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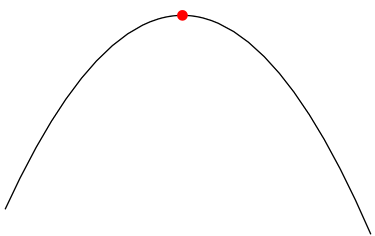

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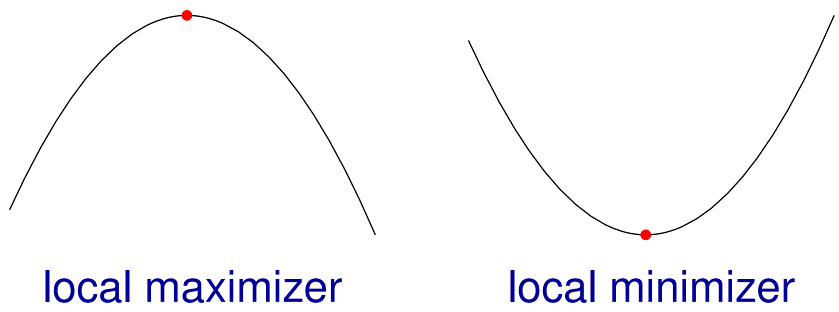

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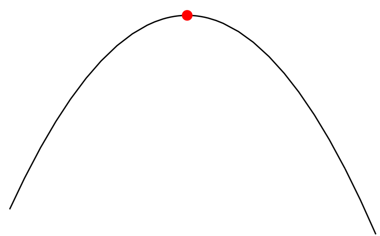
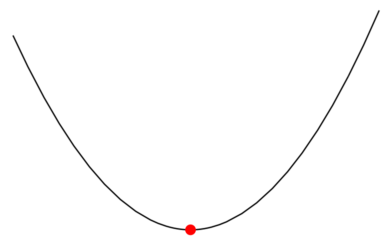
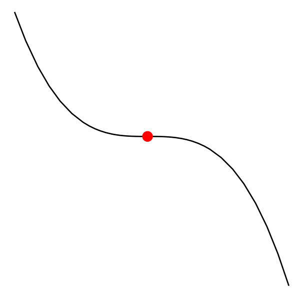


local maximizer



local minimizer



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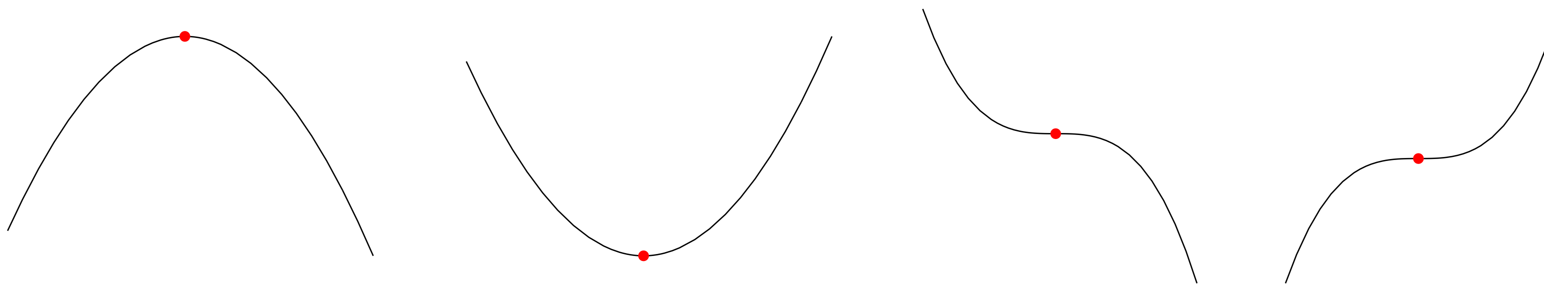

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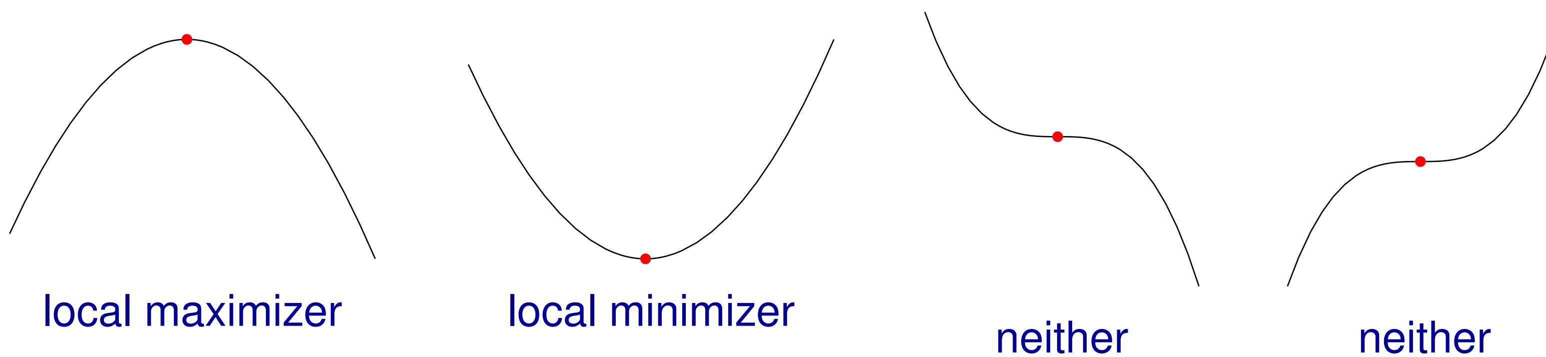

Same meaning

- x_0 is a local maximizer of f
- f has a local maximum at x_0.

Terminology A curve is said to be

- *bending up* if its slope is increasing;

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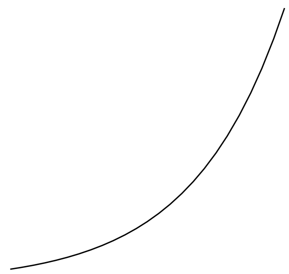

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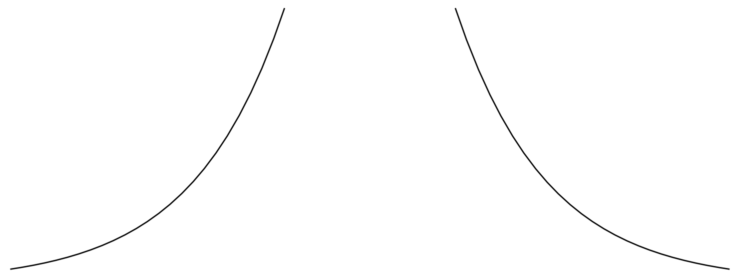

