Consider the curve given by

$$y = f(x)$$

Let $P = \big(x_1, f(x_1)\big)$ be a point belonging to the curve.

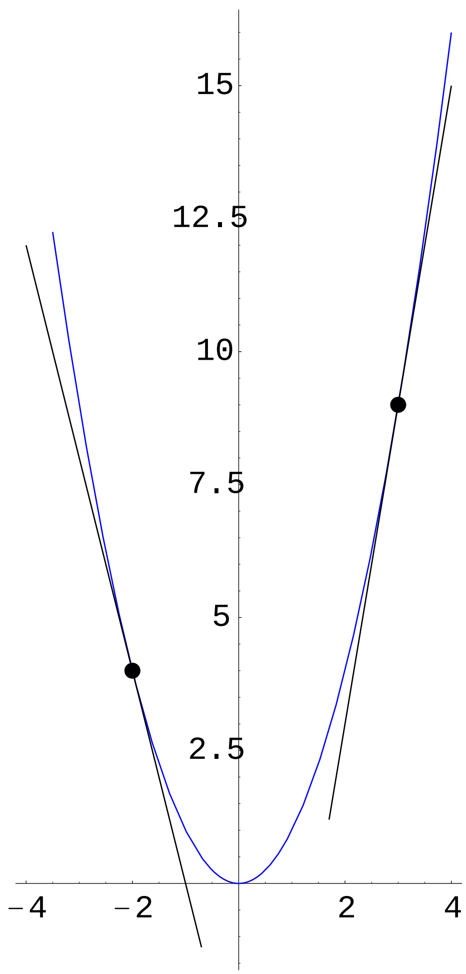
Formula The slope of the curve at P is

$$\lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h} \tag{1}$$

provided that the limit exists (two-sided limit).

Exercise Find the slope of the curve $y = x^2$

- at the point $(3, 9)$;
- at the point $(-2, 4)$.



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Definition The *derivative* of a function f is the function denoted by f' and defined by

$$\boxed{f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}}$$

(provided that this limit exists).

Example For each of the following f, find its derivative

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(2) $f(x) = x^3$

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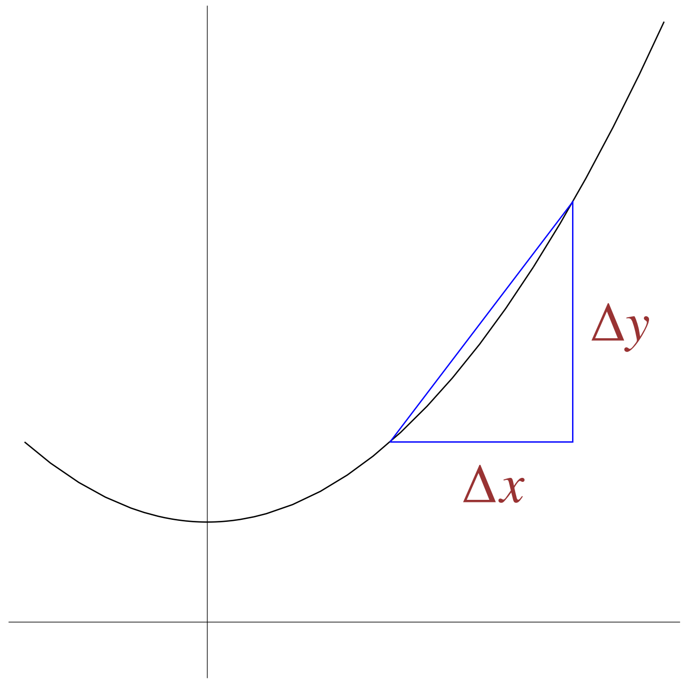

