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Fundamental Theorem of Integral Calculus *Let* $f : [a, b] \longrightarrow \mathbb{R}$ *be a continuous function. Suppose* G *is a continuous function on* $[a, b]$ *such that* $G'(x) = f(x)$ *for all* $x \in (a, b)$. *Then*

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Terminology

- To *integrate* a function f *(with respect to x)* means to find $\int f(x)\,\mathrm{d}x$

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Solution Since

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}(x^3 + 5x) &= \frac{\mathrm{d}}{\mathrm{d}x}x^3 + \frac{\mathrm{d}}{\mathrm{d}x}5x \\ &= 3x^2 + 5\end{aligned}$$

$F(x) = x^3 + 5x$ is a primitive for $f(x) = 3x^2 + 5$.

Therefore $\int (3x^2 + 5)\,\mathrm{d}x = x^3 + 5x + C$, where *$C$ is an arbitrary constant*

Terminology

- To *integrate* a function f *(with respect to x)* means to find $\int f(x)\,\mathrm{d}x$

  the function f is called the *integrand*.

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Integration Formulas / Rules

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Integration Formulas / Rules

(1) $\displaystyle\int k \, \mathrm{d}x = kx + C$ where k is a constant (function)

Proof

$$\frac{\mathrm{d}}{\mathrm{d}x}(kx) = k \cdot \frac{\mathrm{d}}{\mathrm{d}x}x$$

$$= k \cdot 1$$

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Example

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(2) **Power Rule**

$$\int x^{r}\,\mathrm{d}x$$

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$$\int x^7 \, dx =$$

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Example

$$\begin{aligned}
\int x^7 \, dx &= \frac{x^{7+1}}{7+1} + C \\
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Example $\displaystyle\int \frac{1}{x^3}\,\mathrm{d}x \quad =$

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Question In power rule, how about $r = -1$? *What is* $\int \frac{1}{x}\,\mathrm{d}x$ *?*

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Wrong solution

$$\int \frac{1}{x}\,\mathrm{d}x = \int x^{-1}\,\mathrm{d}x$$

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Example

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Formula for $\int \frac{1}{x}\,\mathrm{d}x$, see Chapter 10.

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Example

$$\begin{aligned}\int 2x^7\,\mathrm{d}x &= 2\int x^7\,\mathrm{d}x \\ &= 2\cdot\frac{x^{7+1}}{7+1} + C \\ &= \frac{x^8}{4} + C\end{aligned}$$

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Remark Can also write

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Reason $\frac{x^8}{4} + C$ and $\frac{x^8}{4} + 2C$ represent the *same family* of functions.

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Proof $$\frac{\mathrm{d}}{\mathrm{d}x}\left(F(x) \pm G(x) \right) = \frac{\mathrm{d}}{\mathrm{d}x}F(x) \pm \frac{\mathrm{d}}{\mathrm{d}x}G(x)$$

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$$= \quad x +$$

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$$= \frac{x^{\sqrt{2}+1}}{\sqrt{2}+1} + 3\cdot\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 11x + C = \cdots$$