Chapter 5: Applications of Differentiation

- Curve Sketching
- Applied Extremum Problems

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For convenience, functions under consideration are assumed to have continuous second-order derivatives *(that is, f'' is continuous)*.

Increasing and Decreasing Functions

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- *strictly increasing* on I if

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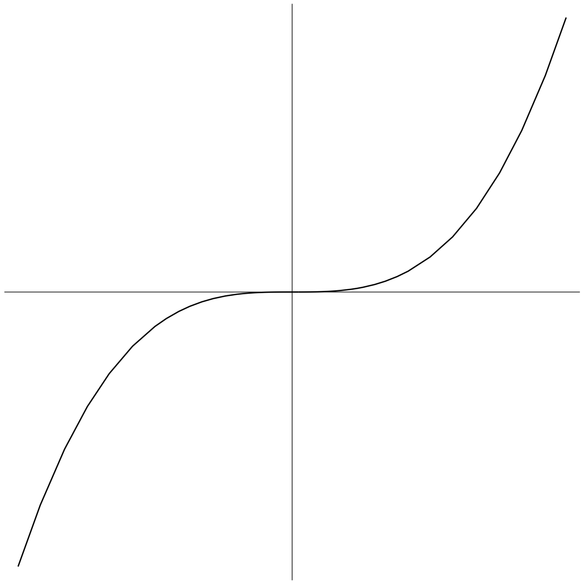

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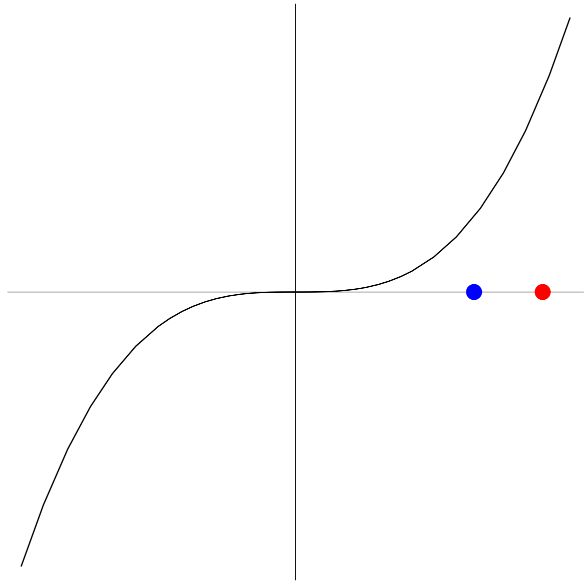

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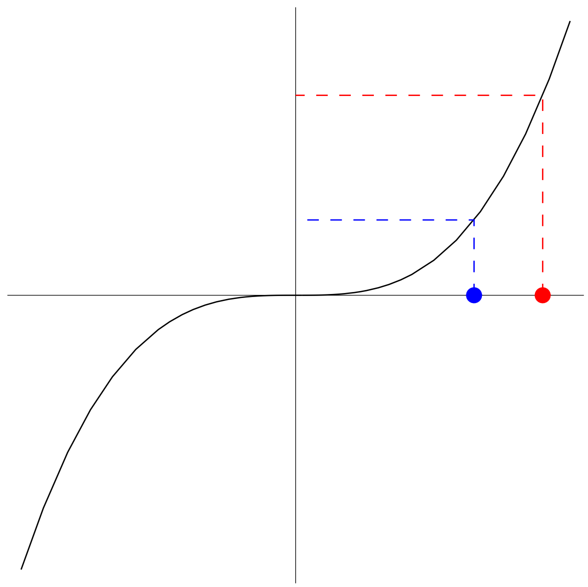

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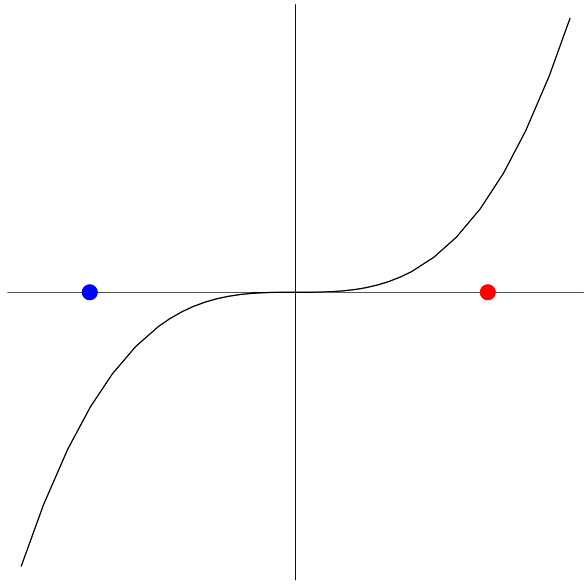

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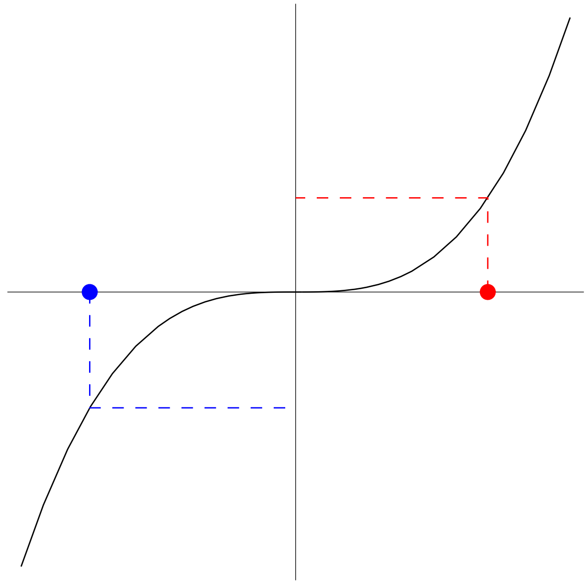

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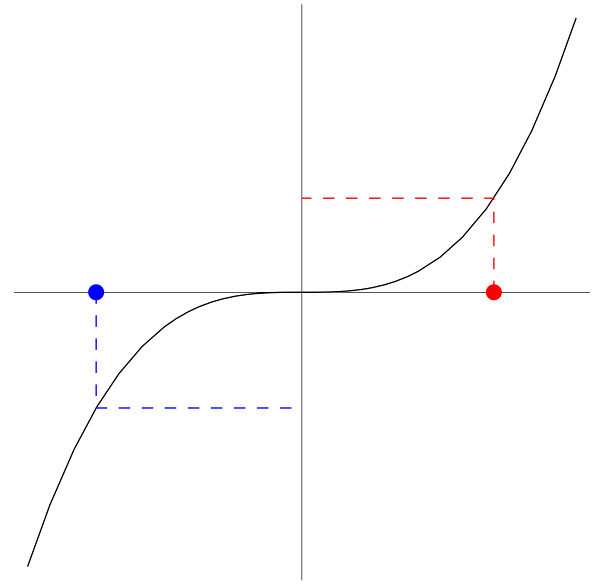

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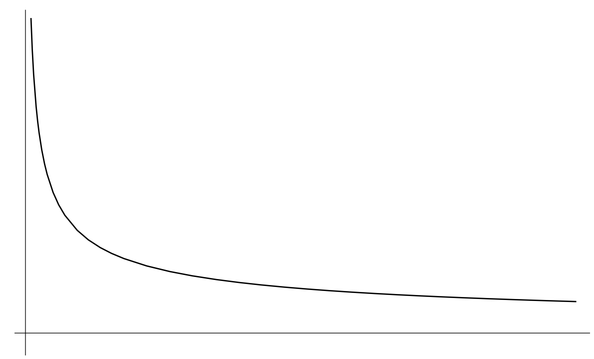

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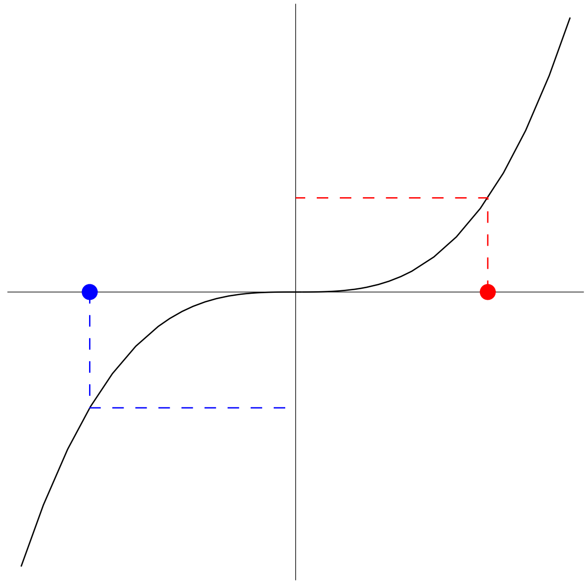

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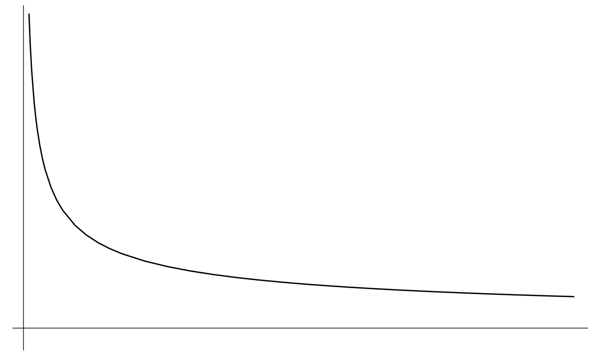

Terminology *Strictly increasing* will be called *increasing*

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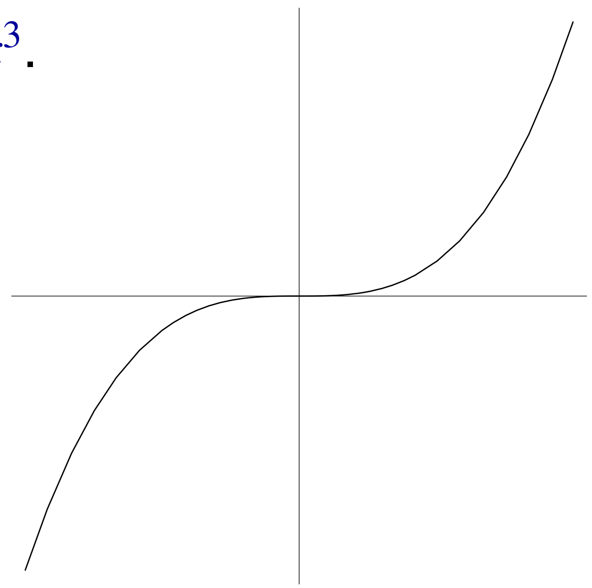

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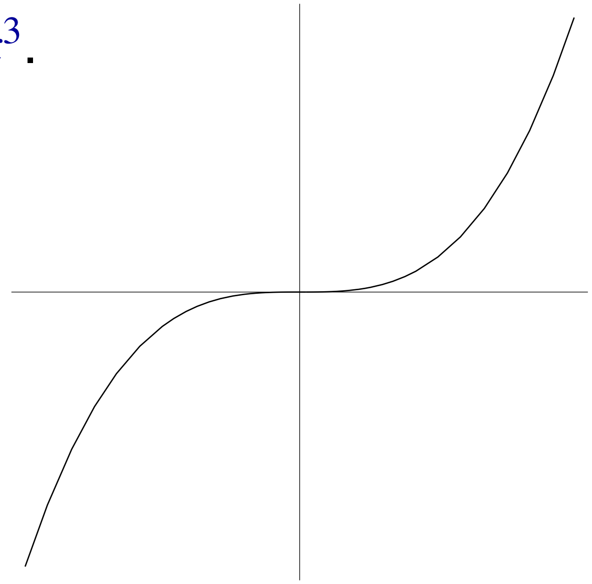