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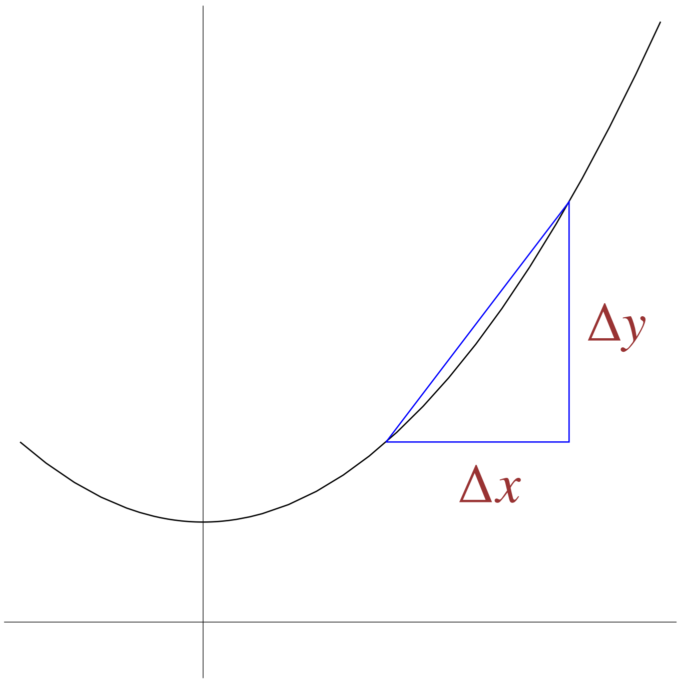

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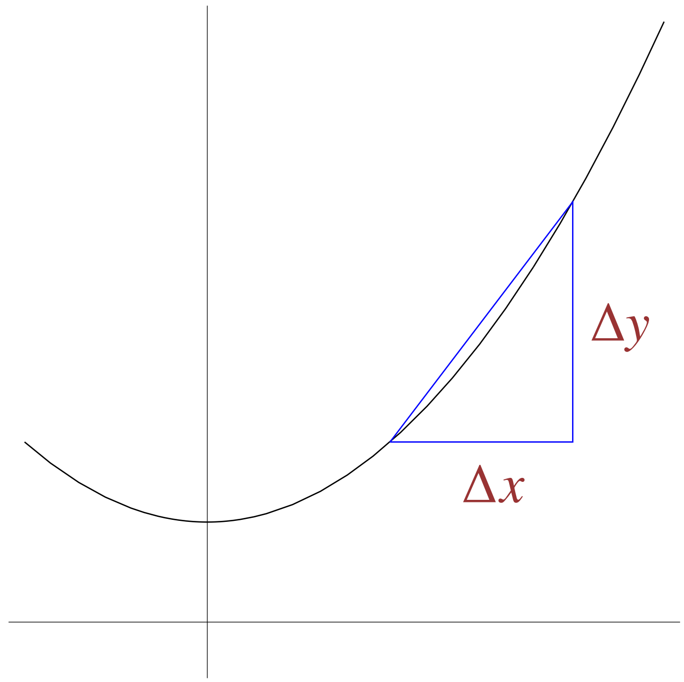

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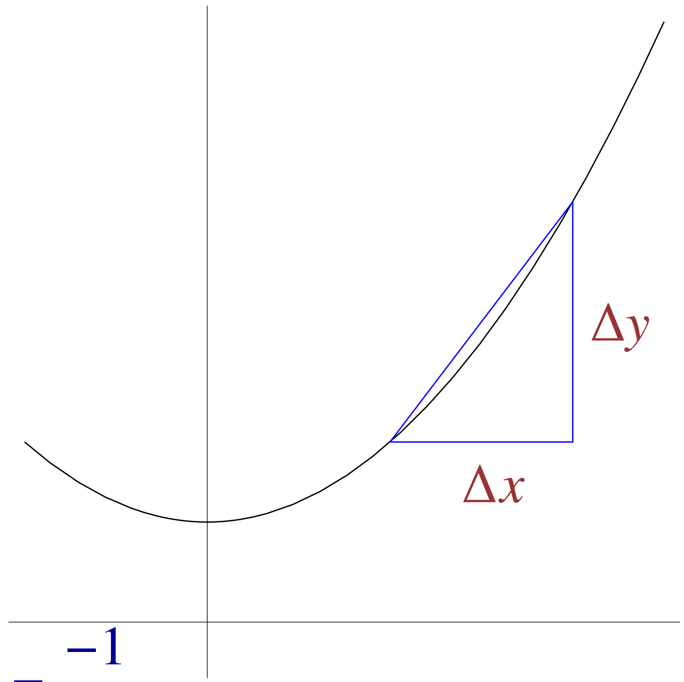

