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Both methods need *chain rule*.

Chapter 9: More Differentiation

- Chain Rule
- Implicit Differentiation
- More Curve Sketching
- More Extremum Problems

Objectives

- To use Chain Rule to do differentiation.
- To use Implicit Differentiation to find $\frac{\mathrm{d}y}{\mathrm{d}x}$.
- To apply differentiation.

Up to this moment, can differentiate "simple" functions like

(1) $f(x) = x^5 + 1$

(2) $f(x) = \dfrac{x-1}{x+1}$

(3) $f(x) = \sin x$

(4) $f(x) = \mathrm{e}^x + 2\tan x$

(5) $f(x) = \dfrac{\ln x}{\cos x} - \dfrac{\mathrm{e}^x}{x^2+1}$

using *simple rules* and *formulas* derived in the last few chapters.

How about

(1) $g(x) = \sin(x^2)$?

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Recall $(g \circ f)(x) = g\big(f(x)\big)$

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$$\begin{aligned}(f \circ g)(x) &= f\big(g(x)\big)\\ &= f(x^2)\end{aligned}$$

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(g \circ f)(x) &= g\big(f(x)\big) \\
&= g(\sin x) \\
&= (\sin x)^2 = \sin^2 x
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Chain Rule in alternative form

Put $y = f(u)$ and $u = g(x)$.

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$$\boxed{(f \circ g)'(x) = f'(g(x)) \cdot g'(x)}$$

Example Find $\dfrac{\mathrm{d}}{\mathrm{d}x}(x^2+5)^3$

(1) without using chain rule;

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Solution

(1) (without chain rule) Expanding

$$\begin{aligned}(x^2+5)^3 &= (x^2)^3 + 3(x^2)^2(5) + 3(x^2)(5^2) + 5^3 \\ &= x^6 + 15x^4 + 75x^2 + 125\end{aligned}$$

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Differentiating term by term:

$$\begin{aligned}\frac{d}{dx}(x^2+5)^3 &= \frac{d}{dx}(x^6 + 15x^4 + 75x^2 + 125) \\ &= 6x^5 + 15\cdot 4x^3 + 75\cdot 2x \\ &= 6x^5 + 60x^3 + 150x\end{aligned}$$

Example Find $\frac{\mathrm{d}}{\mathrm{d}x}(x^2+5)^3$

(2) using chain rule.

Solution

(2) (using chain rule)

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Method 1 Answer is $6x^5 + 60x^3 + 150x$

Method 2 Answer is $6x(x^2 + 5)^2$

Remark 1

- The above two results are the same.

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- If change the function to $y = (x^2 + 5)^{\frac{1}{3}}$, first method can't be applied.

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- Second method makes use of
  - ◇ the chain rule
  - ◇ together with the power rule.

Simple Form	General Form
$\frac{\mathrm{d}}{\mathrm{d}x}x^r = rx^{r-1}$	$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)]^r = r[f(x)]^{r-1} \cdot \frac{\mathrm{d}}{\mathrm{d}x}f(x)$
$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$	$\frac{\mathrm{d}}{\mathrm{d}x}\sin[f(x)] = \cos[f(x)] \cdot \frac{\mathrm{d}}{\mathrm{d}x}f(x)$
$\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$	$\frac{\mathrm{d}}{\mathrm{d}x}\cos[f(x)] = -\sin[f(x)] \cdot \frac{\mathrm{d}}{\mathrm{d}x}f(x)$
$\frac{\mathrm{d}}{\mathrm{d}x}\tan x = \sec^2 x$	$\frac{\mathrm{d}}{\mathrm{d}x}\tan[f(x)] = \sec^2[f(x)] \cdot \frac{\mathrm{d}}{\mathrm{d}x}f(x)$
$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^x = \mathrm{e}^x$	$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{f(x)} = \mathrm{e}^{f(x)} \cdot \frac{\mathrm{d}}{\mathrm{d}x}f(x)$
$\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}$	$\frac{\mathrm{d}}{\mathrm{d}x}\ln[f(x)] = \frac{1}{f(x)} \cdot \frac{\mathrm{d}}{\mathrm{d}x}f(x)$

(3)
$$\frac{\mathrm{d}}{\mathrm{d}x}\cos[f(x)] = -\sin[f(x)] \cdot \frac{\mathrm{d}}{\mathrm{d}x} f(x)$$

Proof

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Proof Put $u = f(x)$ and $y = \cos u$.

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$$= \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x} \qquad \textit{chain rule}$$

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&= \frac{d}{du}\cos u \cdot \frac{du}{dx} \\
&= -\sin u \cdot \frac{du}{dx} \\
&= -\sin[f(x)] \cdot \frac{d}{dx}f(x)
\end{aligned}$$

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&= \frac{1}{f(x)} \cdot \frac{\mathrm{d}}{\mathrm{d}x} f(x)
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Simple Form	General Form
$\frac{\mathrm{d}}{\mathrm{d}x}x^r = rx^{r-1}$	

Simple Form	General Form
$\frac{d}{dx}x^r = rx^{r-1}$	$\frac{d}{dx}[f(x)]^r = r[f(x)]^{r-1} \cdot \frac{d}{dx}f(x)$

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$\frac{\mathrm{d}}{\mathrm{d}x} \sin x = \cos x$	

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Example Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ for the following:

(1) $y = \sin(x^2 + 1)$

(2) $y = \mathrm{e}^{x^3+2}$

(3) $y = \ln(x^4 - 3x + 2)$

(4) $y = \mathrm{e}^{x^5+\tan x^5}$

(5) $y = \ln[\sin^2(2x + 3)]$

(6) $y = \dfrac{1}{(x^2 + 3)^{40}} - 3\mathrm{e}^{4x+1}$

(7) $y = \mathrm{e}^{x+1}\ln(x^2 + 1)$

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Example Find $\frac{dy}{dx}$ for the following:

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$$= \frac{1}{x^4 - 3x + 2} \cdot \frac{d}{dx}(x^4 - 3x + 2) = \frac{4x^3 - 3}{x^4 - 3x + 2}$$

(4) $y = e^{x^5 + \tan x^5}$

Solution $$\frac{dy}{dx} = \frac{d}{dx}e^{x^5 + \tan x^5} = e^{x^5 + \tan x^5} \cdot \frac{d}{dx}(x^5 + \tan x^5)$$

Example Find $\frac{dy}{dx}$ for the following:

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Solution

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$$= e^{x^5+\tan x^5} \cdot (5x^4 + 5x^4 \sec^2 x^5) = 5x^4(1 + \sec^2 x^5)e^{x^5+\tan x^5}$$