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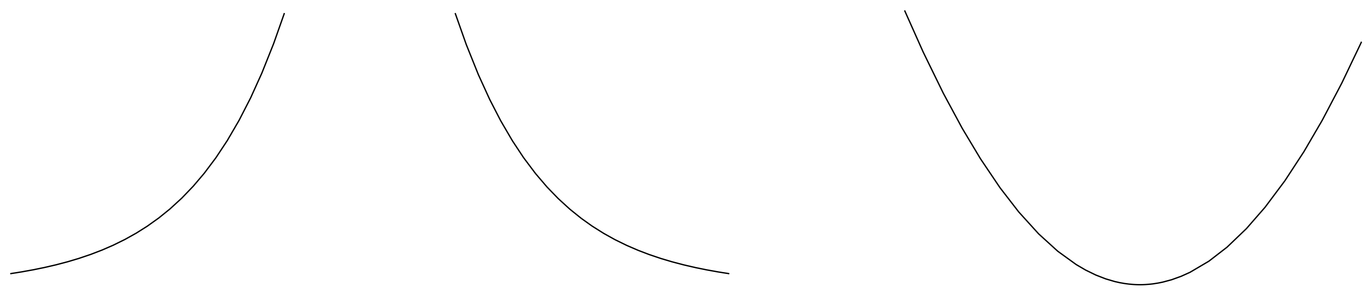

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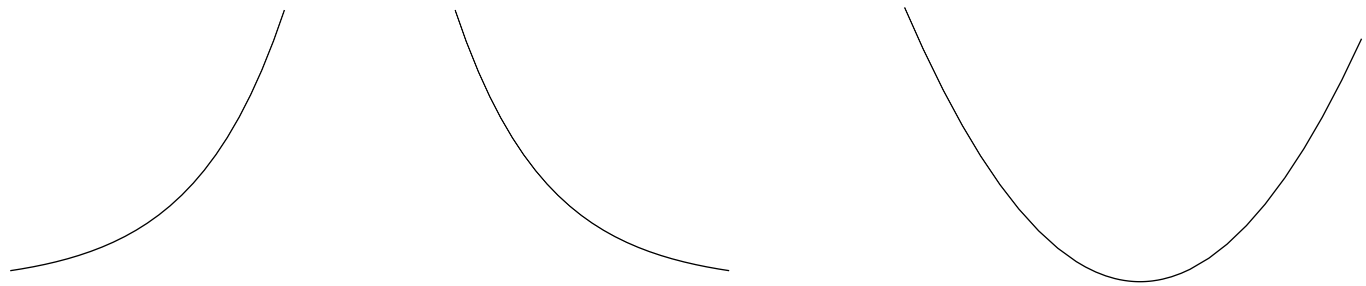

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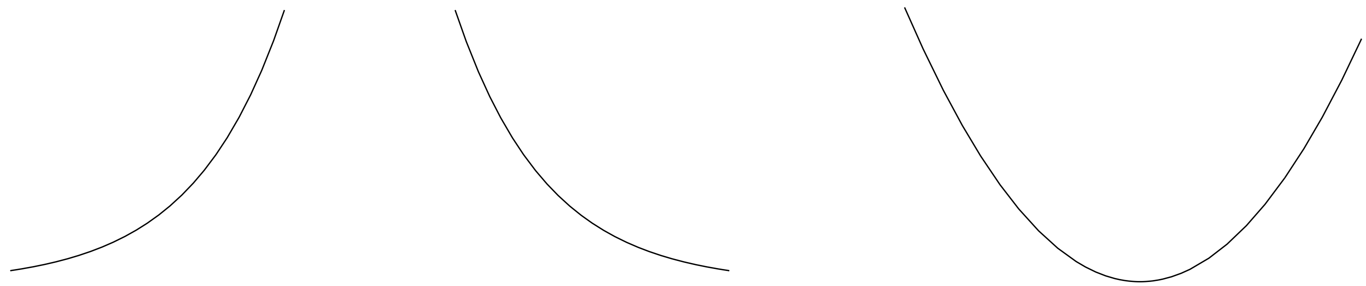

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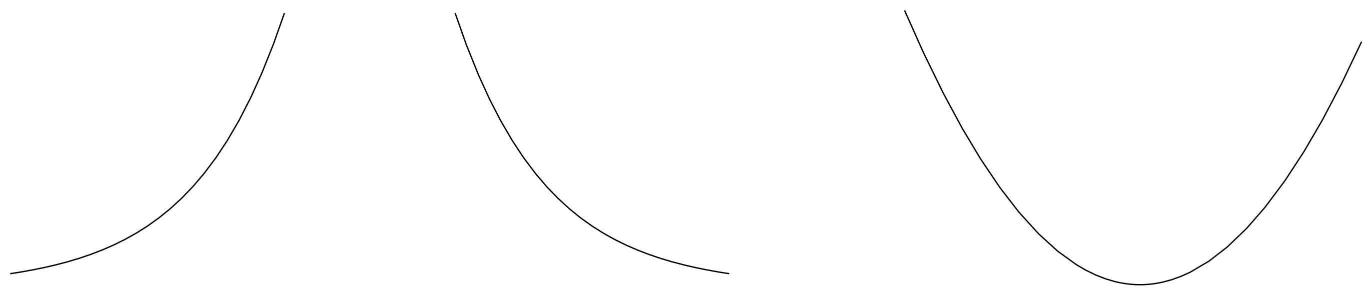

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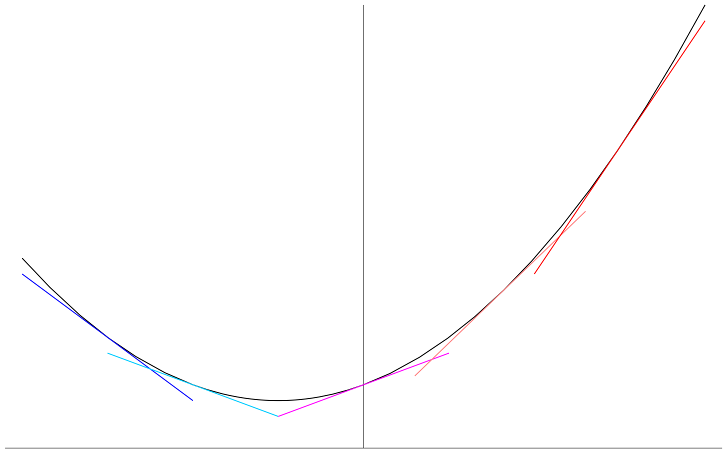
Definition A function is said to be

- *convex* of an open interval (a,b) if f' is increasing on (a,b);
- *concave* of an open interval (a,b) if f' is decreasing on (a,b).