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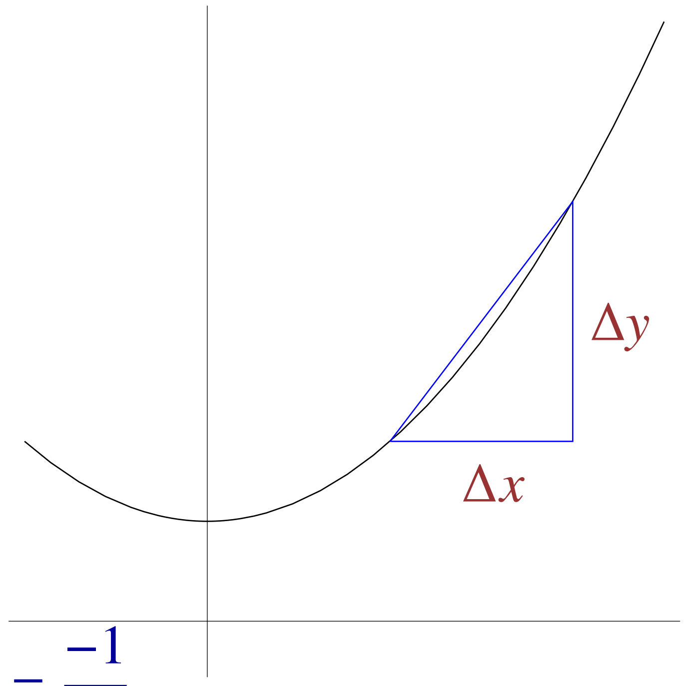

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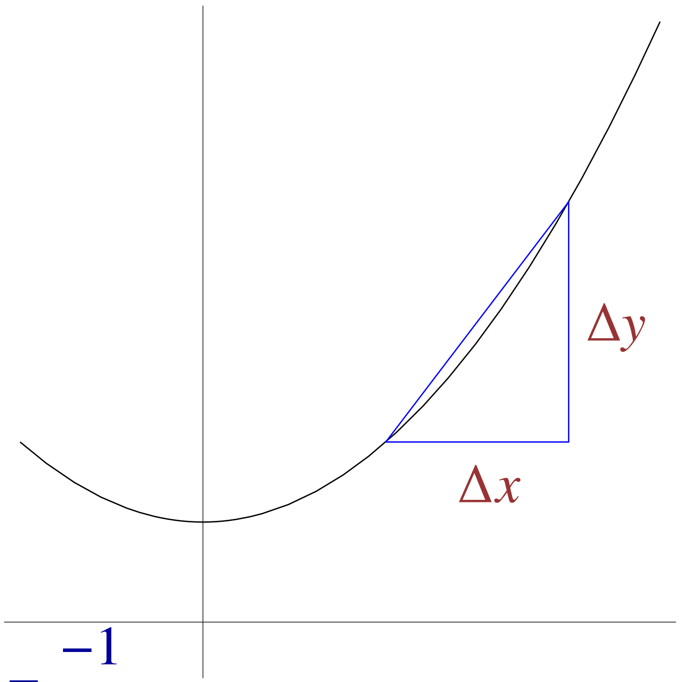

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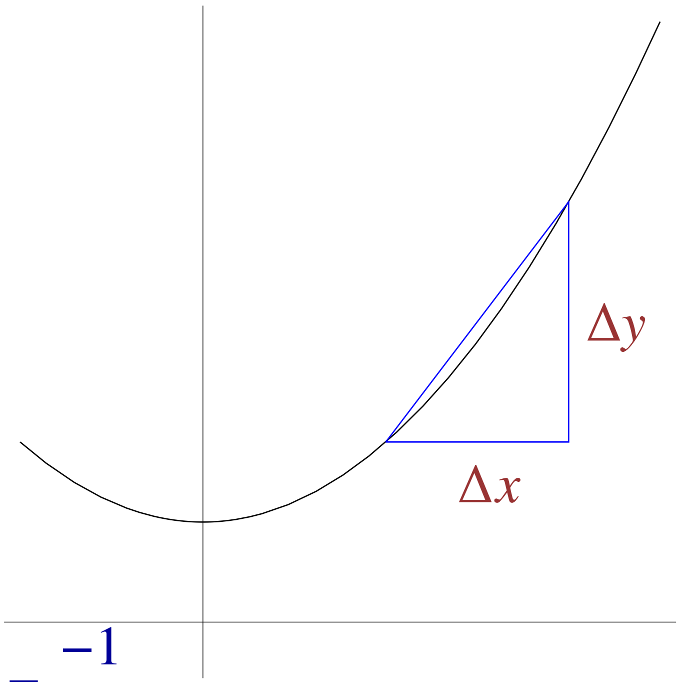

