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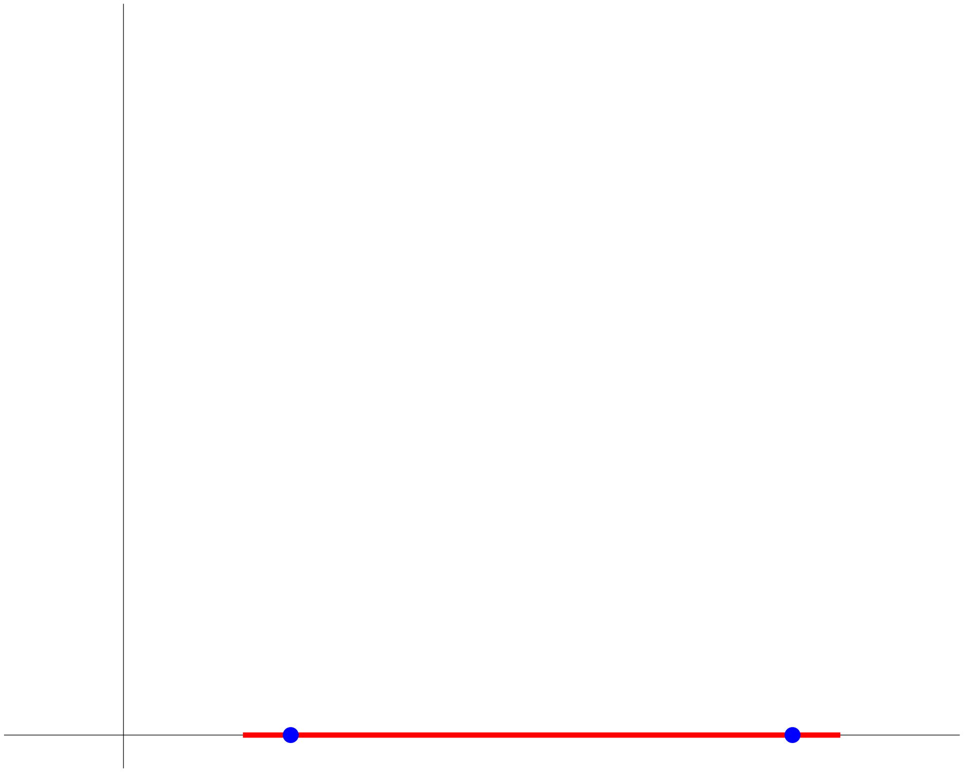

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