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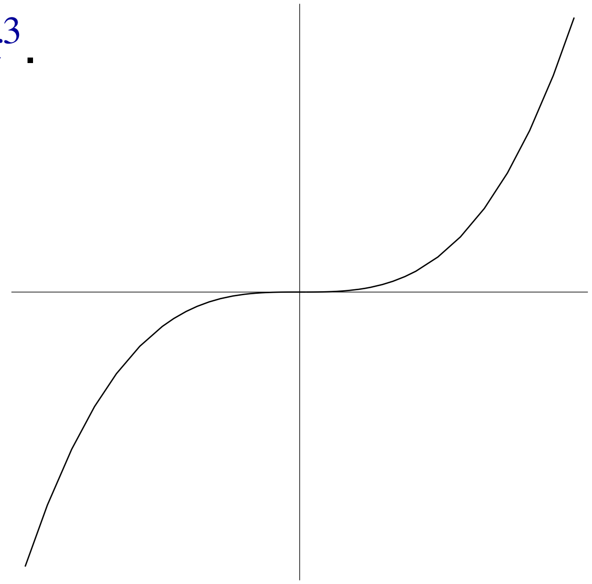

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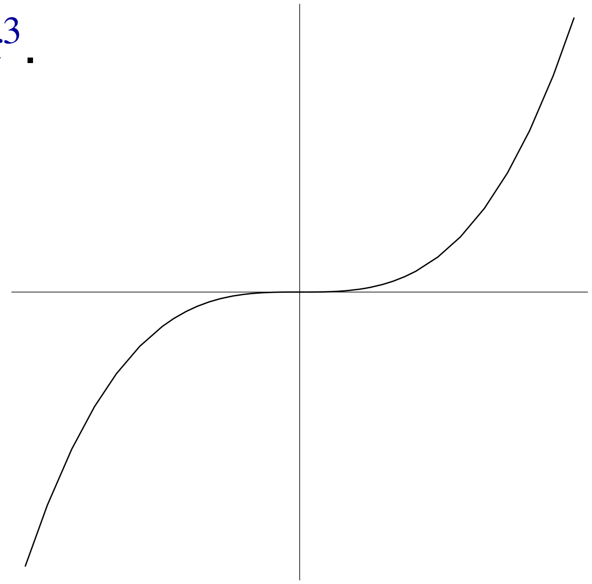

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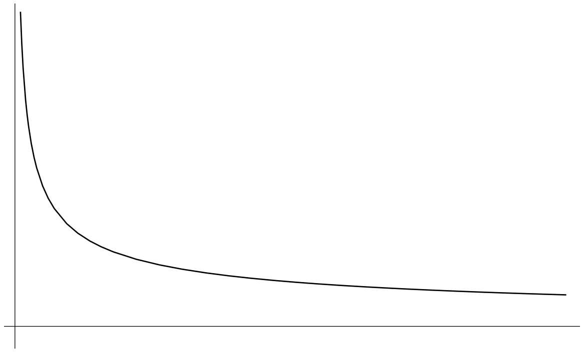

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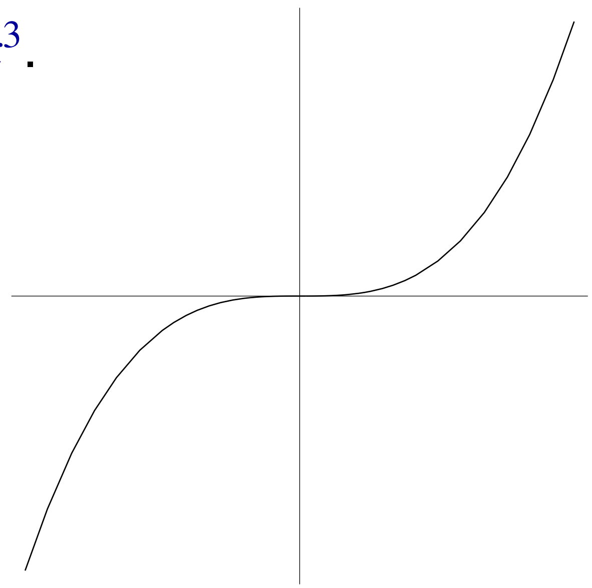

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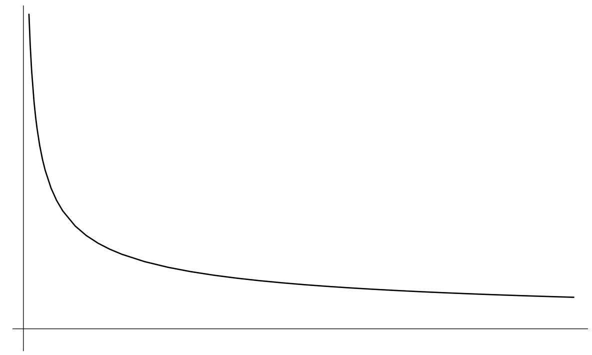

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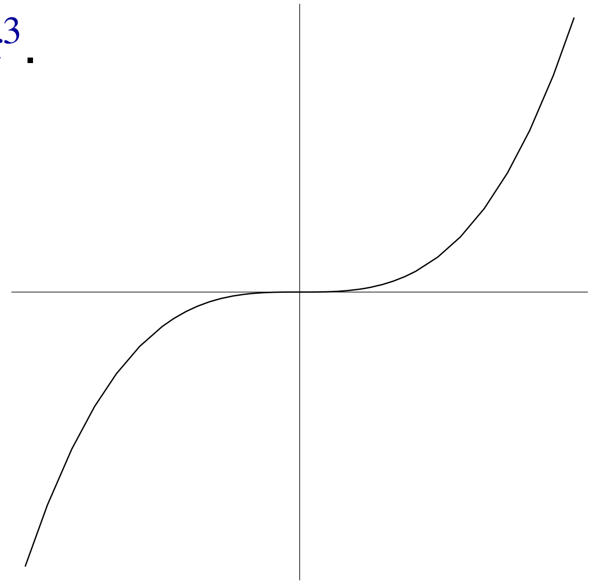

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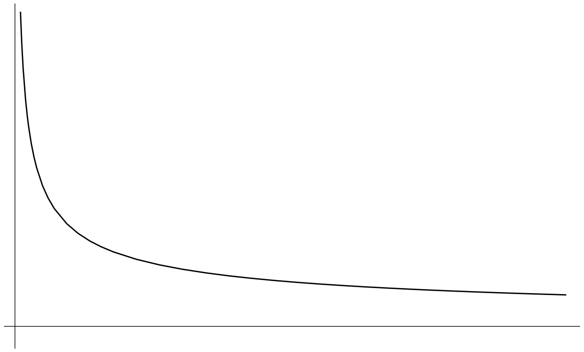

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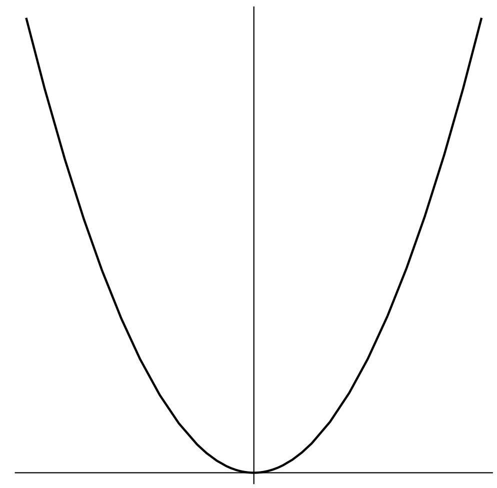

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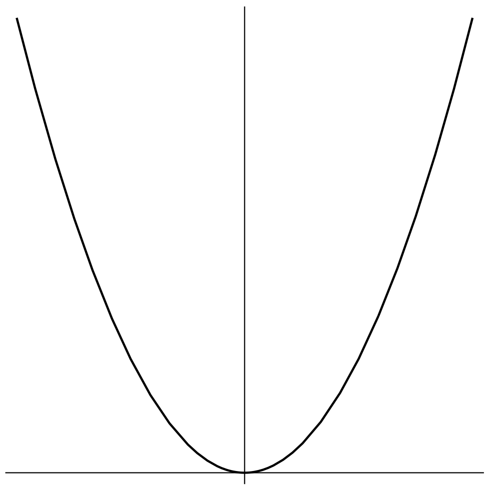

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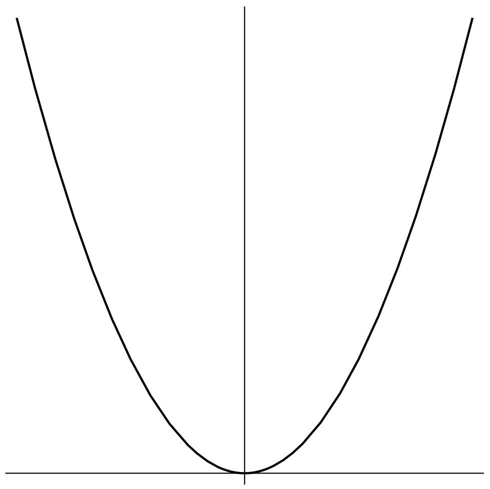

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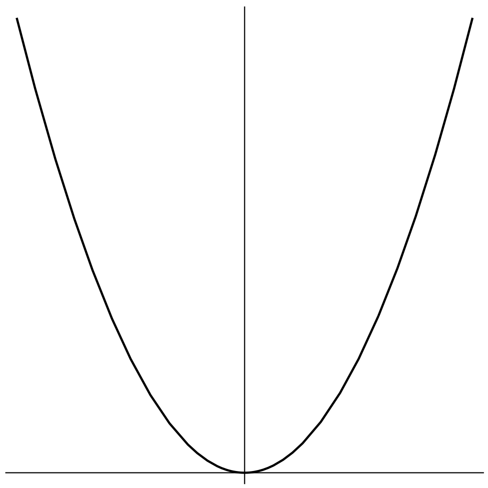

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Question Given a function $f : \mathbb{R} \longrightarrow \mathbb{R}$, for example, $f(x) = 27x - x^3$, how to find *intervals on which f is increasing / decreasing* ?

