dfrac{du}{dx}$
$\dfrac{d}{dx}e^{f(x)} = e^{f(x)} \cdot \dfrac{d}{dx}f(x)$	$\dfrac{d}{dx}e^u = e^u\,\dfrac{du}{dx}$

General Form	Alternative Form
$\frac{d}{dx}[f(x)]^r = r[f(x)]^{r-1} \cdot \frac{d}{dx}f(x)$	$\frac{d}{dx}u^r = ru^{r-1}\,\frac{du}{dx}$
$\frac{d}{dx}\sin[f(x)] = \cos[f(x)] \cdot \frac{d}{dx}f(x)$	$\frac{d}{dx}\sin u = \cos u\,\frac{du}{dx}$
$\frac{d}{dx}\cos[f(x)] = -\sin[f(x)] \cdot \frac{d}{dx}f(x)$	$\frac{d}{dx}\cos u = -\sin u\,\frac{du}{dx}$
$\frac{d}{dx}\tan[f(x)] = \sec^2[f(x)] \cdot \frac{d}{dx}f(x)$	$\frac{d}{dx}\tan u = \sec^2 u\,\frac{du}{dx}$
$\frac{d}{dx}e^{f(x)} = e^{f(x)} \cdot \frac{d}{dx}f(x)$	$\frac{d}{dx}e^u = e^u\,\frac{du}{dx}$
$\frac{d}{dx}\ln[f(x)] = \frac{1}{f(x)} \cdot \frac{d}{dx}f(x)$	$\frac{d}{dx}\ln u = \frac{1}{u}\,\frac{du}{dx}$

Example For each of the following, express it using x, y and $\frac{\mathrm{d}y}{\mathrm{d}x}$

(1) $\frac{\mathrm{d}}{\mathrm{d}x} 5 \ln y \quad =$

(2) $\frac{\mathrm{d}}{\mathrm{d}x} (x^2 + \sin y) \quad =$

Example For each of the following, express it using x, y and $\frac{\mathrm{d}y}{\mathrm{d}x}$

(1) $$\frac{\mathrm{d}}{\mathrm{d}x}5\ln y = 5 \cdot \frac{\mathrm{d}}{\mathrm{d}x}\ln y$$

(2) $$\frac{\mathrm{d}}{\mathrm{d}x}(x^2 + \sin y) =$$

Example For each of the following, express it using x, y and $\frac{\mathrm{d}y}{\mathrm{d}x}$

(1)
$$\frac{\mathrm{d}}{\mathrm{d}x}5\ln y = 5\cdot\frac{\mathrm{d}}{\mathrm{d}x}\ln y$$
$$= 5\cdot\frac{1}{y}\,\frac{\mathrm{d}y}{\mathrm{d}x}$$

(2)
$$\frac{\mathrm{d}}{\mathrm{d}x}(x^2+\sin y) =$$

Example For each of the following, express it using x, y and $\frac{\mathrm{d}y}{\mathrm{d}x}$

(1) $$\frac{\mathrm{d}}{\mathrm{d}x} 5 \ln y = 5 \cdot \frac{\mathrm{d}}{\mathrm{d}x} \ln y$$
$$= 5 \cdot \frac{1}{y} \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5}{y} \frac{\mathrm{d}y}{\mathrm{d}x}$$

(2) $$\frac{\mathrm{d}}{\mathrm{d}x}(x^2 + \sin y) =$$

Example For each of the following, express it using x, y and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(1)
$$\frac{\mathrm{d}}{\mathrm{d}x}5\ln y = 5\cdot\frac{\mathrm{d}}{\mathrm{d}x}\ln y$$
$$= 5\cdot\frac{1}{y}\,\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5}{y}\,\frac{\mathrm{d}y}{\mathrm{d}x}$$

(2)
$$\frac{\mathrm{d}}{\mathrm{d}x}(x^2+\sin y) = \frac{\mathrm{d}}{\mathrm{d}x}x^2 + \frac{\mathrm{d}}{\mathrm{d}x}\sin y$$

Example For each of the following, express it using x, y and $\frac{\mathrm{d}y}{\mathrm{d}x}$

(1) $$\frac{\mathrm{d}}{\mathrm{d}x}5\ln y = 5\cdot\frac{\mathrm{d}}{\mathrm{d}x}\ln y$$

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(2) $$\frac{\mathrm{d}}{\mathrm{d}x}(x^2+\sin y) = \frac{\mathrm{d}}{\mathrm{d}x}x^2 + \frac{\mathrm{d}}{\mathrm{d}x}\sin y$$

$$= 2x +$$

Example For each of the following, express it using x, y and $\frac{\mathrm{d}y}{\mathrm{d}x}$

(1)
$$\frac{\mathrm{d}}{\mathrm{d}x}5\ln y = 5\cdot\frac{\mathrm{d}}{\mathrm{d}x}\ln y = 5\cdot\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5}{y}\frac{\mathrm{d}y}{\mathrm{d}x}$$

(2)
$$\frac{\mathrm{d}}{\mathrm{d}x}(x^2+\sin y) = \frac{\mathrm{d}}{\mathrm{d}x}x^2 + \frac{\mathrm{d}}{\mathrm{d}x}\sin y = 2x + \cos y\,\frac{\mathrm{d}y}{\mathrm{d}x}$$

Example For each of the following, express it using x, y and $\frac{\mathrm{d}y}{\mathrm{d}x}$