Criteria for Convex/Concave Functions



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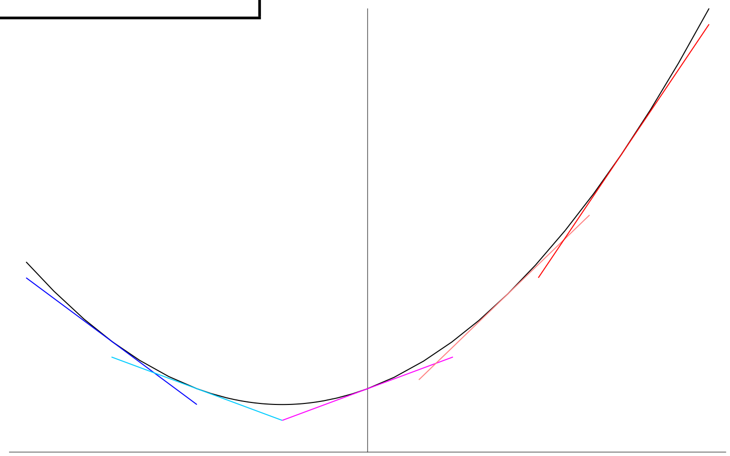

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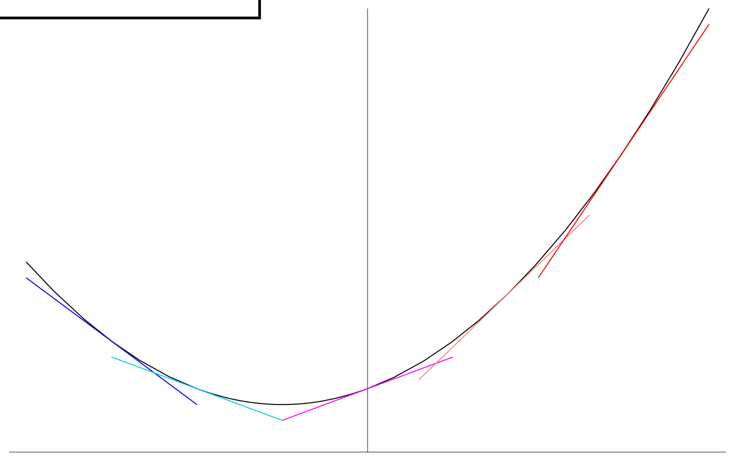

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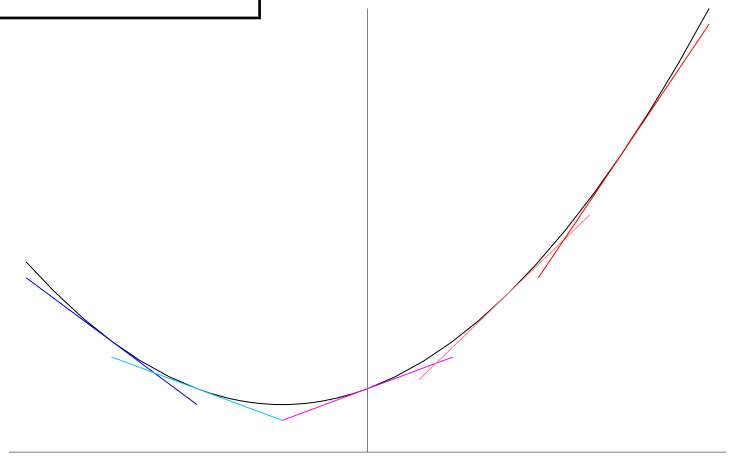

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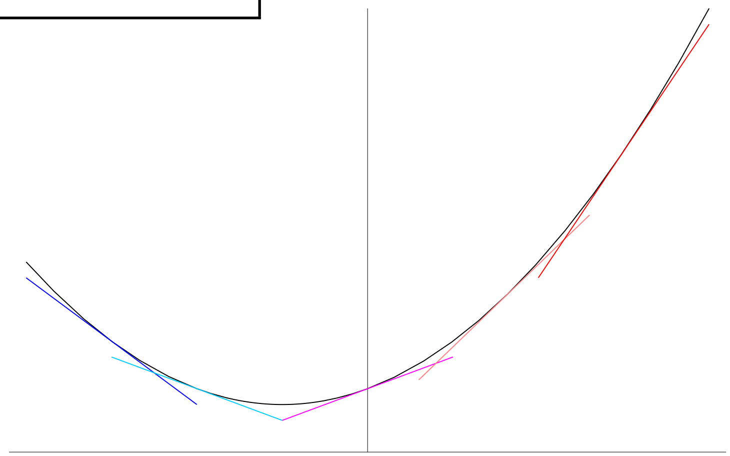

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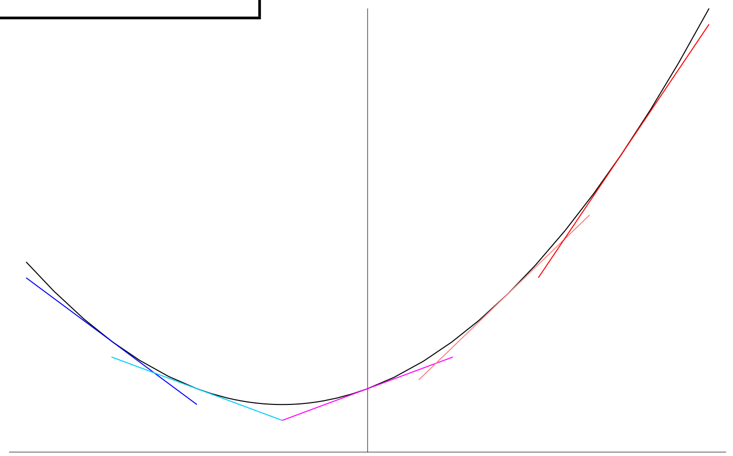

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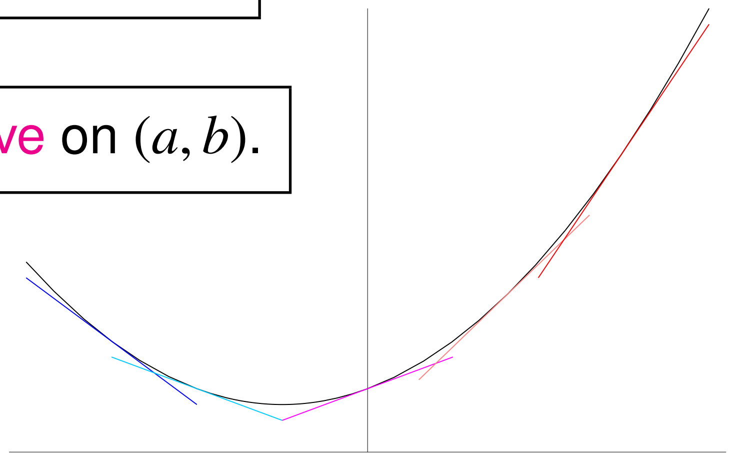

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Summary

Criteria	$f' > 0$	$f' < 0$
Function	increasing	decreasing
Graph	going up	going down
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Criteria	$f'' > 0$	$f'' < 0$
Function	convex	concave
Graph	bending up	bending down
	$\smile$	$\frown$

Example Consider the function f given by

$$f(x) = 27x - x^3.$$

Find the interval(s) on which its graph is

(a) bending up; (b) bending down.