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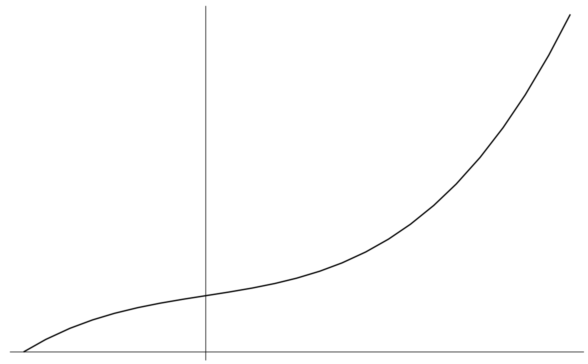

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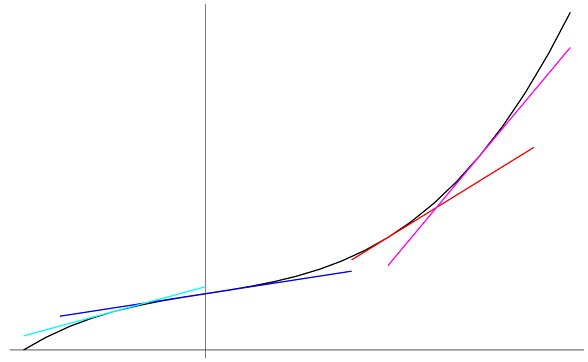

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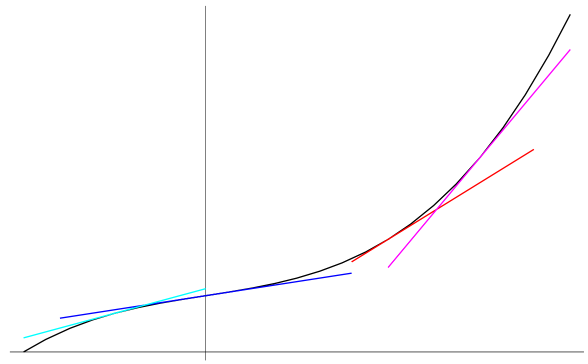

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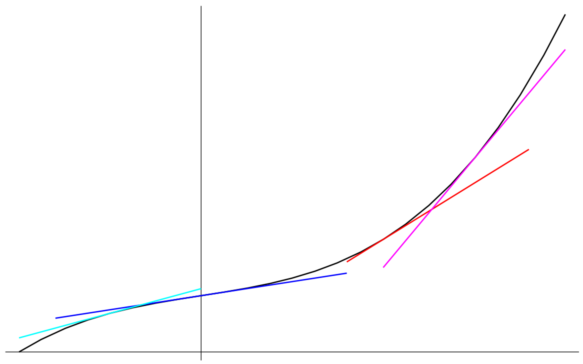

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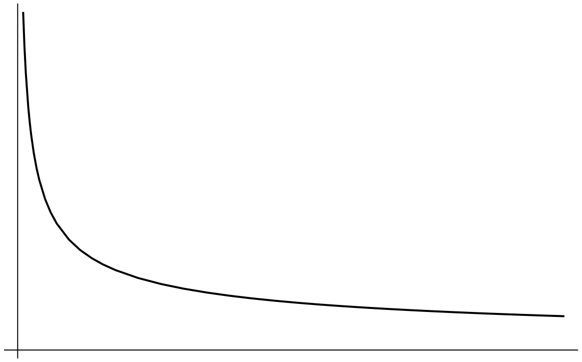

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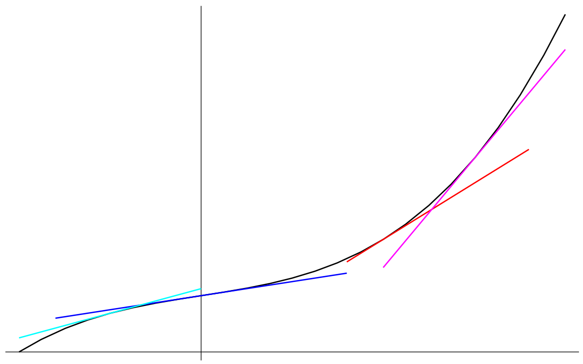

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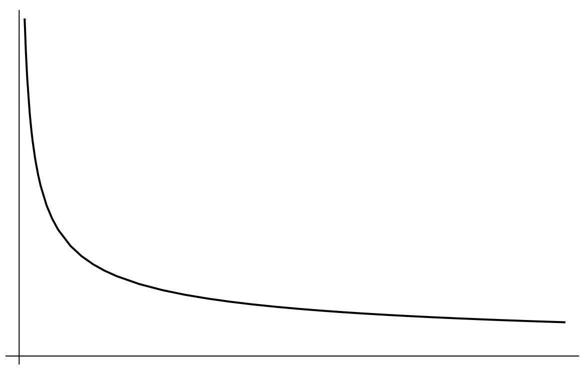

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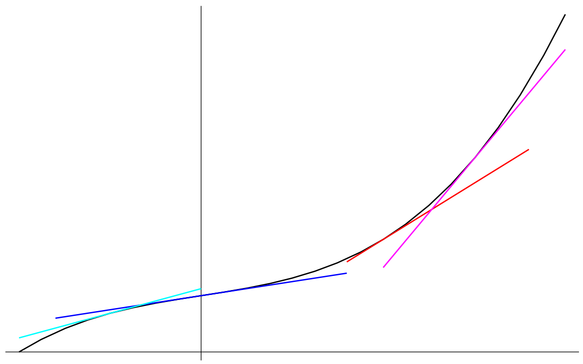

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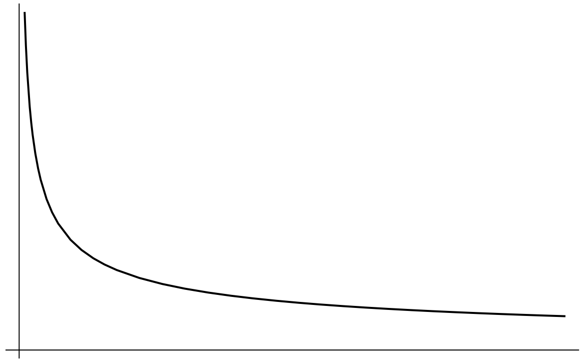

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Remark The proofs require *Mean Value Theorem* (see supplementary notes).

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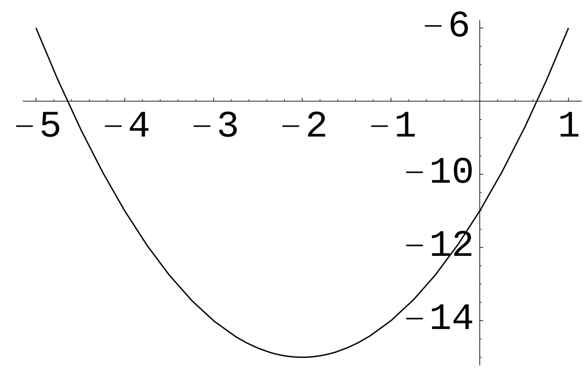

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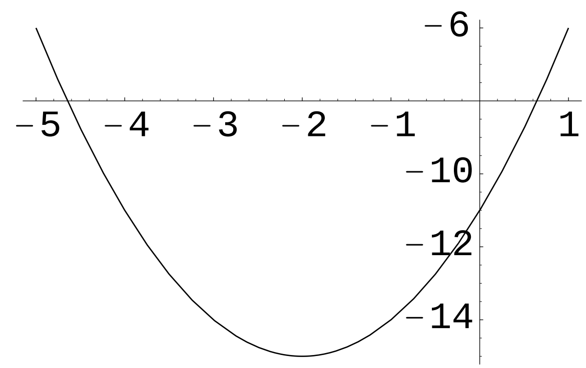

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Remark Can use differentiation to find vertex of parabola.

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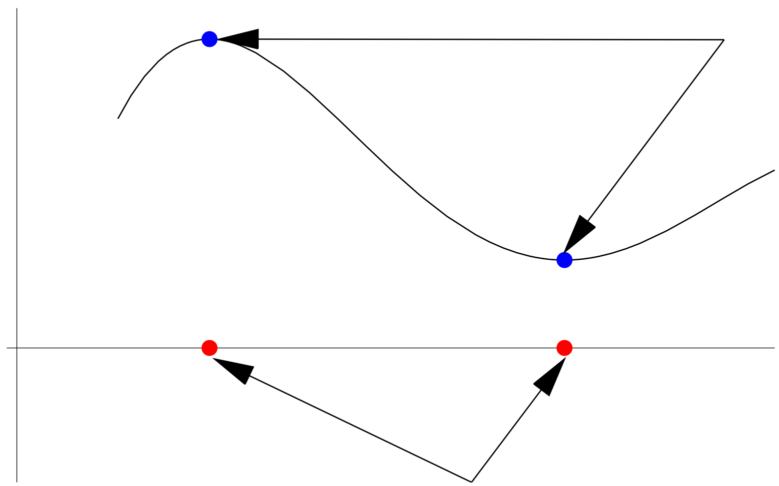

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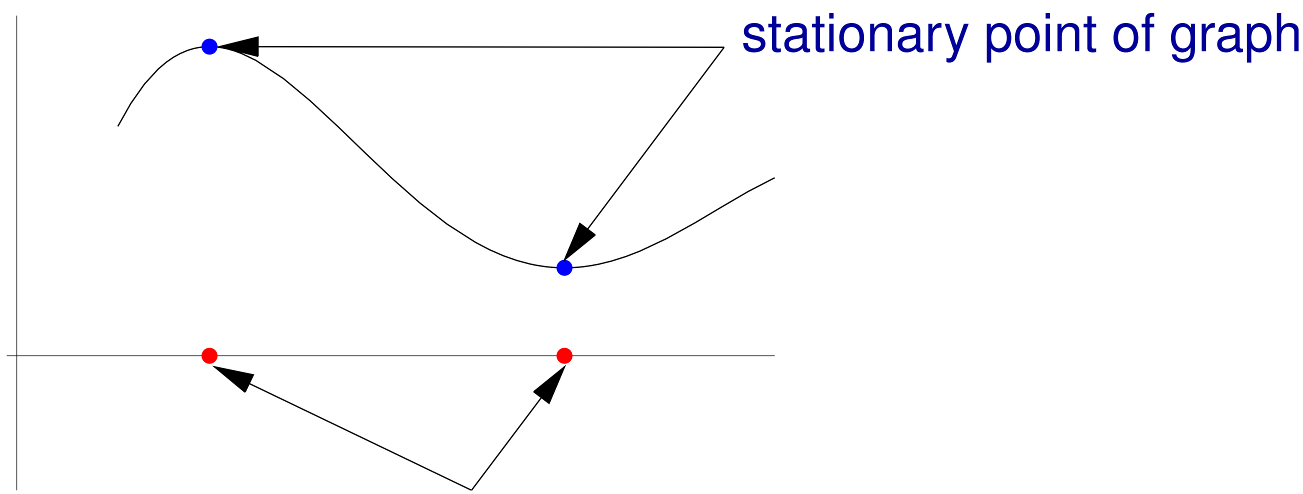

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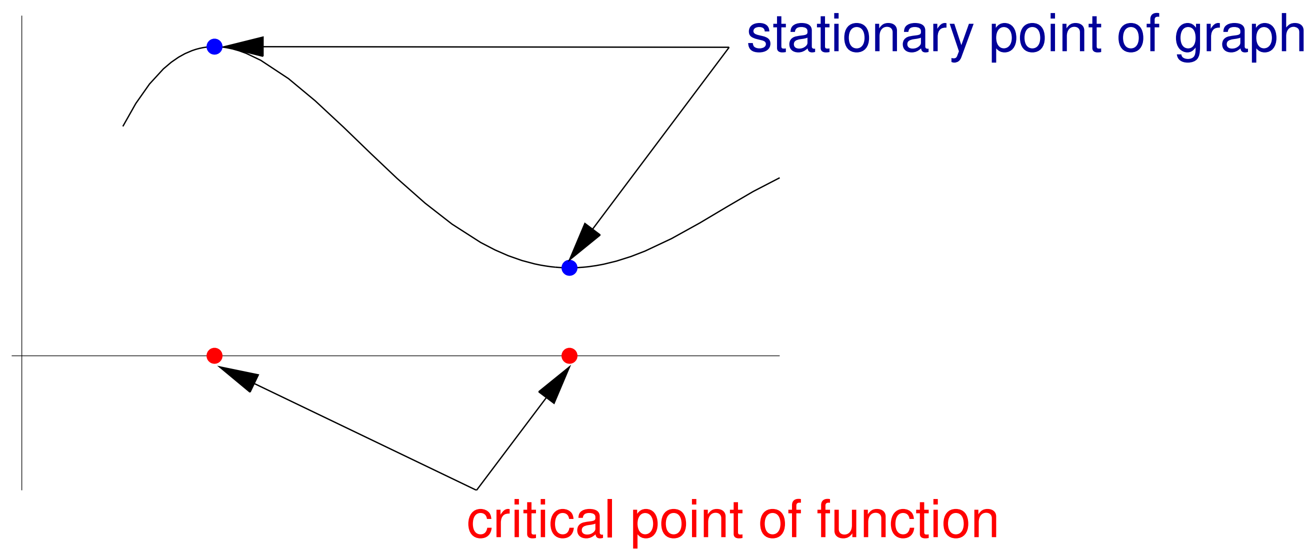