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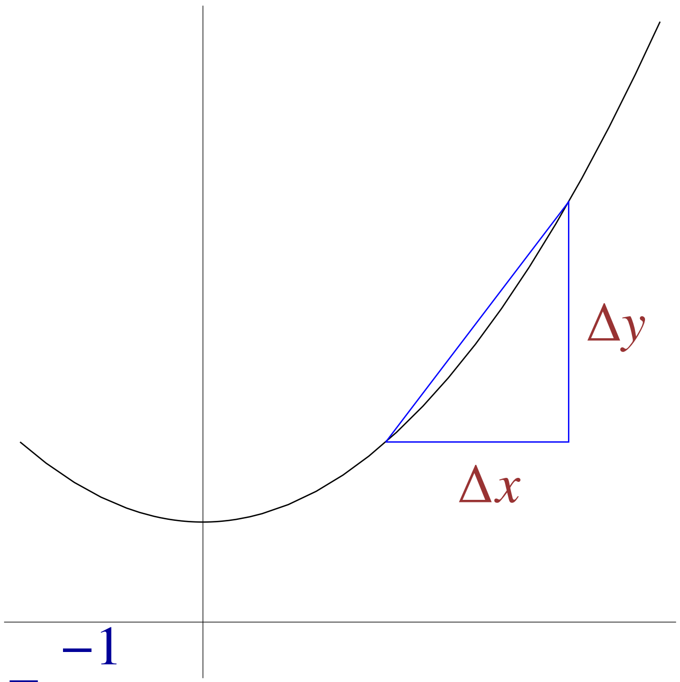

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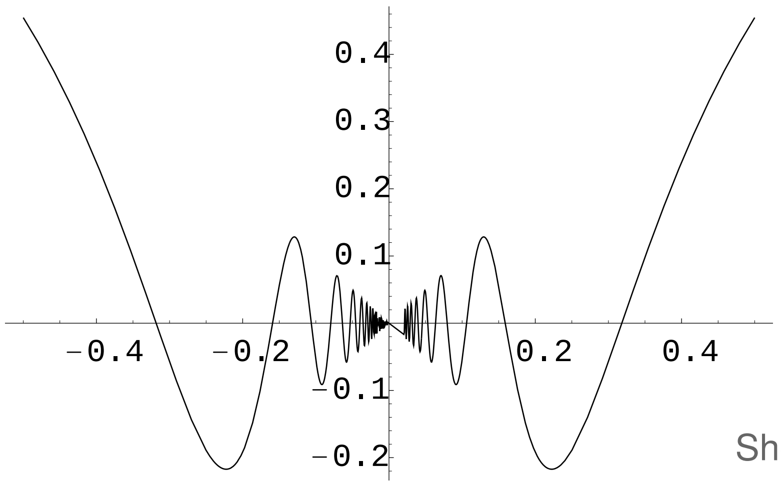

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Rule 1: If $f(x) = c$ is a constant function, then

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$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{c - c}{h} \\ &= \lim_{h\to 0} \frac{0}{h} \\ &= \lim_{h\to 0} 0 \\ &= 0 \end{aligned}$$

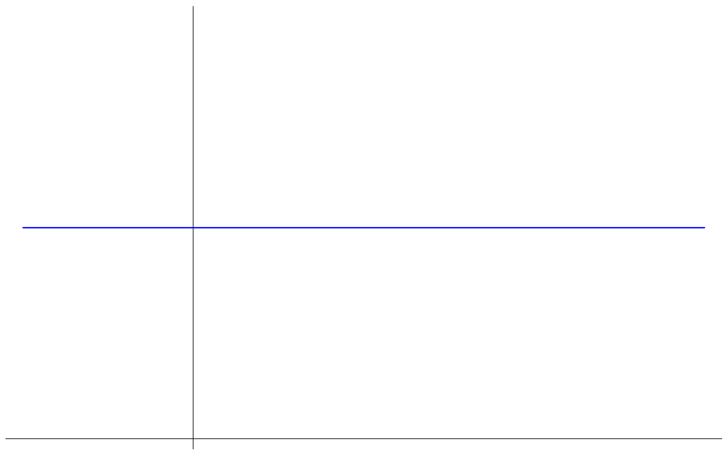
Rules for Differentiation

Rule 1: If $f(x) = c$ is a constant function, then

$$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}c = 0$$

Proof

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{c - c}{h} \\ &= \lim_{h\to 0} \frac{0}{h} \\ &= \lim_{h\to 0} 0 \\ &= 0 \end{aligned}$$



Geometrical meaning Graph of constant function is a horizontal line.