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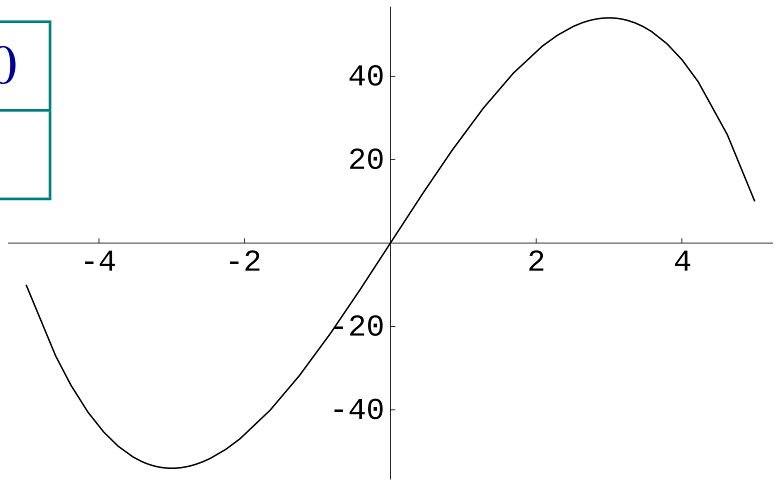

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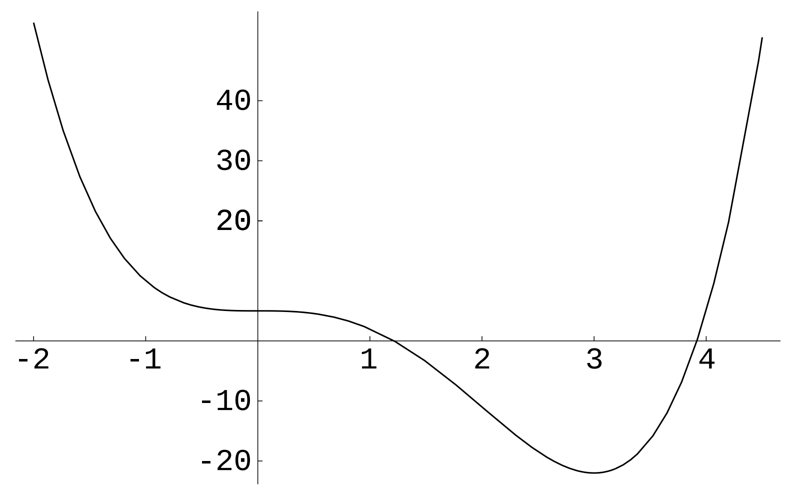

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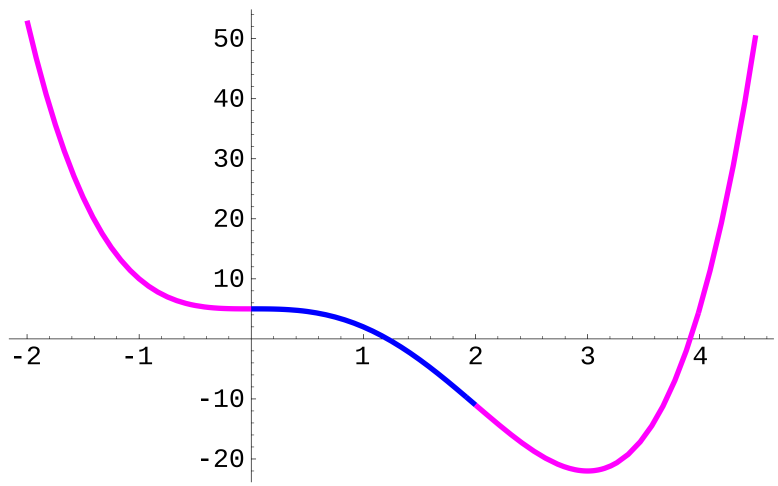

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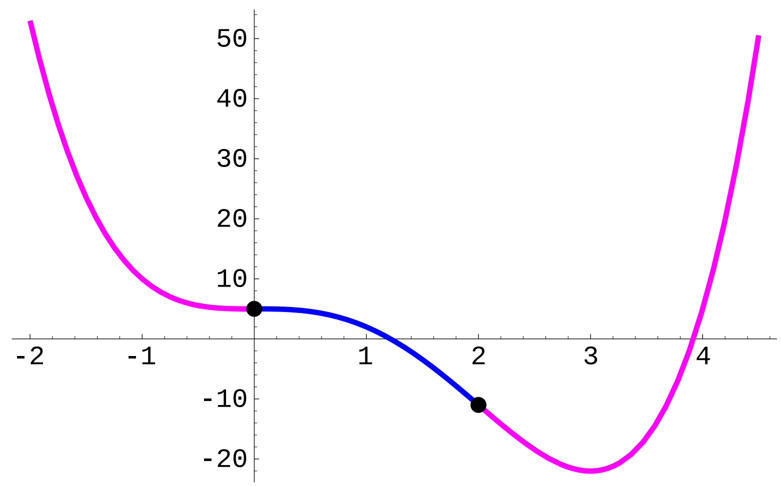

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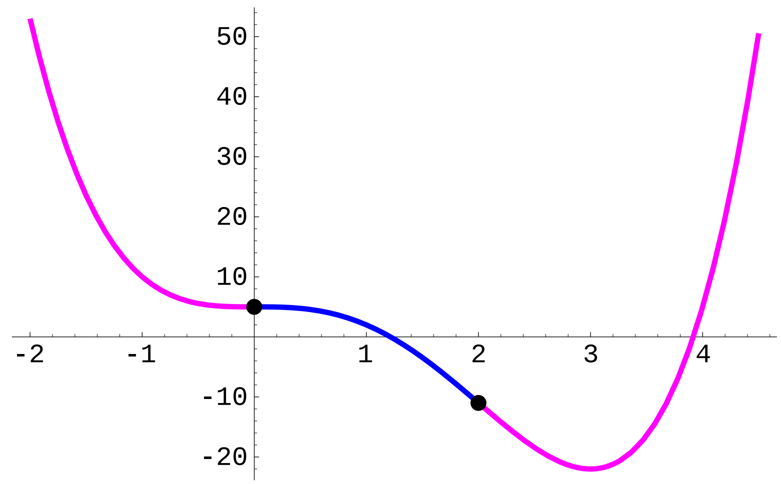