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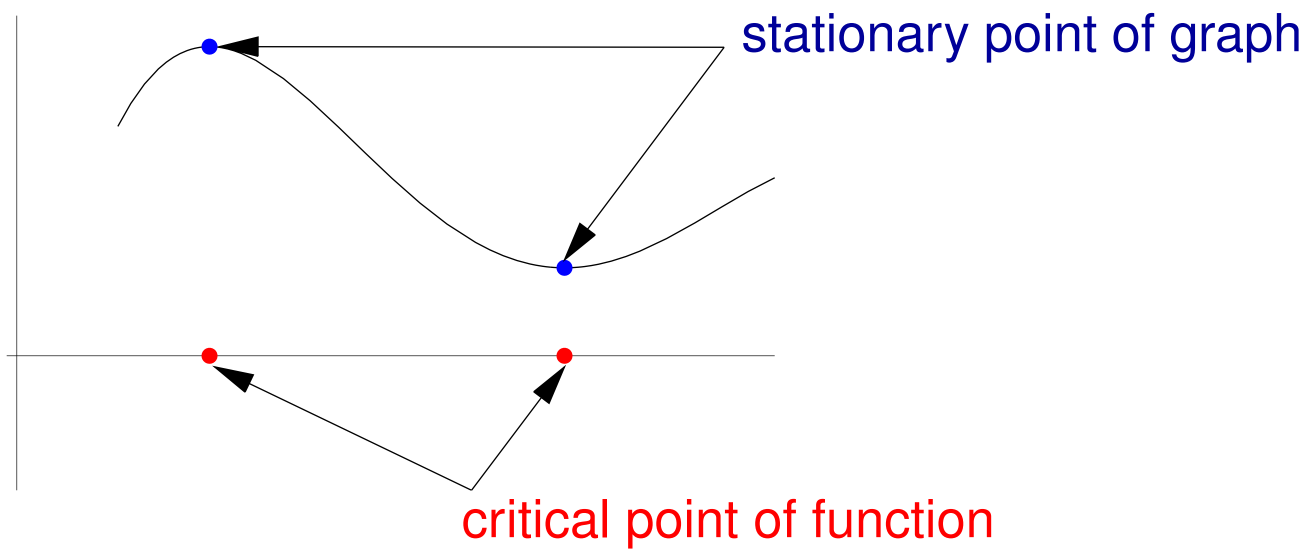

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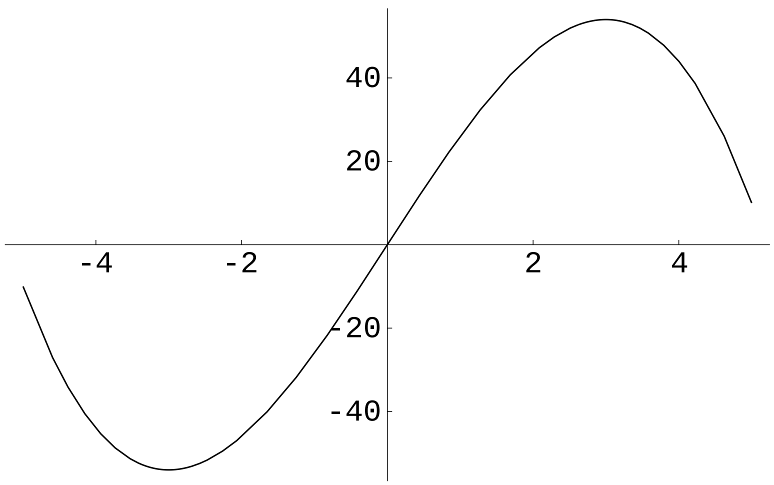

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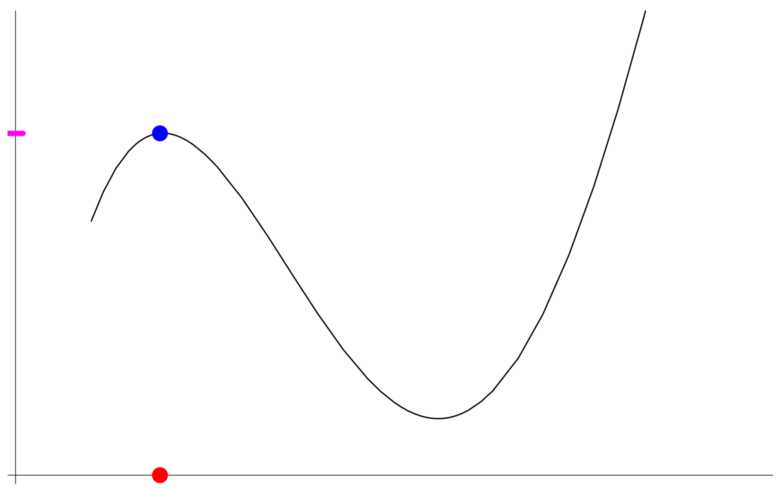

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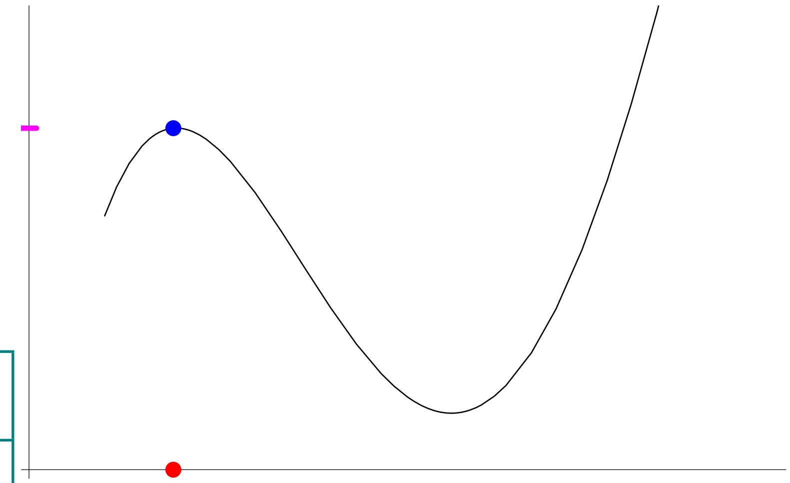

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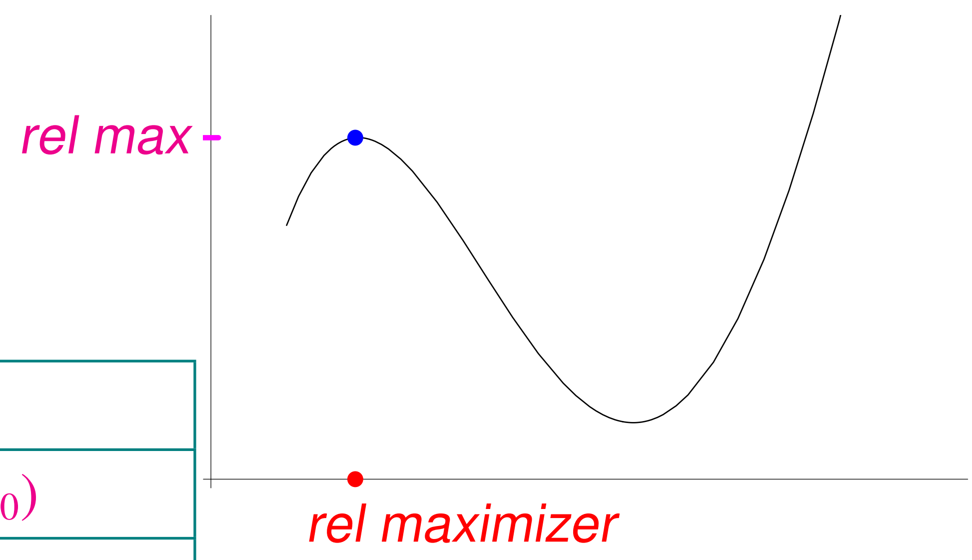

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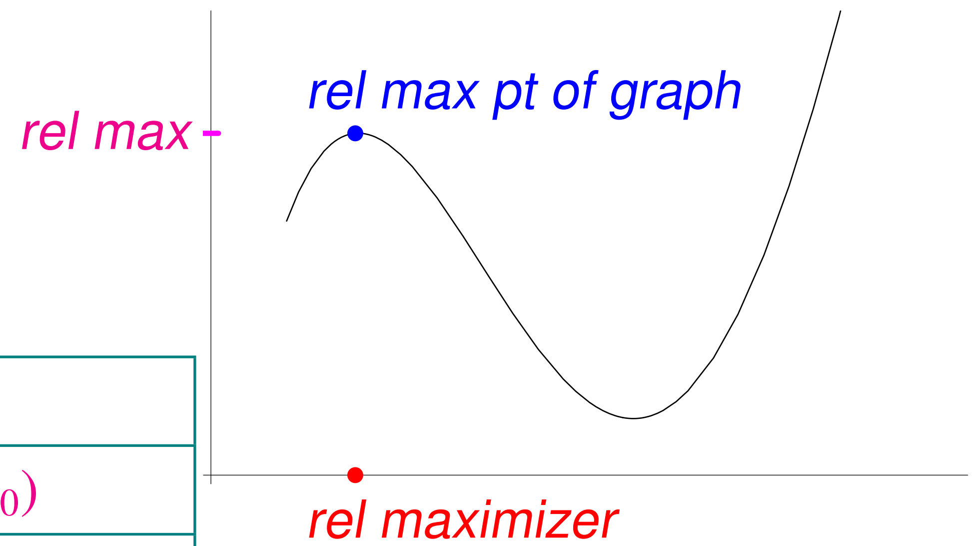

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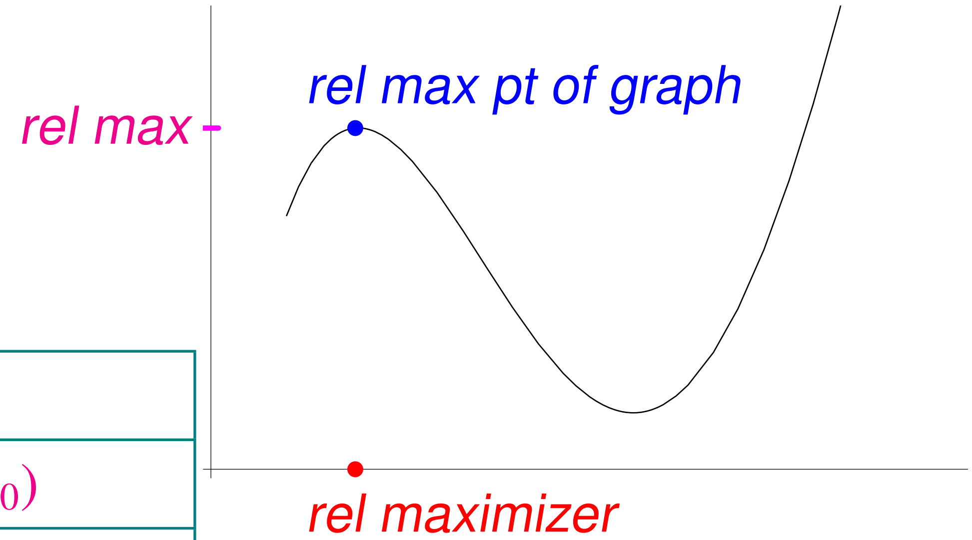

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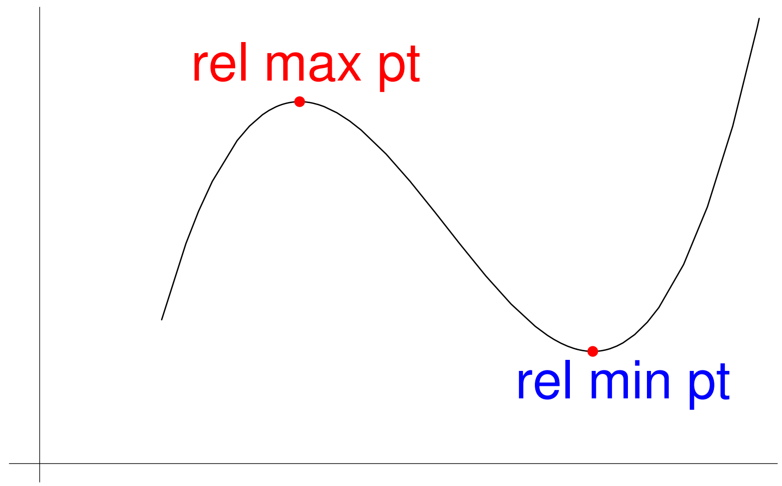
Following figure shows graph of a function.