Slope at every point on the line is 0.

Rule 2: Let $f(x) = x$ be the identity function. Then

$$\boxed{f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}x = 1}$$

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Proof

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Rule 2: Let $f(x) = x$ be the identity function. Then

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Proof

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0}\frac{x+h-x}{h} \end{aligned}$$

Rule 2: Let $f(x) = x$ be the identity function. Then

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Proof

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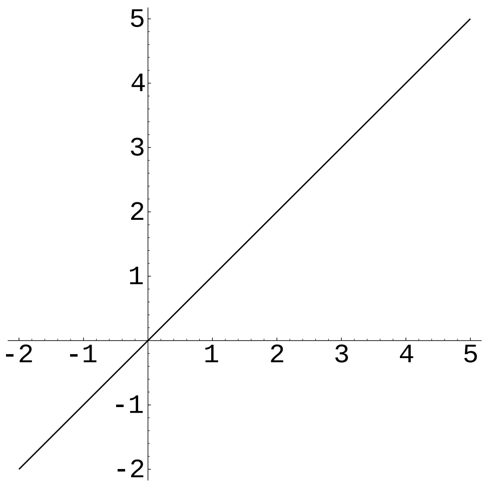

Geometrical meaning The line $y = x$ has slope equal to 1 *(slope at every point* $= 1$*)*.

$$\frac{d}{dx}x^2 = 2x \qquad \frac{d}{dx}\frac{1}{x} = \frac{-1}{x^2}$$

$$\frac{d}{dx}x^3 = 3x^2 \qquad \frac{d}{dx}\frac{1}{x^2} = \frac{-2}{x^3}$$

Rule 3: (Power Rule)

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Rule 3: (Power Rule)

$$\frac{d}{dx}x^r =$$

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Rule 3: (Power Rule) $$\frac{d}{dx}x^r = rx^{r-1}$$

provided that x^{r-1} is defined, where r is a fixed real number.

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Rule 3: (Power Rule) $\boxed{\frac{\mathrm{d}}{\mathrm{d}x}x^r = rx^{r-1}}$

provided that x^{r-1} is defined, where r is a fixed real number.

Proof *For the case where $r = n$ is a positive integer (deferred to next lecture)*

The general case can be proved using logarithmic differentiation (discussed in Chapter 9)

Notation $\frac{\mathrm{d}}{\mathrm{d}x}$ as a function or operator

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Example $y = x^2, \qquad \frac{\mathrm{d}y}{\mathrm{d}x} = 2x$

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Notation $\frac{\mathrm{d}}{\mathrm{d}x}$ as a function or operator

Example

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$$y = x^3, \qquad \frac{\mathrm{d}}{\mathrm{d}x}(x^3) = 3x^2$$

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Example $y = x^2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x$ $\frac{\mathrm{d}}{\mathrm{d}x}(x^2) = 2x$

$y = x^3$, $\frac{\mathrm{d}}{\mathrm{d}x}(x^3) = 3x^2$

Compare *square root function* $9 \xrightarrow{\sqrt{\ }} 3$

differentiation operator $x^2 \xrightarrow{\frac{\mathrm{d}}{\mathrm{d}x}} 2x$

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Function notation $\sqrt{}(9) = 3$ taking principle square root of 9, we get 3

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Operator notation $\frac{\mathrm{d}}{\mathrm{d}x}(x^2) = 2x$ differentiating x^2, we get $2x$

Notation $\dfrac{\mathrm{d}}{\mathrm{d}x}$ as a function or operator

Example $y = x^2,\quad \dfrac{\mathrm{d}y}{\mathrm{d}x} = 2x \quad \dfrac{\mathrm{d}}{\mathrm{d}x}(x^2) = 2x$