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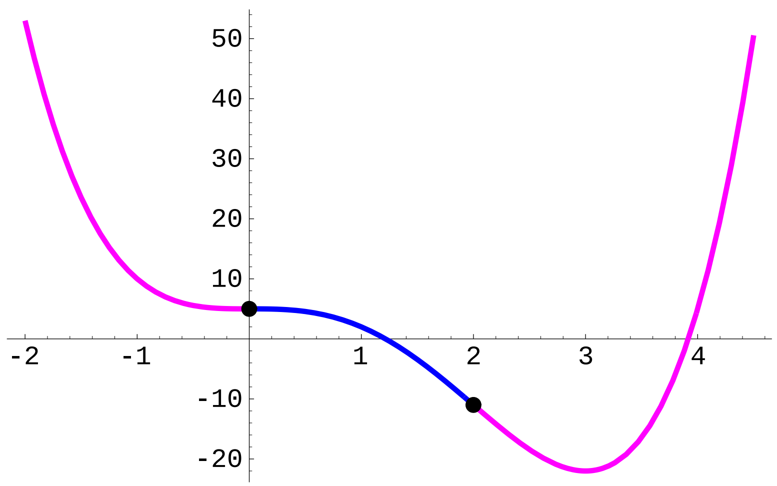

- The graph of g has two inflection points $(0, 5)$ and $(2, -11)$

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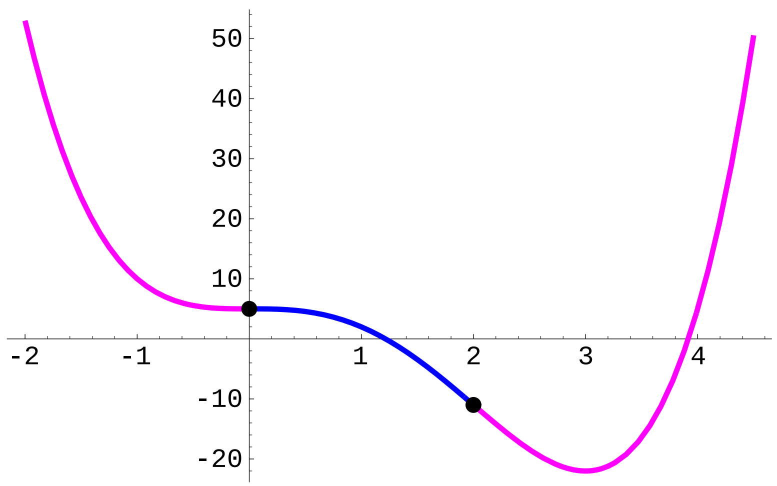
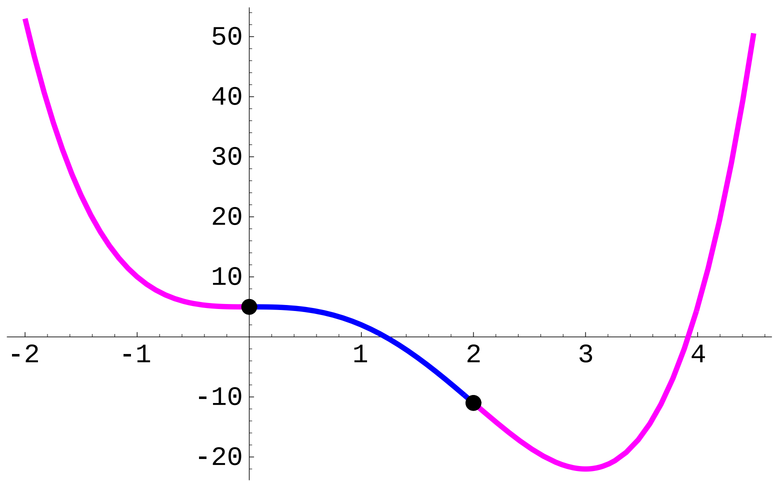


- The graph of g has two inflection points $(0, 5)$ and $(2, -11)$

Definition

- A point (x_0, y_0) at which a curve $y = f(x)$ changes bending (from bending up to bending down or vice versa) is called an *inflection point* of the curve
- x_0 is called an *inflection point* of the function f.

$g(x) = x^4 - 4x^3 + 5$



- The graph of g has two inflection points $(0, 5)$ and $(2, -11)$
- The function g has two inflection points: 0 and 2.

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- If f has an inflection point at x_0, then $f''(x_0) = 0$.

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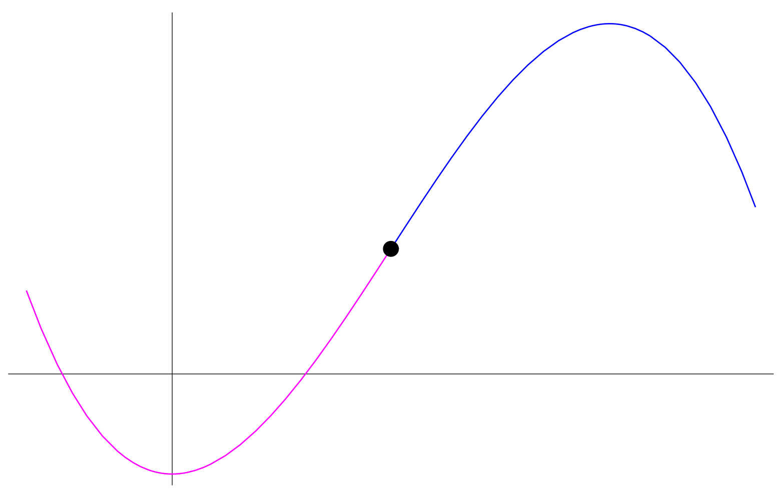

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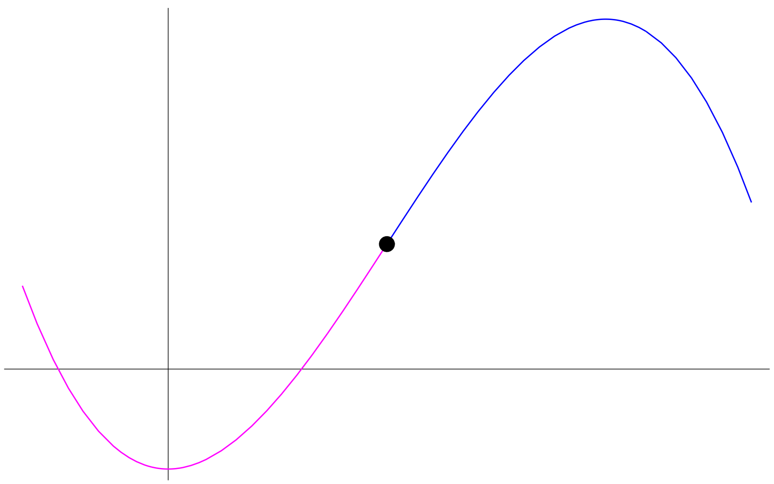