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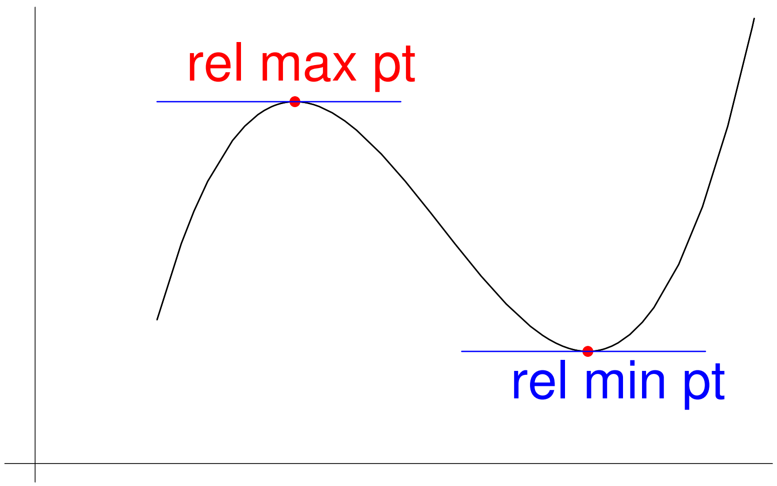

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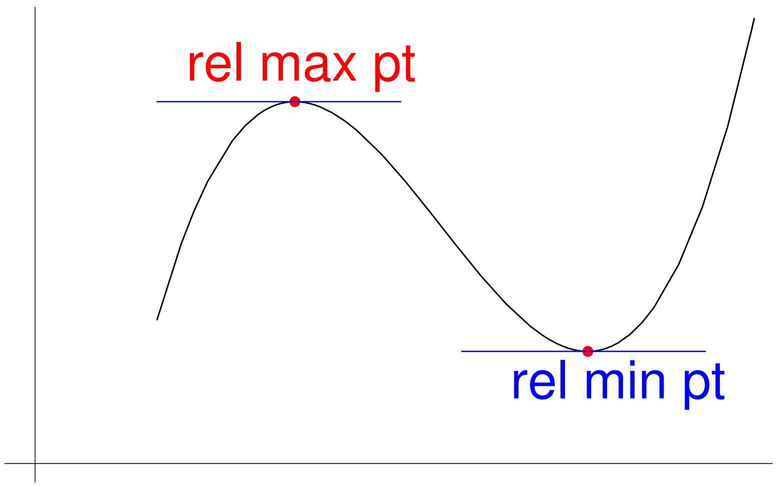

Note *Tangents at the relative extremum points are horizontal.*

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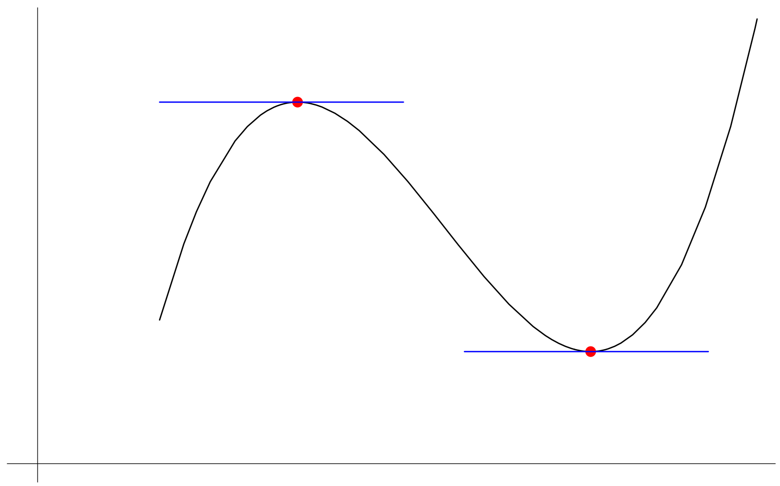

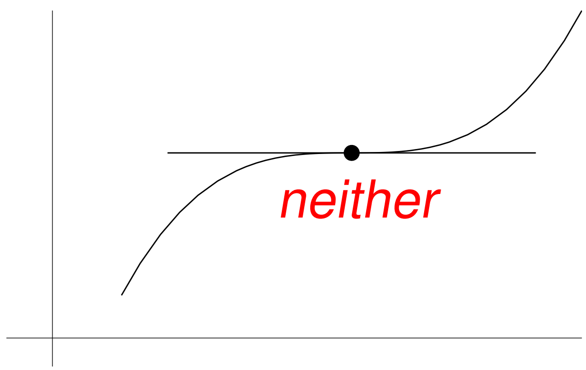

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- If $f'(x)$ *does not change sign* as x increases through x_0, then x_0 is neither a local maximizer nor local minimizer.

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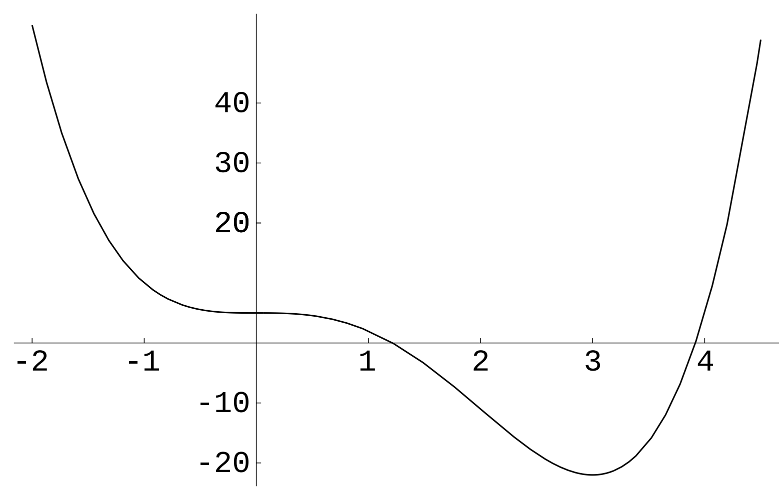
g	↘	↘	↗



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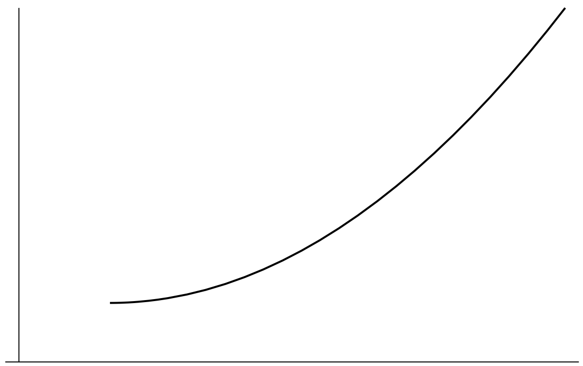

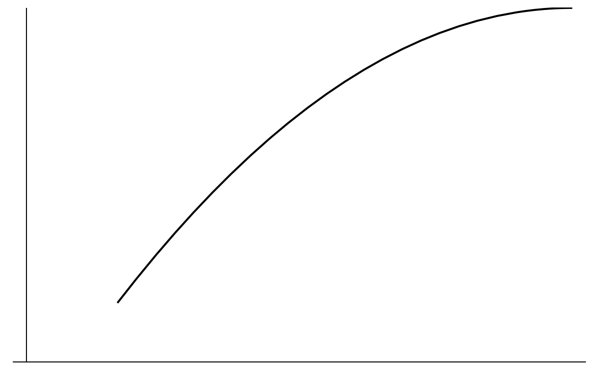

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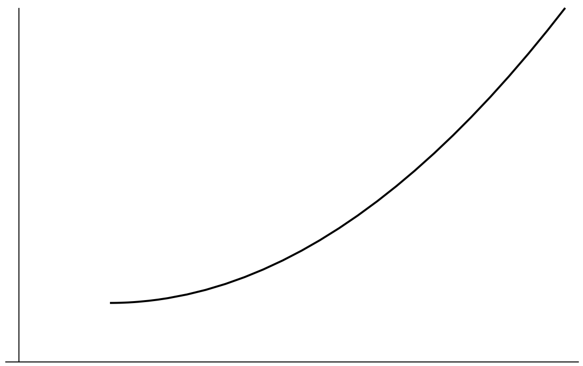

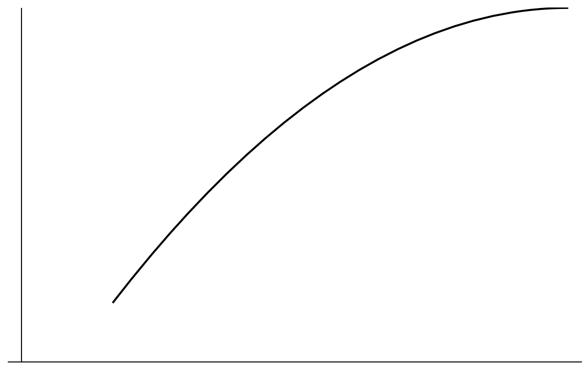

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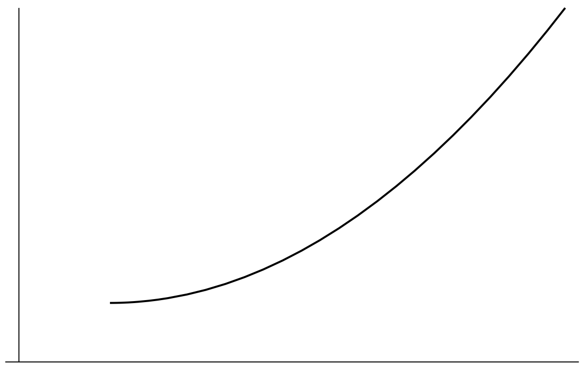