(3) $\frac{\mathrm{d}}{\mathrm{d}x}\left(y \ln x + x\,\mathrm{e}^{y}\right) \quad =$

Example For each of the following, express it using x, y and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(3) $$\frac{\mathrm{d}}{\mathrm{d}x}\left(y\ln x + x\,\mathrm{e}^{y}\right) \quad = \quad \left(y\cdot\frac{\mathrm{d}}{\mathrm{d}x}\ln x + \ln x\cdot\frac{\mathrm{d}}{\mathrm{d}x}y\right)$$

Example For each of the following, express it using x, y and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(3) $$\frac{\mathrm{d}}{\mathrm{d}x}\left(y \ln x + x\,\mathrm{e}^{y}\right) = \left(y \cdot \frac{\mathrm{d}}{\mathrm{d}x}\ln x + \ln x \cdot \frac{\mathrm{d}}{\mathrm{d}x}y\right) + \left(x \cdot \frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{y} + \mathrm{e}^{y} \cdot \frac{\mathrm{d}}{\mathrm{d}x}x\right)$$

Example For each of the following, express it using x, y and $\frac{dy}{dx}$

(3) $$\frac{d}{dx}\left(y \ln x + x e^y\right) = \left(y \cdot \frac{d}{dx}\ln x + \ln x \cdot \frac{d}{dx}y\right) + \left(x \cdot \frac{d}{dx}e^y + e^y \cdot \frac{d}{dx}x\right)$$

$$= \left(y \cdot \frac{1}{x} + \right.$$

Example For each of the following, express it using x, y and $\dfrac{dy}{dx}$

(3) $$\frac{d}{dx}\left(y\ln x + x e^y\right) = \left(y \cdot \frac{d}{dx}\ln x + \ln x \cdot \frac{d}{dx}y\right) + \left(x \cdot \frac{d}{dx}e^y + e^y \cdot \frac{d}{dx}x\right)$$

$$= \left(y \cdot \frac{1}{x} + \ln x \cdot \frac{dy}{dx}\right)$$

Example For each of the following, express it using x, y and $\frac{\mathrm{d}y}{\mathrm{d}x}$

(3) $$\frac{\mathrm{d}}{\mathrm{d}x}\left(y\ln x + x\,\mathrm{e}^{y}\right) = \left(y\cdot\frac{\mathrm{d}}{\mathrm{d}x}\ln x + \ln x\cdot\frac{\mathrm{d}}{\mathrm{d}x}y\right) + \left(x\cdot\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{y} + \mathrm{e}^{y}\cdot\frac{\mathrm{d}}{\mathrm{d}x}x\right)$$

$$= \left(y\cdot\frac{1}{x} + \ln x\cdot\frac{\mathrm{d}y}{\mathrm{d}x}\right) + \left(x\cdot\mathrm{e}^{y}\cdot\frac{\mathrm{d}}{\mathrm{d}x}y + \right.$$

Example For each of the following, express it using x, y and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(3)
$$\frac{\mathrm{d}}{\mathrm{d}x}\left(y\ln x + x\,\mathrm{e}^y\right) = \left(y\cdot\frac{\mathrm{d}}{\mathrm{d}x}\ln x + \ln x\cdot\frac{\mathrm{d}}{\mathrm{d}x}y\right) + \left(x\cdot\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^y + \mathrm{e}^y\cdot\frac{\mathrm{d}}{\mathrm{d}x}x\right)$$
$$= \left(y\cdot\frac{1}{x} + \ln x\cdot\frac{\mathrm{d}y}{\mathrm{d}x}\right) + \left(x\cdot\mathrm{e}^y\cdot\frac{\mathrm{d}}{\mathrm{d}x}y + \mathrm{e}^y\cdot 1\right)$$

Example For each of the following, express it using x, y and $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(3)

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(y\ln x + x\,\mathrm{e}^y\right) &= \left(y\cdot\frac{\mathrm{d}}{\mathrm{d}x}\ln x + \ln x\cdot\frac{\mathrm{d}}{\mathrm{d}x}y\right) + \left(x\cdot\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^y + \mathrm{e}^y\cdot\frac{\mathrm{d}}{\mathrm{d}x}x\right) \\
&= \left(y\cdot\frac{1}{x} + \ln x\cdot\frac{\mathrm{d}y}{\mathrm{d}x}\right) + \left(x\cdot\mathrm{e}^y\cdot\frac{\mathrm{d}}{\mathrm{d}x}y + \mathrm{e}^y\cdot 1\right) \\
&= \left(\ln x + x\,\mathrm{e}^y\right)\frac{\mathrm{d}y}{\mathrm{d}x} +
\end{aligned}$$

Example For each of the following, express it using x, y and $\dfrac{dy}{dx}$

(3)
$$\begin{aligned}\frac{d}{dx}\left(y\ln x + x\,e^y\right) &= \left(y\cdot\frac{d}{dx}\ln x + \ln x\cdot\frac{d}{dx}y\right) + \left(x\cdot\frac{d}{dx}e^y + e^y\cdot\frac{d}{dx}x\right)\\ &= \left(y\cdot\frac{1}{x} + \ln x\cdot\frac{dy}{dx}\right) + \left(x\cdot e^y\cdot\frac{d}{dx}y + e^y\cdot 1\right)\\ &= \left(\ln x + x\,e^y\right)\frac{dy}{dx} + \left(\frac{y}{x} + e^y\right)\end{aligned}$$