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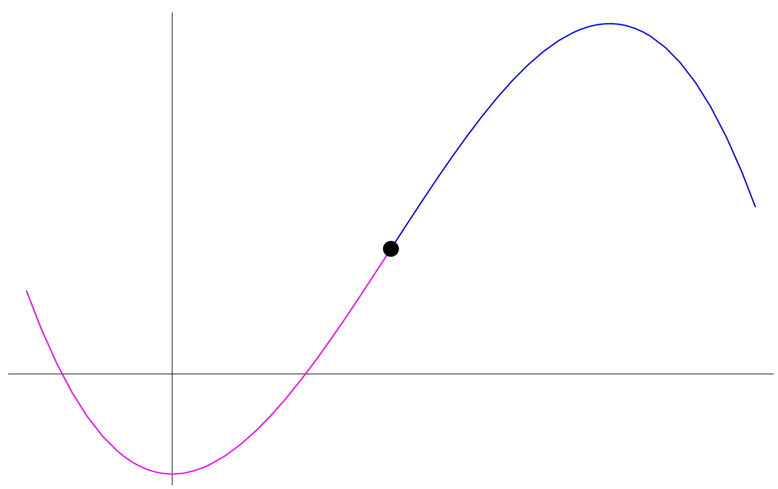

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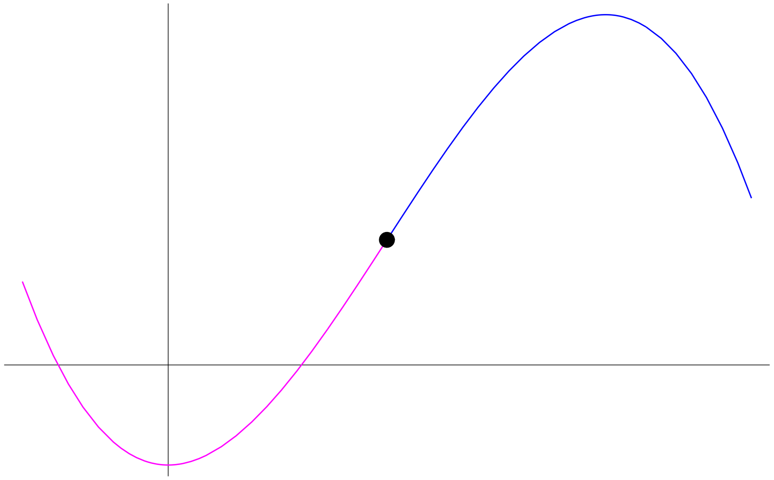
Therefore $f''(x_0) = 0$.



Example Consider the function h given by

$$h(x) = x^4 - 6x^2 + 5x - 6$$

Find interval(s) on which h is convex or concave and inflection point(s) of h.

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- $h'(x) \quad = \quad 4x^3 - 12x + 5$

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	$x < -1$	$-1 < x < 1$	$x > 1$
$x + 1$			
$x - 1$			
$h''(x)$			
h			

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h	convex		

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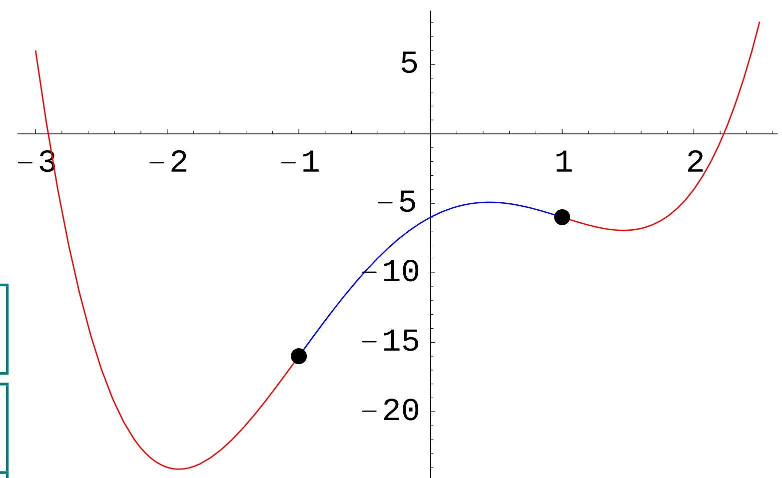

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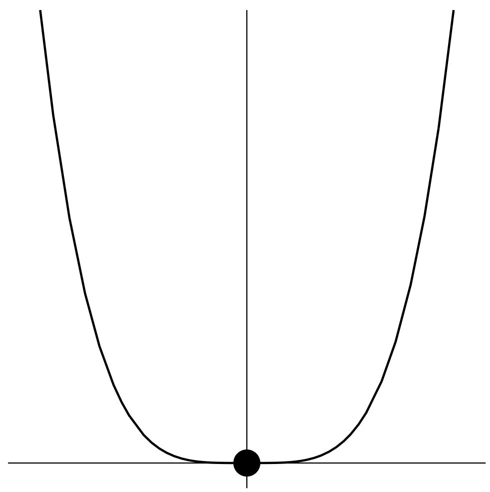
  because f is convex on $\mathbb{R}$



Second-Derivative Test For Relative Extrema

Suppose f has a critical point at $x = x_0$ (that is, $f'(x_0) = 0$)

- | If then x_0 is a local maximizer of f | f has a local max. at x_0

Second-Derivative Test For Relative Extrema

Suppose f has a critical point at $x = x_0$ (that is, $f'(x_0) = 0$)

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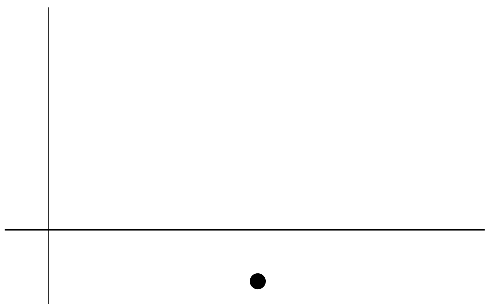
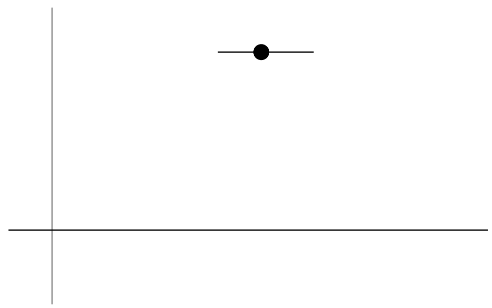


$y = f''(x)$



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Reason If $f''(x_0) < 0$,

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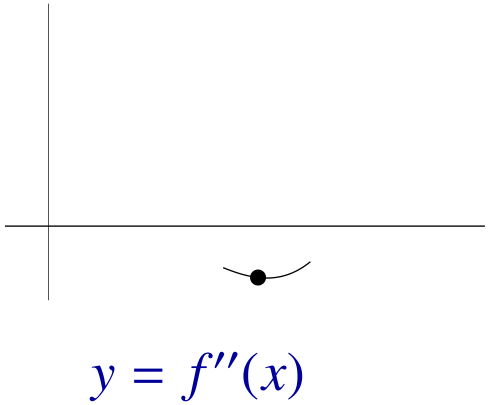