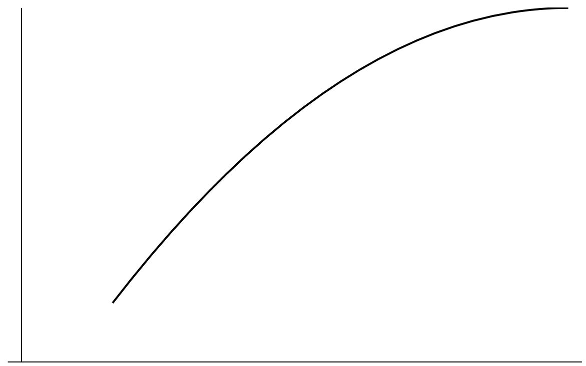

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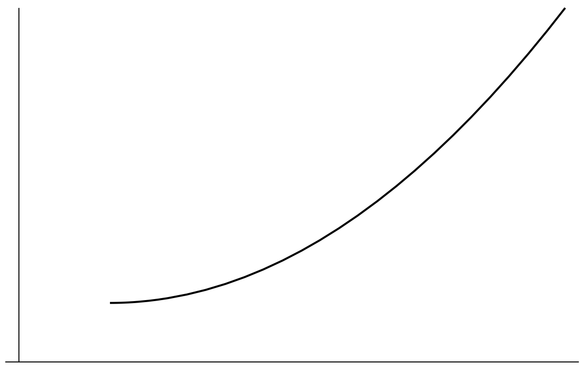

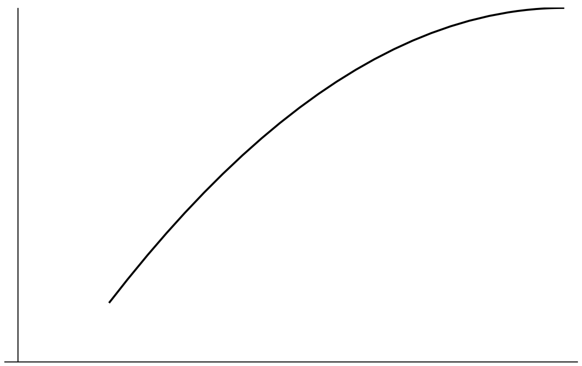

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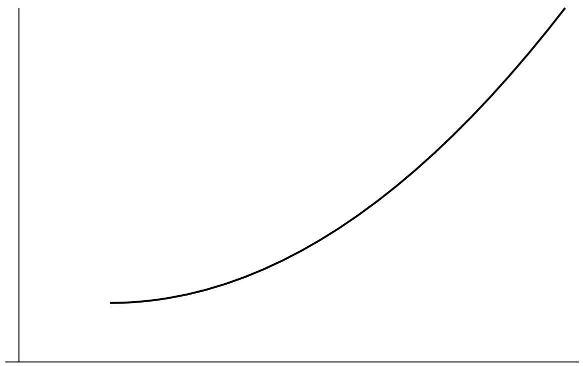

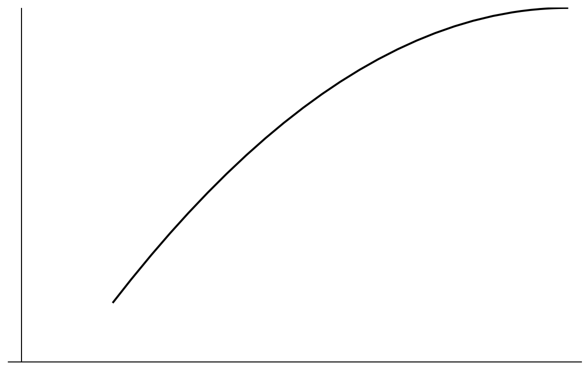

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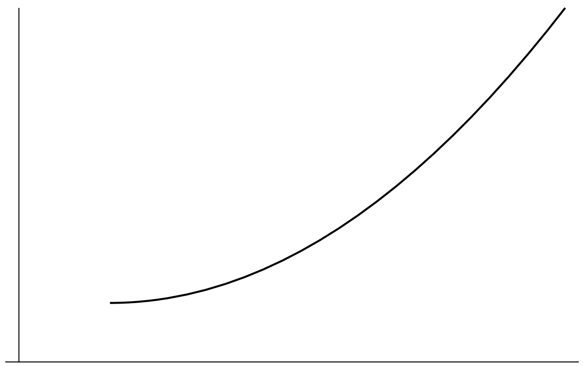

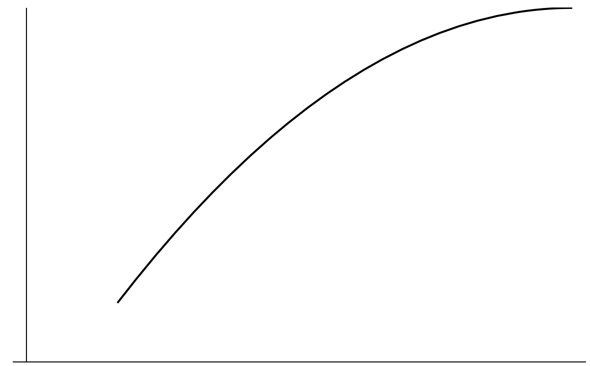

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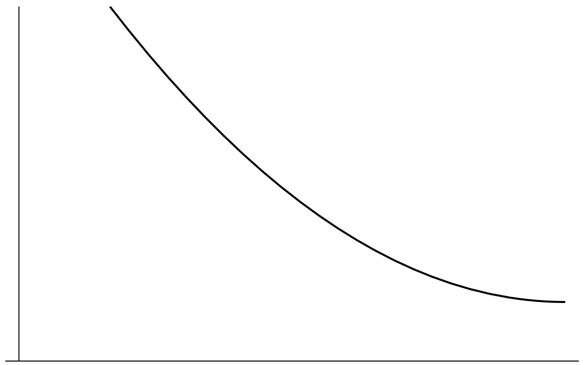

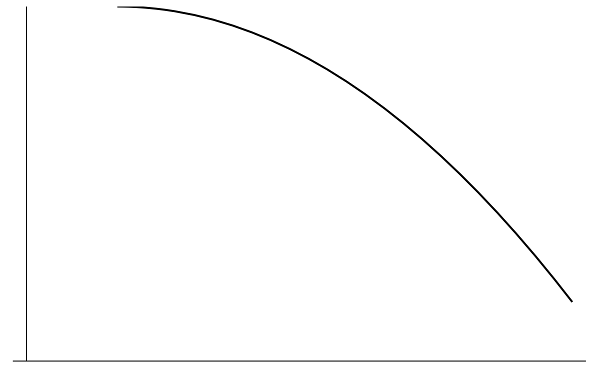

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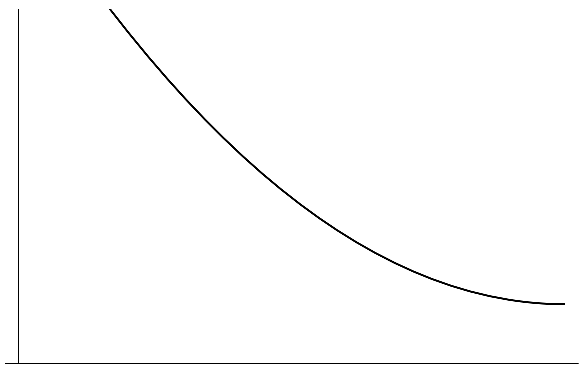

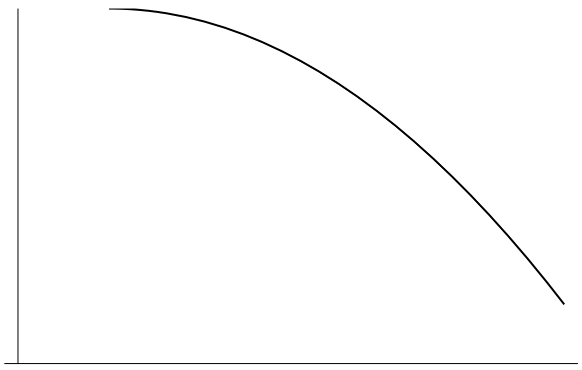

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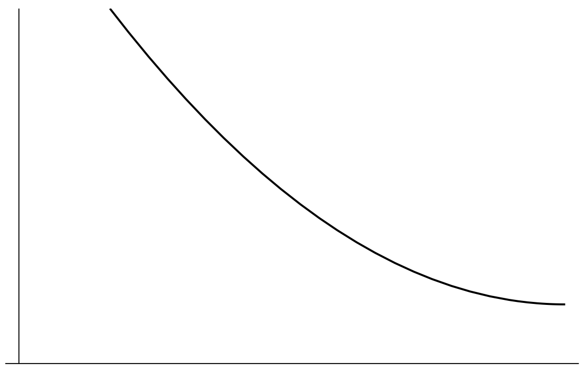

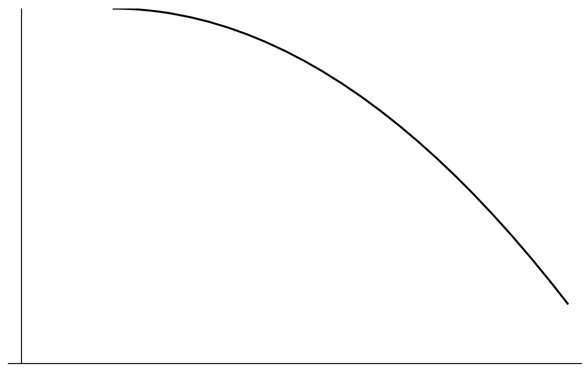

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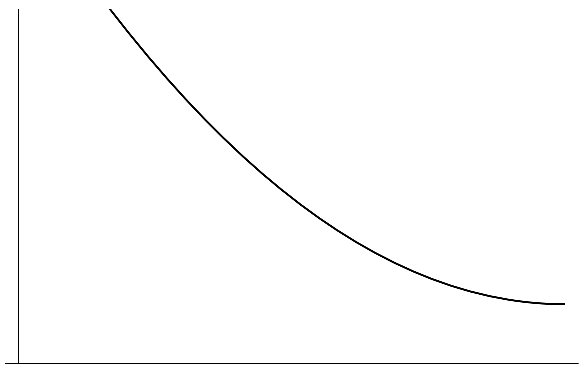

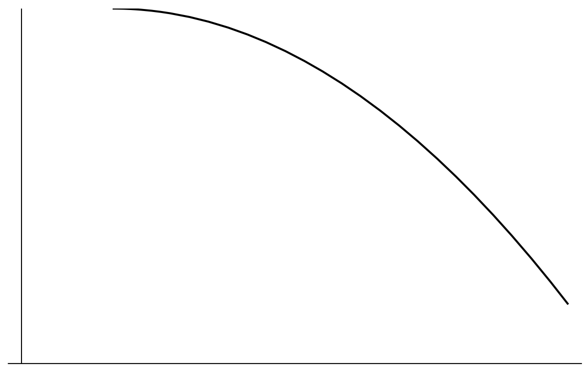

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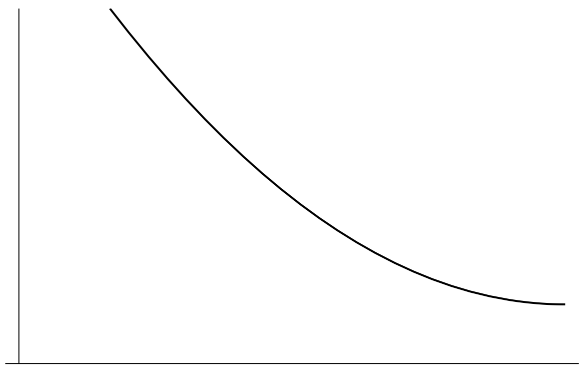