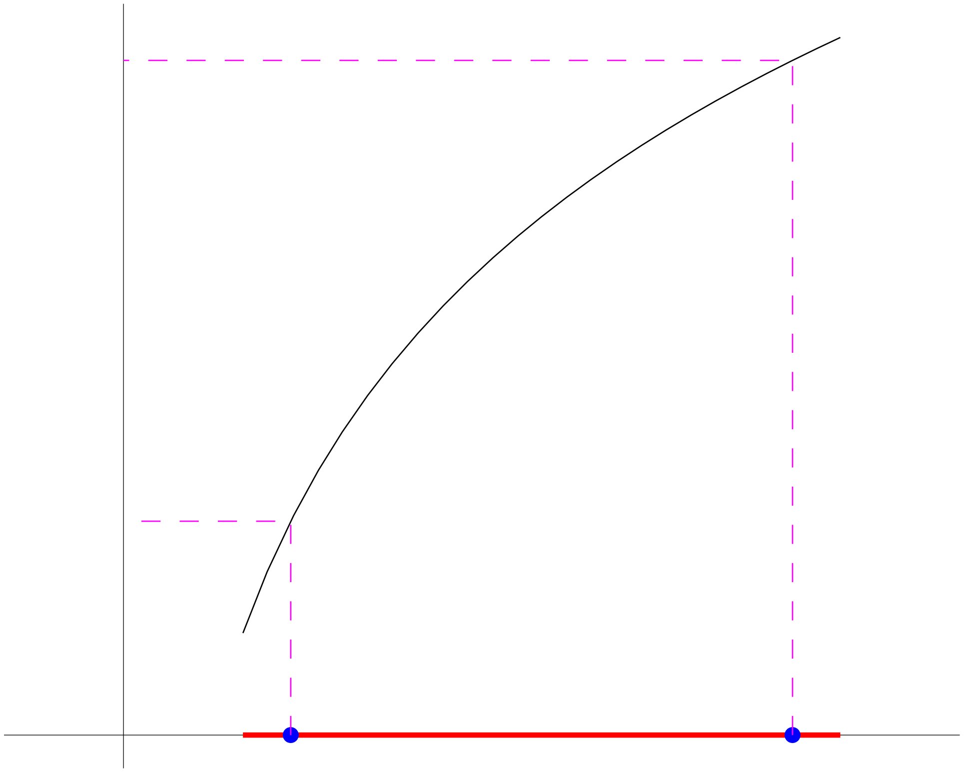

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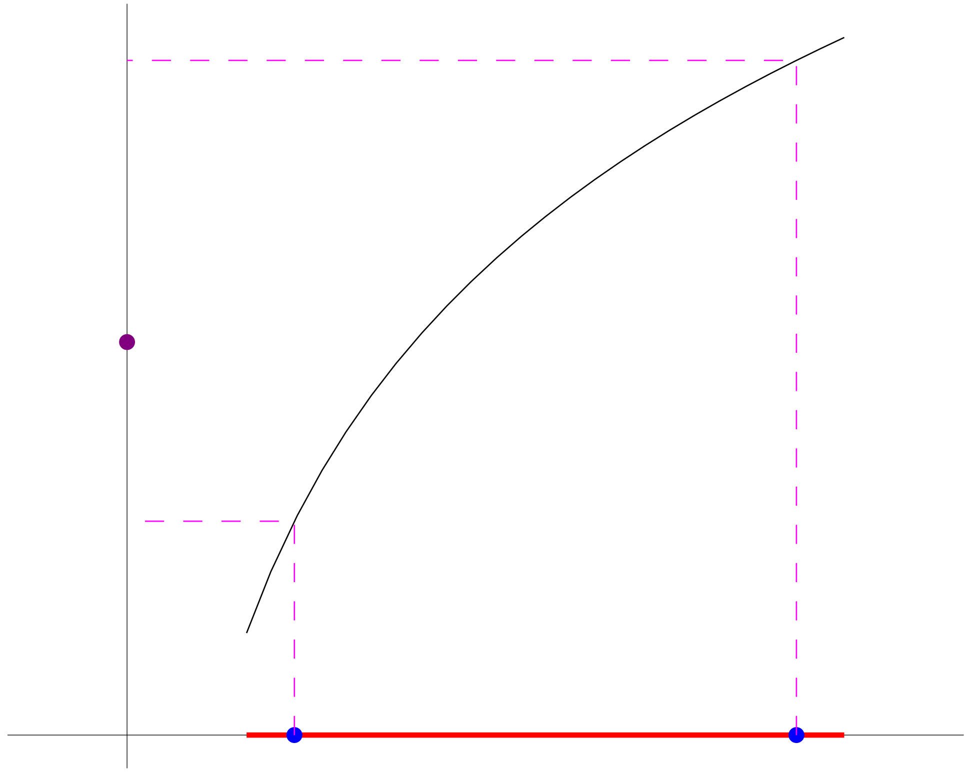

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