$y = x^3,\quad \dfrac{\mathrm{d}}{\mathrm{d}x}(x^3) = 3x^2$

Compare *square root function* $9 \xrightarrow{\sqrt{\ }} 3$

Function notation $\sqrt{\ }(9) = 3$ taking principle square root of 9, we get 3

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differentiation operator $x^2 \xrightarrow{\frac{\mathrm{d}}{\mathrm{d}x}} 2x$

Operator notation $\dfrac{\mathrm{d}}{\mathrm{d}x}(x^2) = 2x$ differentiating x^2, we get $2x$

Compact notation $\dfrac{\mathrm{d}x^2}{\mathrm{d}x} = 2x$ the derivative of x^2 is $2x$

Example

(1) $\dfrac{\mathrm{d}}{\mathrm{d}x}98765 =$

Example

(1) $\frac{\mathrm{d}}{\mathrm{d}x}98765 = 0$ Rule 1

Example

(1) $$\frac{\mathrm{d}}{\mathrm{d}x}98765 = 0$$ Rule 1

(2) $$\frac{\mathrm{d}}{\mathrm{d}x}x^{123} =$$

Example

(1) $\frac{\mathrm{d}}{\mathrm{d}x}98765 = 0$ Rule 1

(2) $\frac{\mathrm{d}}{\mathrm{d}x}x^{123} = 123x^{123-1}$ Power Rule

Example

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$= 123x^{122}$ Simplify answer

Example

(1) $\frac{\mathrm{d}}{\mathrm{d}x} 98765 \quad = \quad 0$ Rule 1

(2) $\frac{\mathrm{d}}{\mathrm{d}x} x^{123} \quad = \quad 123x^{123-1}$ Power Rule

$= \quad 123x^{122}$ Simplify answer

(3) $\frac{\mathrm{d}}{\mathrm{d}x} \frac{1}{\sqrt{x}} \quad =$

Example

(1) $$\frac{\mathrm{d}}{\mathrm{d}x}98765 = 0$$ Rule 1

(2) $$\frac{\mathrm{d}}{\mathrm{d}x}x^{123} = 123x^{123-1}$$ Power Rule

$$= 123x^{122}$$ Simplify answer

(3) $$\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{\sqrt{x}} = \frac{\mathrm{d}}{\mathrm{d}x}x^{-\frac{1}{2}}$$ Rewrite function

Example

(1) $$\frac{d}{dx}98765 = 0$$ Rule 1

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$$= -\frac{1}{2} \cdot x^{-\frac{1}{2}-1}$$ Power Rule

Example

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$$= -\frac{1}{2} \cdot x^{-\frac{1}{2}-1}$$ Power Rule

$$= -\frac{1}{2}x^{-\frac{3}{2}}$$ Simplify answer

Rule 4: If f is differentiable (at x), then cf is differentiable (at x) (where c is a constant) and

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}x}\big(cf(x)\big) = c \cdot \frac{\mathrm{d}}{\mathrm{d}x} f(x)}$$

Proof

Rule 4: If f is differentiable (at x), then cf is differentiable (at x) (where c is a constant) and

$$\frac{\mathrm{d}}{\mathrm{d}x}\big(cf(x)\big) = c \cdot \frac{\mathrm{d}}{\mathrm{d}x}f(x)$$

Proof

$$\frac{\mathrm{d}}{\mathrm{d}x}\big(cf(x)\big) = \lim_{h\to 0}\frac{cf(x+h) - cf(x)}{h}$$

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Proof

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x}\big(cf(x)\big) &= \lim_{h \to 0} \frac{cf(x+h) - cf(x)}{h} \\ &= \lim_{h \to 0} \left(c \cdot \frac{f(x+h) - f(x)}{h} \right) \end{aligned}$$

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Proof

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&= \lim_{h\to 0} \left(c \cdot \frac{f(x+h) - f(x)}{h} \right) \\
&= c \cdot \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}
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Rule 4: If f is differentiable (at x), then cf is differentiable (at x) (where c is a constant) and

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Proof

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\big(cf(x)\big) &= \lim_{h\to 0}\frac{cf(x+h) - cf(x)}{h} \\
&= \lim_{h\to 0}\left(c \cdot \frac{f(x+h) - f(x)}{h}\right) \\
&= c \cdot \lim_{h\to 0}\frac{f(x+h) - f(x)}{h} \\
&= c \cdot f'(x)
\end{aligned}$$