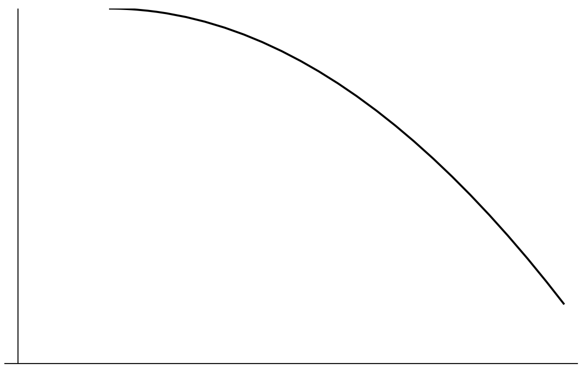

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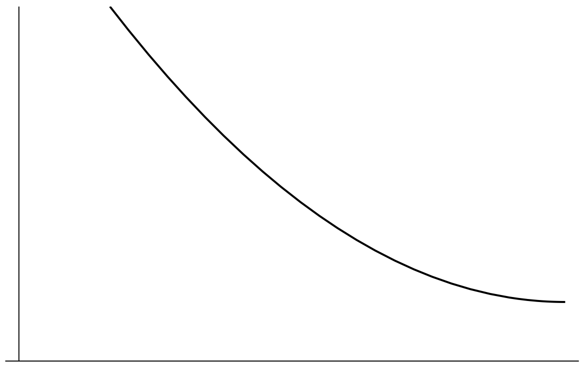

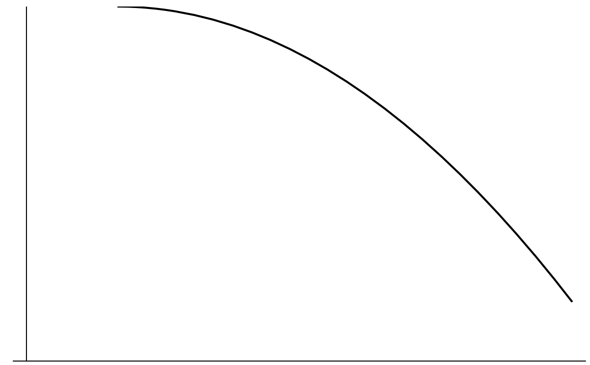

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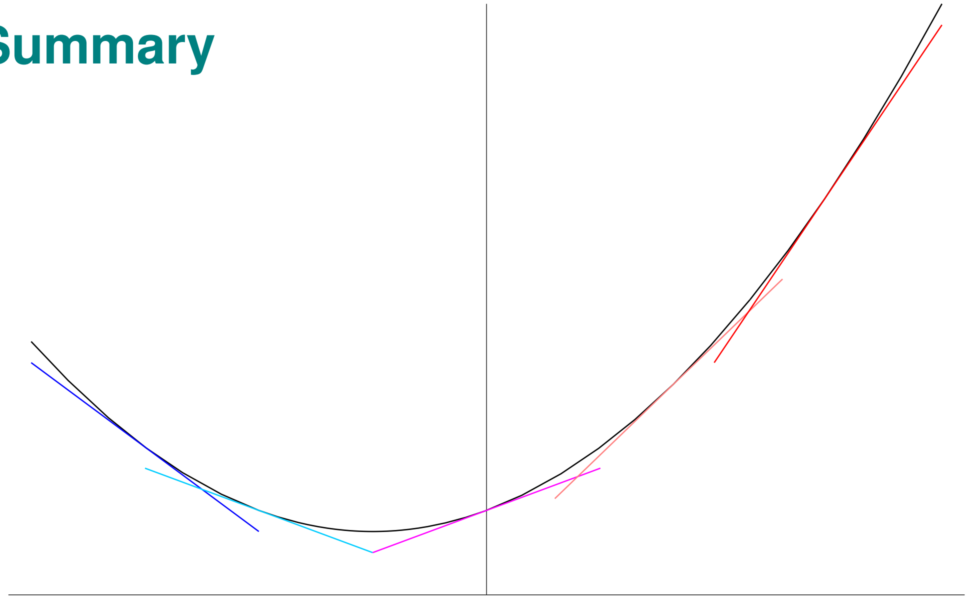

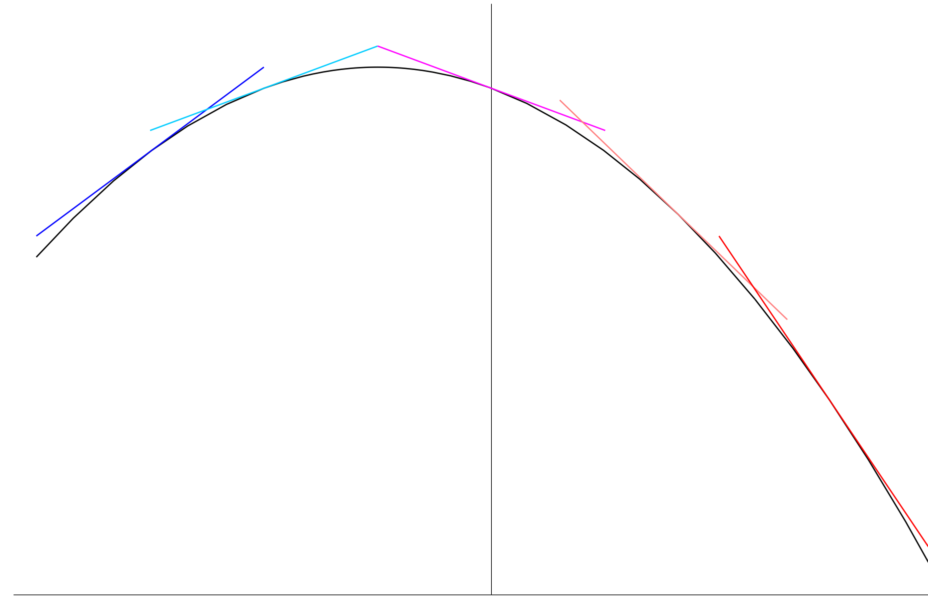

- bending up
- slope increasing

- bending down
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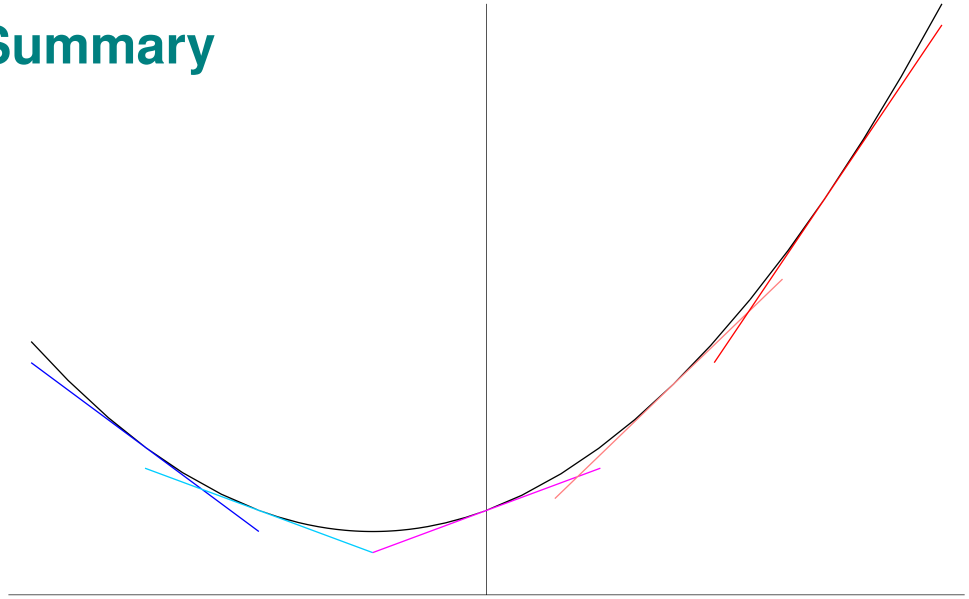

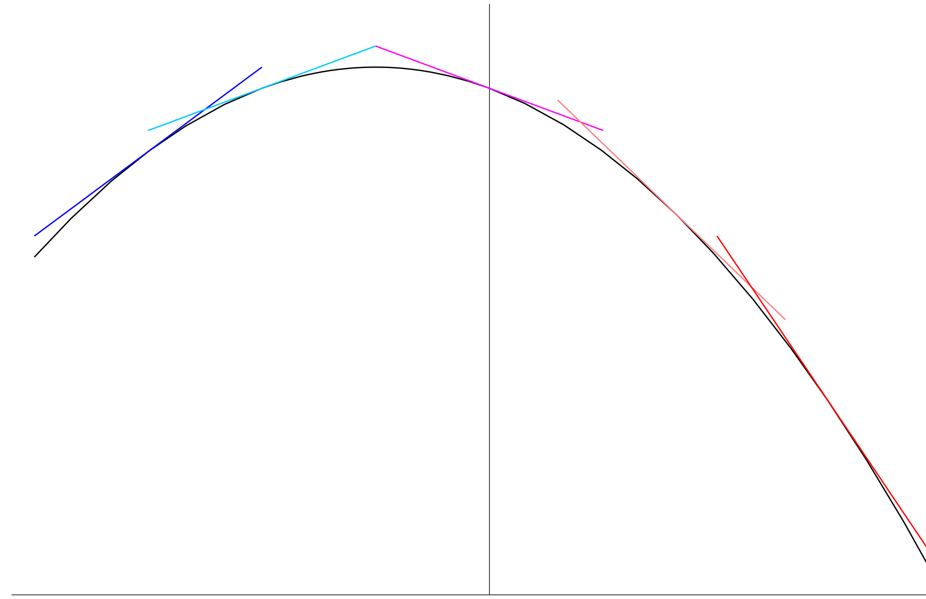

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Remark A graph is

- bending up means the *graph is above the tangent lines*;

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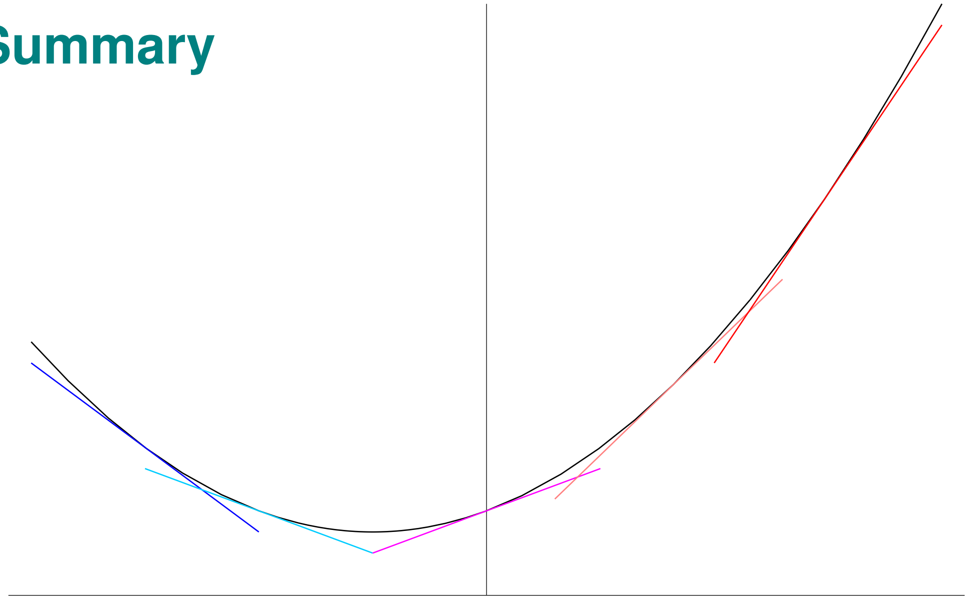