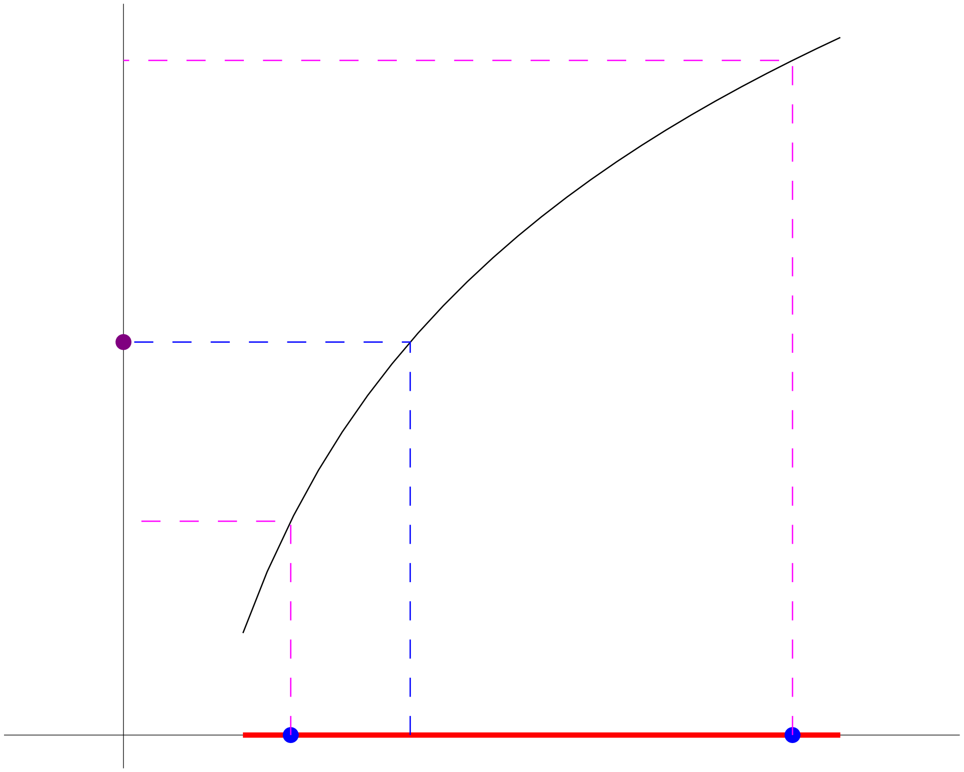

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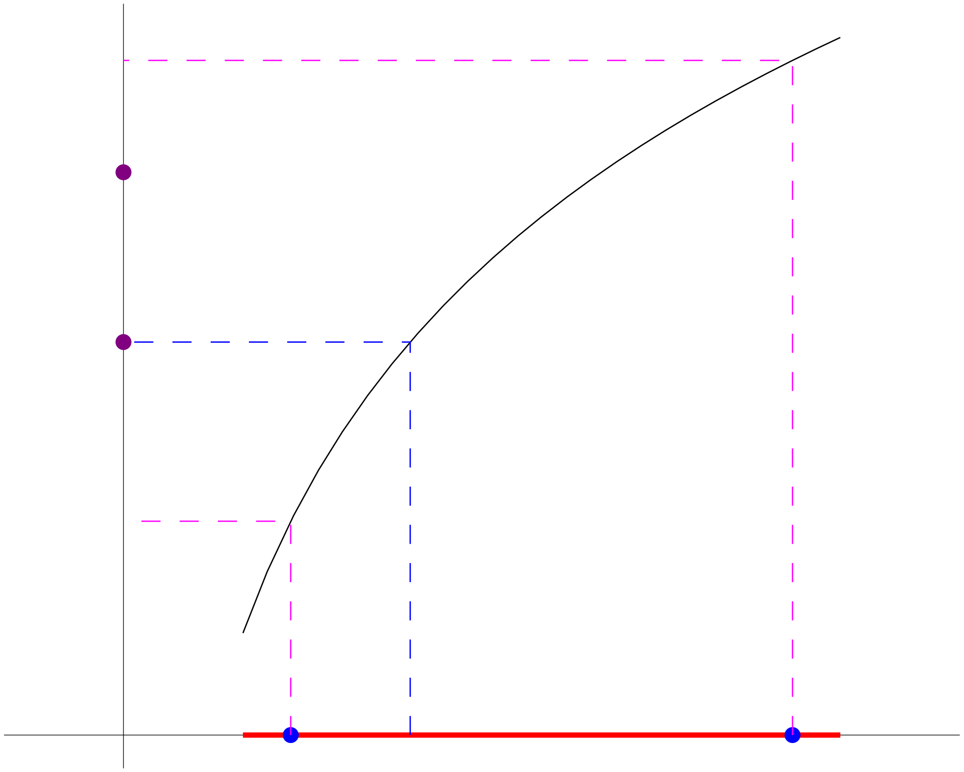