Example

(1) $$\frac{\mathrm{d}}{\mathrm{d}x}\left(7x^{\sqrt{2}}\right) =$$

Example

(1) $$\frac{\mathrm{d}}{\mathrm{d}x}\left(7x^{\sqrt{2}}\right) = 7 \cdot \frac{\mathrm{d}}{\mathrm{d}x}x^{\sqrt{2}}$$ Rule 4

Example

$$(1)\quad \frac{\mathrm{d}}{\mathrm{d}x}\left(7x^{\sqrt{2}}\right) = 7 \cdot \frac{\mathrm{d}}{\mathrm{d}x}x^{\sqrt{2}} \qquad \text{Rule 4}$$

$$= 7 \cdot \sqrt{2}x^{\sqrt{2}-1} \qquad \text{Power Rule}$$

Example

(1) $$\frac{d}{dx}\left(7x^{\sqrt{2}}\right) = 7 \cdot \frac{d}{dx} x^{\sqrt{2}}$$ Rule 4

$$= 7 \cdot \sqrt{2} x^{\sqrt{2}-1}$$ Power Rule

$$= 7\sqrt{2}\, x^{\sqrt{2}-1}$$ Rewrite answer

Example

(1) $\dfrac{d}{dx}\left(7x^{\sqrt{2}}\right) = 7 \cdot \dfrac{d}{dx}x^{\sqrt{2}}$ Rule 4

$= 7 \cdot \sqrt{2}x^{\sqrt{2}-1}$ Power Rule

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(2) $\dfrac{d}{dx}\left(\dfrac{\pi}{x}\right) =$

Example

(1) $\frac{d}{dx}\left(7x^{\sqrt{2}}\right) = 7 \cdot \frac{d}{dx}x^{\sqrt{2}}$ Rule 4

$= 7 \cdot \sqrt{2}x^{\sqrt{2}-1}$ Power Rule

$= 7\sqrt{2}\,x^{\sqrt{2}-1}$ Rewrite answer

(2) $\frac{d}{dx}\left(\frac{\pi}{x}\right) = \frac{d}{dx}\left(\pi \cdot x^{-1}\right)$ Rewrite function

Example

(1) $$\frac{d}{dx}\left(7x^{\sqrt{2}}\right) = 7 \cdot \frac{d}{dx}x^{\sqrt{2}}$$ Rule 4

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(2) $$\frac{d}{dx}\left(\frac{\pi}{x}\right) = \frac{d}{dx}\left(\pi \cdot x^{-1}\right)$$ Rewrite function

$$= \pi \cdot \frac{d}{dx}x^{-1}$$ Rule 4

Example

(1) $$\frac{\mathrm{d}}{\mathrm{d}x}\left(7x^{\sqrt{2}}\right) = 7 \cdot \frac{\mathrm{d}}{\mathrm{d}x}x^{\sqrt{2}}$$ Rule 4

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(2) $$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\pi}{x}\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\pi \cdot x^{-1}\right)$$ Rewrite function

$$= \pi \cdot \frac{\mathrm{d}}{\mathrm{d}x}x^{-1}$$ Rule 4

$$= \pi \cdot (-1)\,x^{-2}$$ Power Rule

Example

(1) $$\frac{d}{dx}\left(7x^{\sqrt{2}}\right) = 7 \cdot \frac{d}{dx}x^{\sqrt{2}}$$ Rule 4

$$= 7 \cdot \sqrt{2}x^{\sqrt{2}-1}$$ Power Rule

$$= 7\sqrt{2}\,x^{\sqrt{2}-1}$$ Rewrite answer

(2) $$\frac{d}{dx}\left(\frac{\pi}{x}\right) = \frac{d}{dx}\left(\pi \cdot x^{-1}\right)$$ Rewrite function

$$= \pi \cdot \frac{d}{dx}x^{-1}$$ Rule 4

$$= \pi \cdot (-1)\,x^{-2}$$ Power Rule

$$= \frac{-\pi}{x^2}$$ Rewrite answer

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Rule 5: If f and g are differentiable (at x), then $f+g$ and $f-g$ are differentiable (at x) and

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Notation **Sum of Functions**

Let $f(x) = \sin x$ and $g(x) = \sqrt{x}$. Put $F(x) = f(x) + g(x)$.