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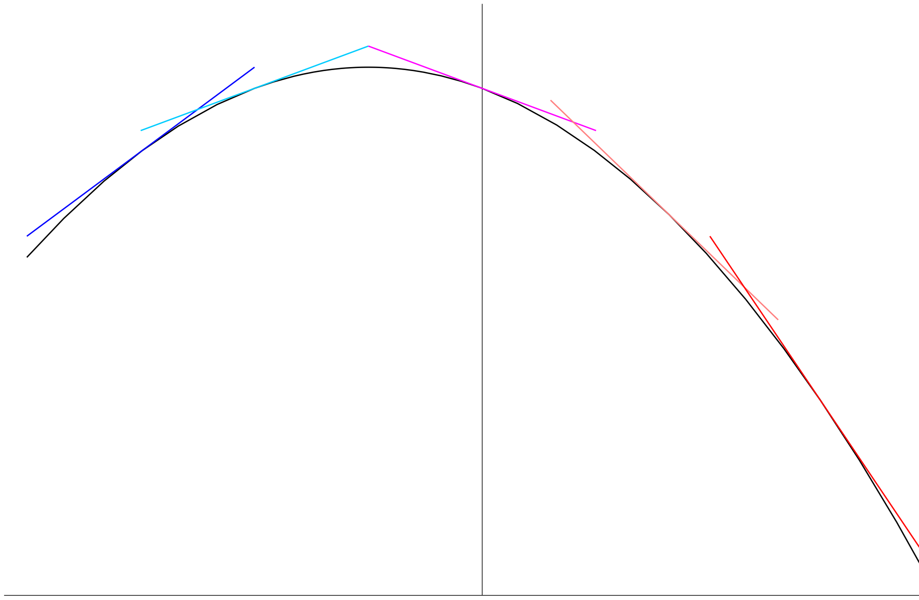
- slope increasing



- bending down
- slope decreasing

Remark A graph is

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- bending down means the graph is *below the tangent lines*.

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- *strictly convex* on an interval I if f' is increasing on I

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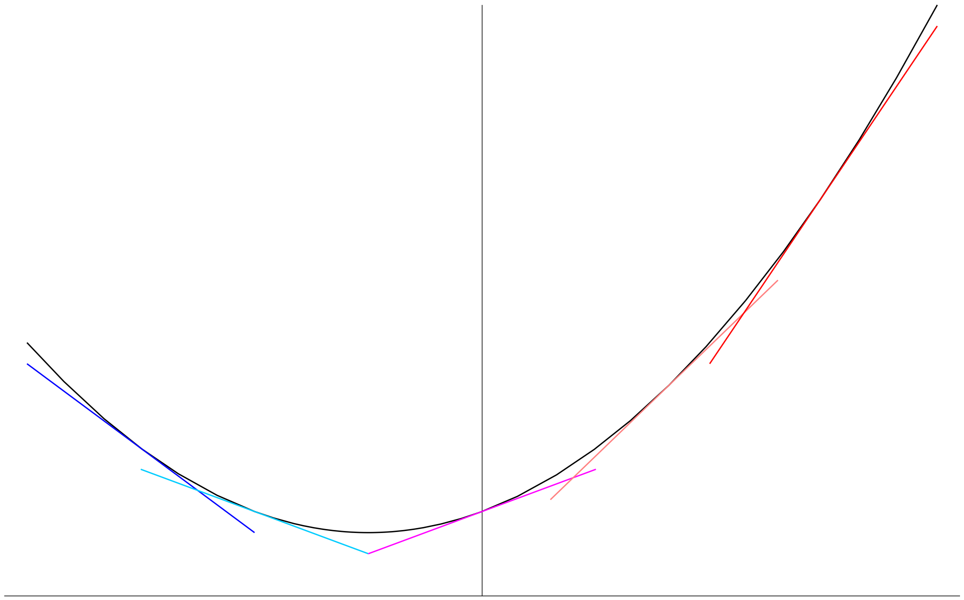

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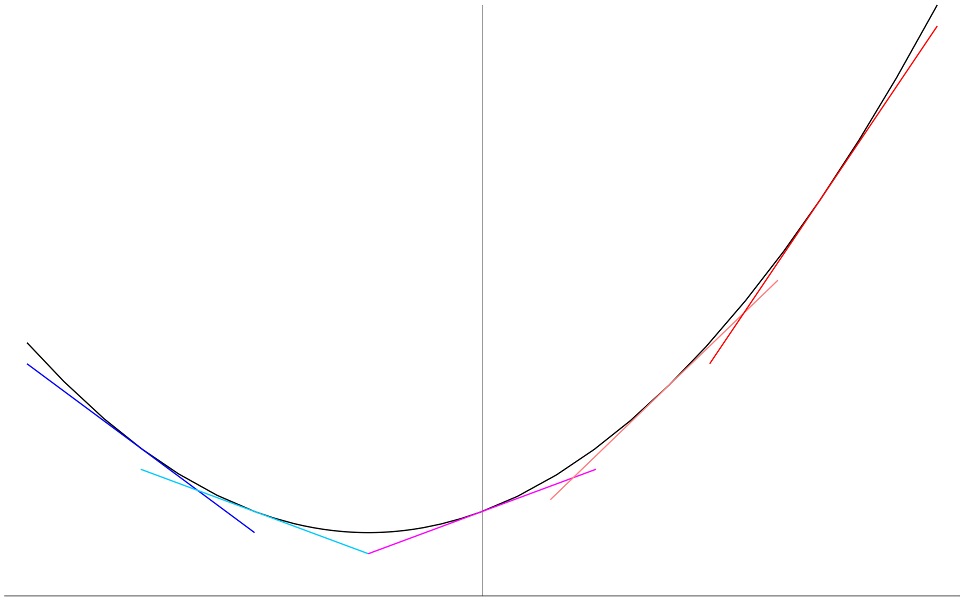

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Terminology For simplicity, *strictly convex* is called *convex* etc.

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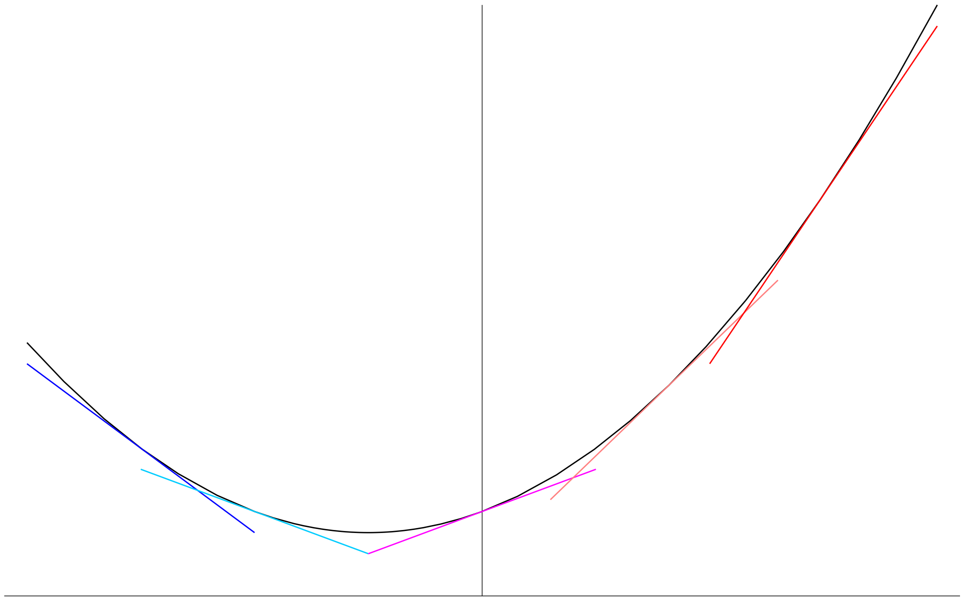

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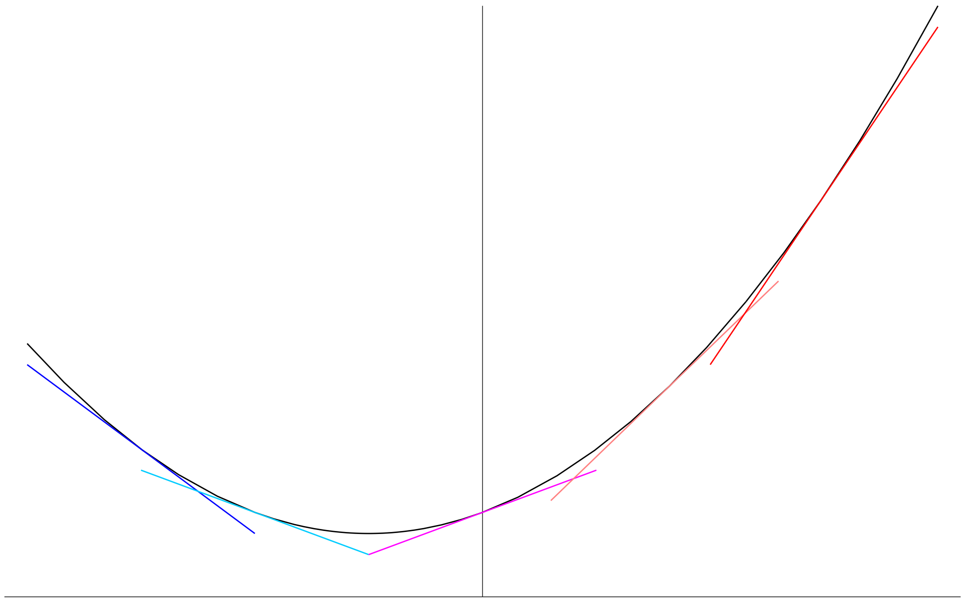

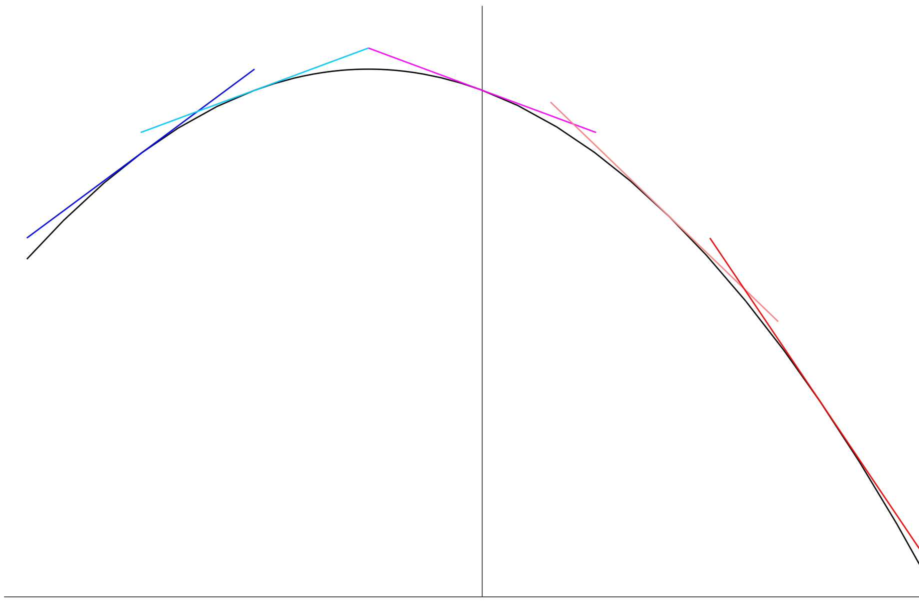

- bending up
- slope increasing *(f is convex)*

- bending down
- slope decreasing *(f is concave)*