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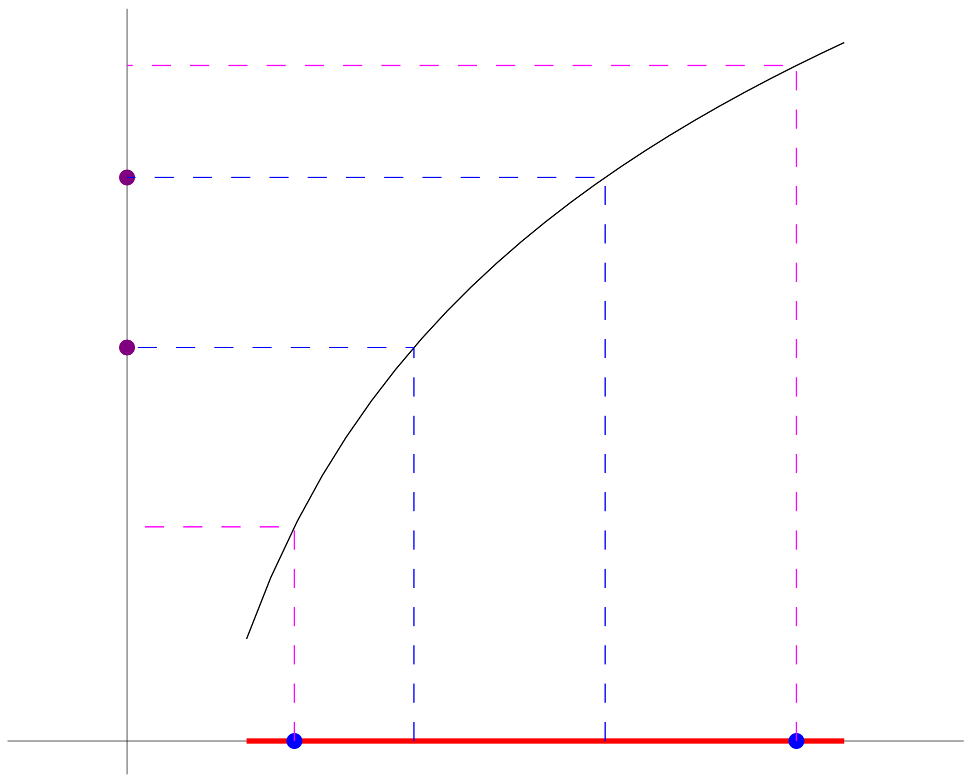

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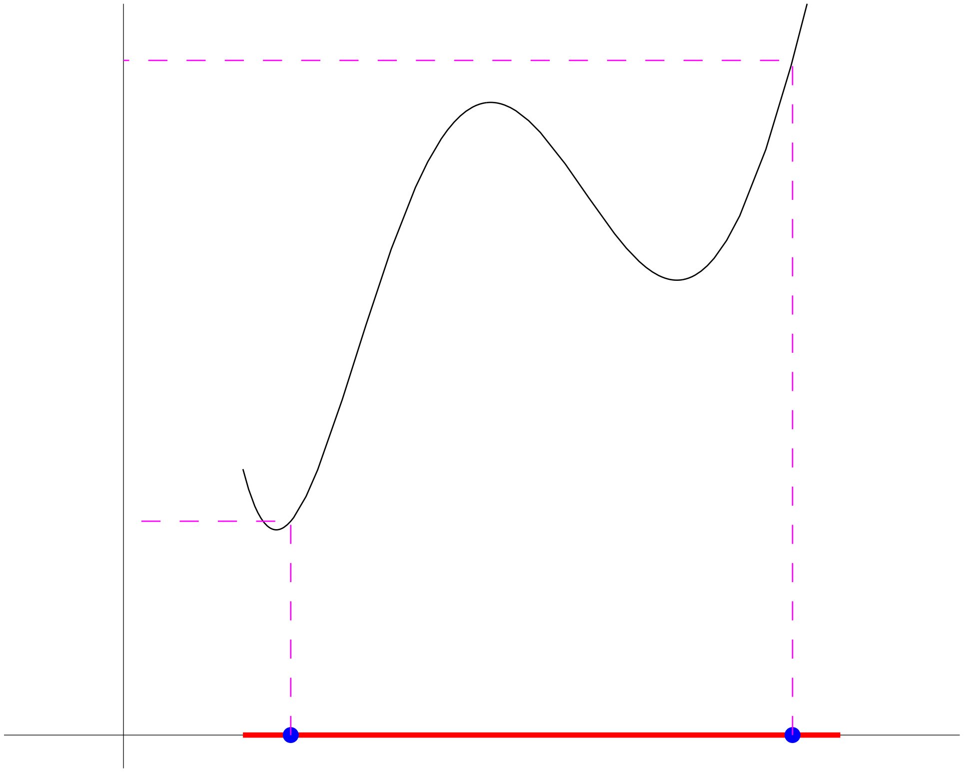