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- $F(x) = \sin x + \sqrt{x}$
- $F(x + h) =$

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- $F(x) = \sin x + \sqrt{x}$
- $F(x+h) = \sin(x+h) + \sqrt{x+h}$

Proof (for sum) Put $F(x) = f(x) + g(x)$

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big(f(x) + g(x)\Big) \quad = \quad \frac{\mathrm{d}}{\mathrm{d}x}F(x)$$

Proof (for sum) Put $F(x) = f(x) + g(x)$

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big(f(x) + g(x)\Big) = \frac{\mathrm{d}}{\mathrm{d}x}F(x)$$

$$= \lim_{h \to 0} \frac{F(x+h) - F(x)}{h}$$

Proof (for sum) Put $F(x) = f(x) + g(x)$

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\Big(f(x) + g(x)\Big) &= \frac{\mathrm{d}}{\mathrm{d}x}F(x) \\
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&= \lim_{h\to 0} \frac{\Big(f(x+h) + g(x+h)\Big) - \Big(f(x) + g(x)\Big)}{h}
\end{aligned}
$$

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$$= \lim_{h\to 0} \frac{\big(f(x+h) + g(x+h)\big) - \big(f(x) + g(x)\big)}{h}$$

$$= \lim_{h\to 0} \left(\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h}\right)$$

Proof (for sum) Put $F(x) = f(x) + g(x)$

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\Big(f(x)+g(x)\Big) &= \frac{\mathrm{d}}{\mathrm{d}x}F(x) \\
&= \lim_{h\to 0}\frac{F(x+h)-F(x)}{h} \\
&= \lim_{h\to 0}\frac{\Big(f(x+h)+g(x+h)\Big)-\Big(f(x)+g(x)\Big)}{h} \\
&= \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}+\frac{g(x+h)-g(x)}{h}\right) \\
&= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}
\end{aligned}
$$

Proof (for sum) Put $F(x) = f(x) + g(x)$

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\big(f(x) + g(x)\big) &= \frac{\mathrm{d}}{\mathrm{d}x}F(x) \\
&= \lim_{h\to 0} \frac{F(x+h) - F(x)}{h} \\
&= \lim_{h\to 0} \frac{\big(f(x+h) + g(x+h)\big) - \big(f(x) + g(x)\big)}{h} \\
&= \lim_{h\to 0} \left(\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right) \\
&= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} \\
&= f'(x) + g'(x)
\end{aligned}
$$

Example Let $f(x) = x^9 - 2\sqrt{x}$. Find $f'(x)$.

Solution

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Solution Differentiate *term by term*

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Solution Differentiate *term by term*

$$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(x^9 - 2\sqrt{x}\right)$$

Use $\frac{\mathrm{d}}{\mathrm{d}x}$ notation, sub. $f(x)$

Example Let $f(x) = x^9 - 2\sqrt{x}$. Find $f'(x)$.

Solution Differentiate *term by term*

$$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(x^9 - 2\sqrt{x}\right)$$

Use $\frac{\mathrm{d}}{\mathrm{d}x}$ notation, sub. $f(x)$

$$= \frac{\mathrm{d}}{\mathrm{d}x}x^9 - \frac{\mathrm{d}}{\mathrm{d}x}\left(2x^{\frac{1}{2}}\right)$$

Rule 5, rewrite 2nd function

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$$= \frac{\mathrm{d}}{\mathrm{d}x}x^9 - \frac{\mathrm{d}}{\mathrm{d}x}\left(2x^{\frac{1}{2}}\right) \qquad \text{Rule 5, rewrite 2nd function}$$

$$= 9x^8 - 2 \cdot \frac{\mathrm{d}}{\mathrm{d}x}x^{\frac{1}{2}} \qquad \text{Power Rule, Rule 4}$$

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	$=$	$\frac{\mathrm{d}}{\mathrm{d}x}x^9 - \frac{\mathrm{d}}{\mathrm{d}x}\left(2x^{\frac{1}{2}}\right)$	Rule 5, rewrite 2nd function
	$=$	$9x^8 - 2 \cdot \frac{\mathrm{d}}{\mathrm{d}x}x^{\frac{1}{2}}$	Power Rule, Rule 4
	$=$	$9x^8 - 2 \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}}$	Power Rule

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Power Rule, Rule 4

$$= 9x^8 - 2 \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}}$$

Power Rule

$$= 9x^8 - x^{-\frac{1}{2}}$$

Simplify answer