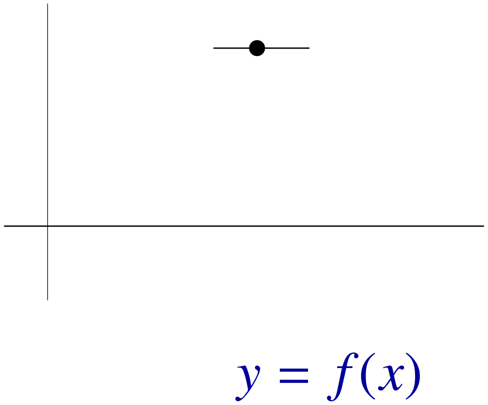

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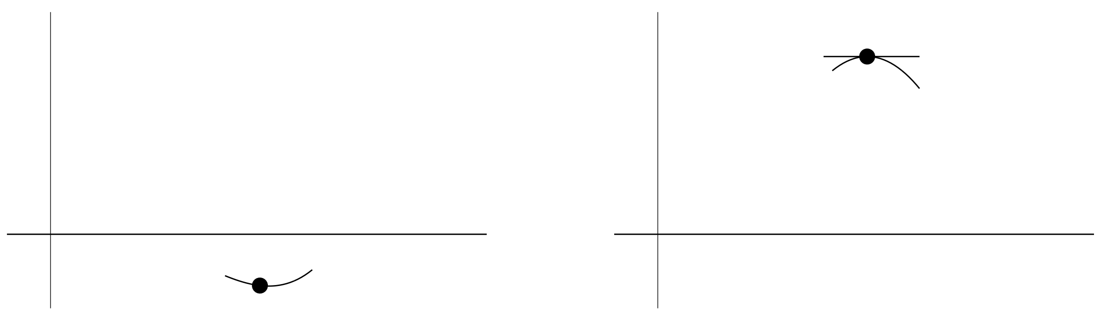

$y = f''(x)$ $y = f(x)$

Second-Derivative Test For Relative Extrema

Suppose f has a critical point at $x = x_0$ (that is, $f'(x_0) = 0$)

- If $f''(x_0) < 0$, then x_0 is a local maximizer of f — f has a local max. at x_0
- If $f''(x_0) > 0$, then x_0 is a local minimizer of f

Reason If $f''(x_0) < 0$,

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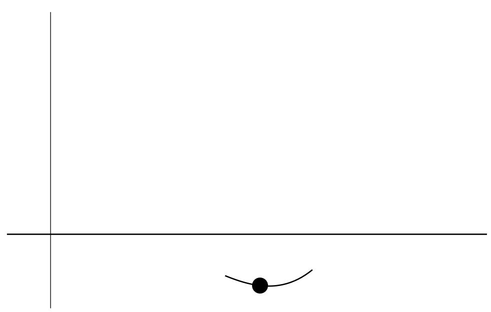
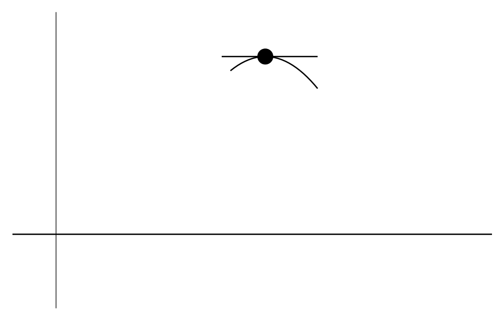


$y = f''(x)$



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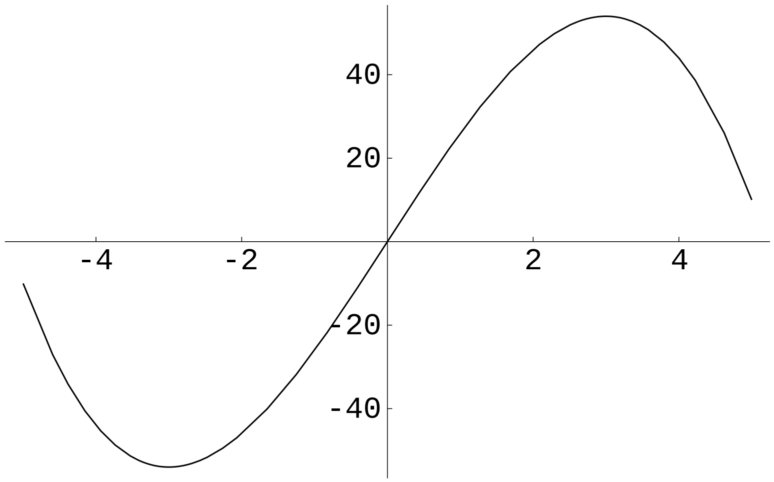

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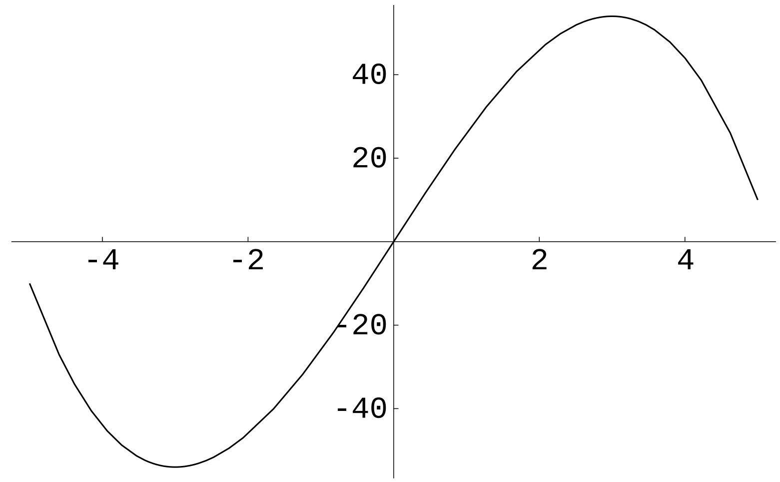

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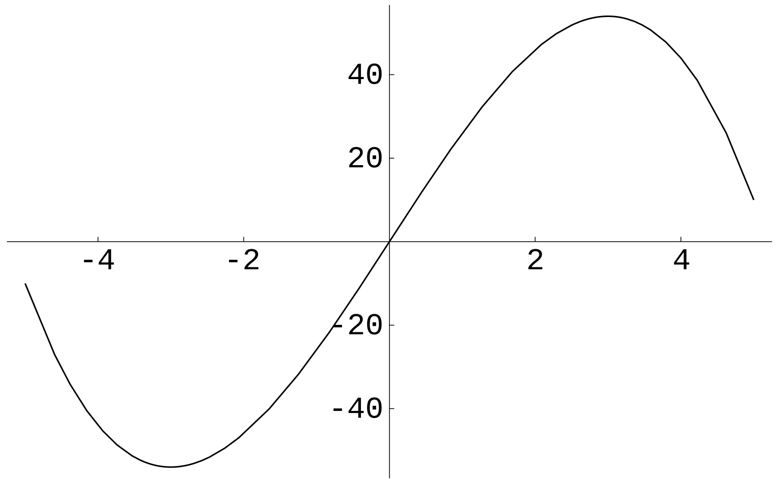

- $f'(x) = 27 - 3x^2$
- $f''(x) = -6x$
- Critical points of f are $x_1 = 3$ and $x_2 = -3$.

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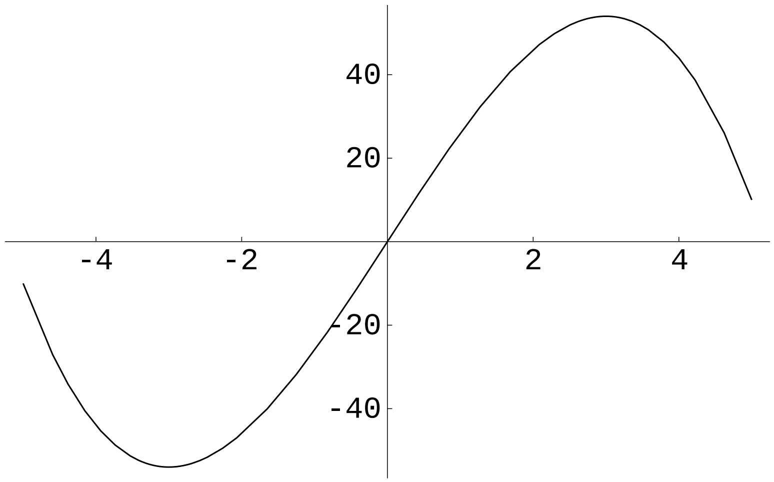

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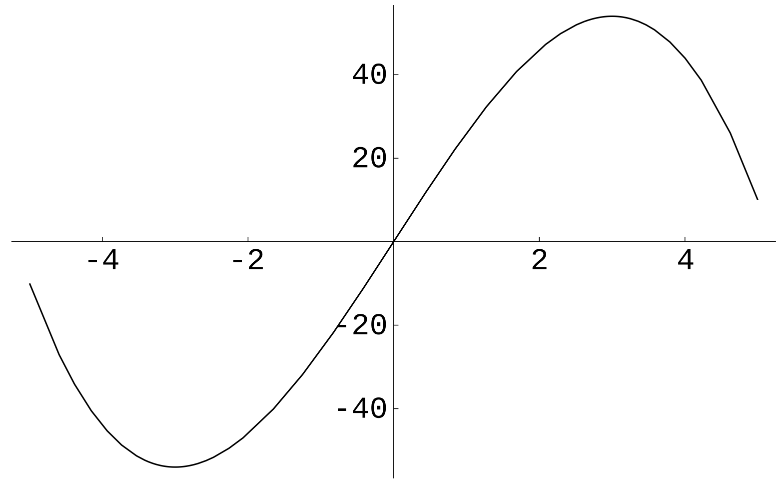

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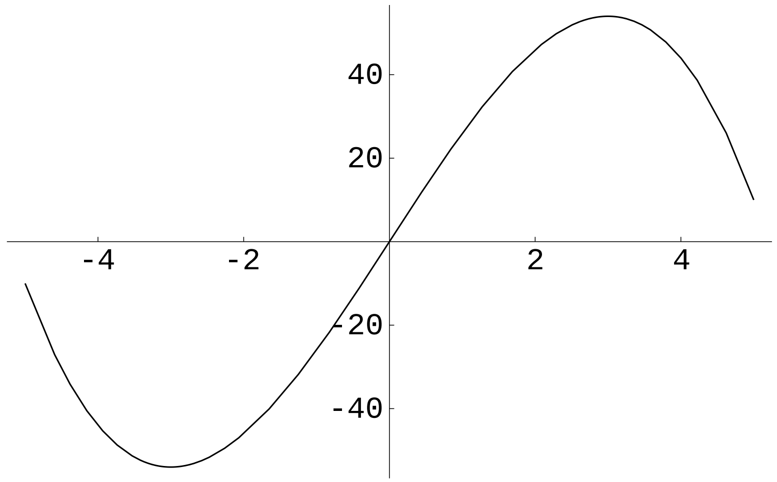

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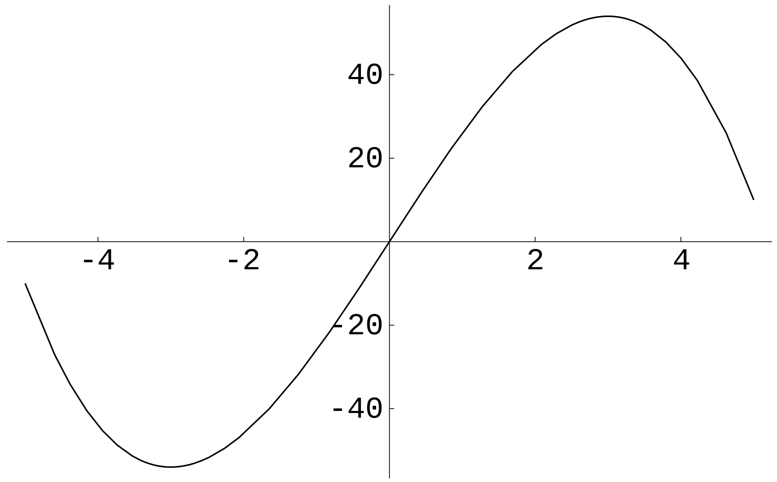

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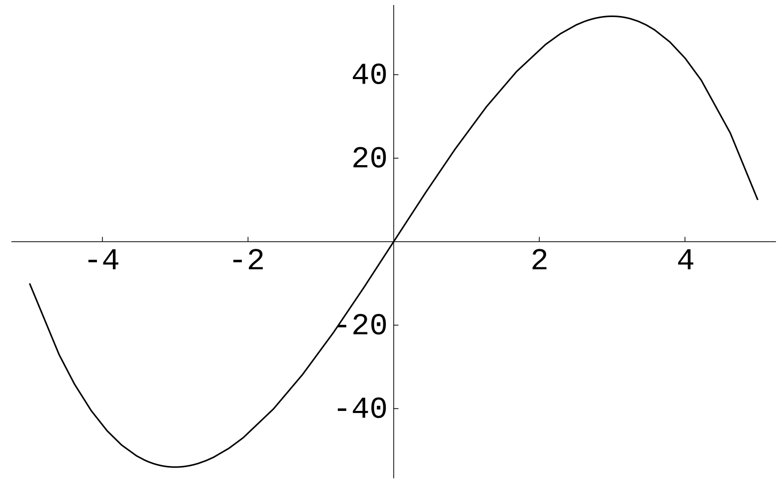

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Remark *No conclusion* from the 2nd-derivative test *if* $f''(x_0) = 0$. May have

- local maximum,
- local minimum,
- neither.