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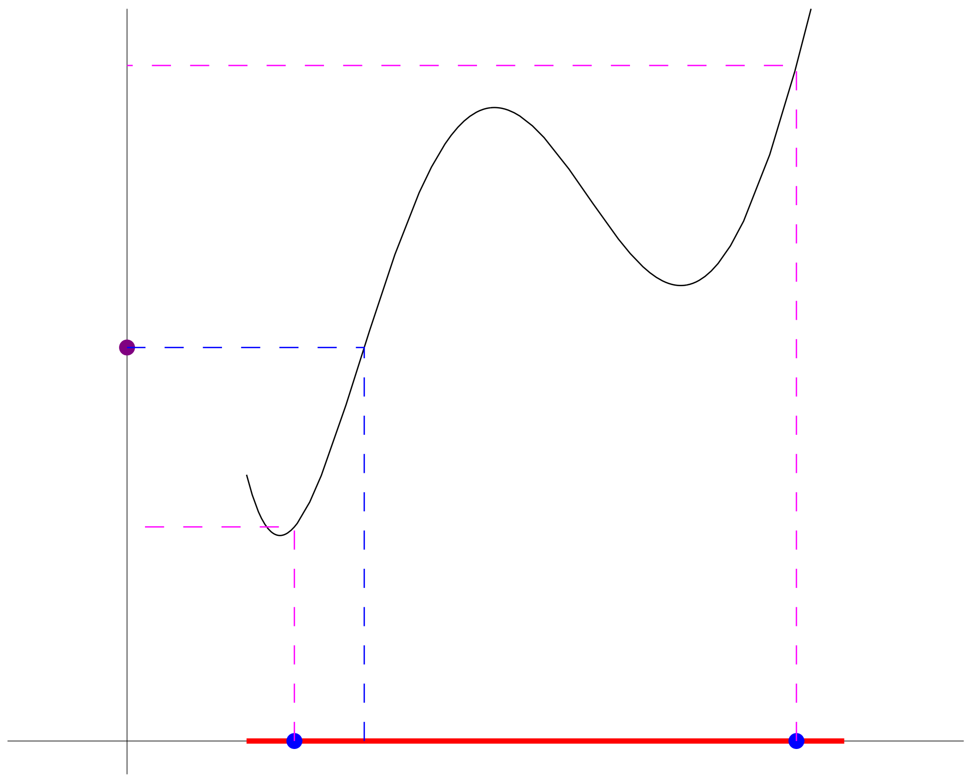

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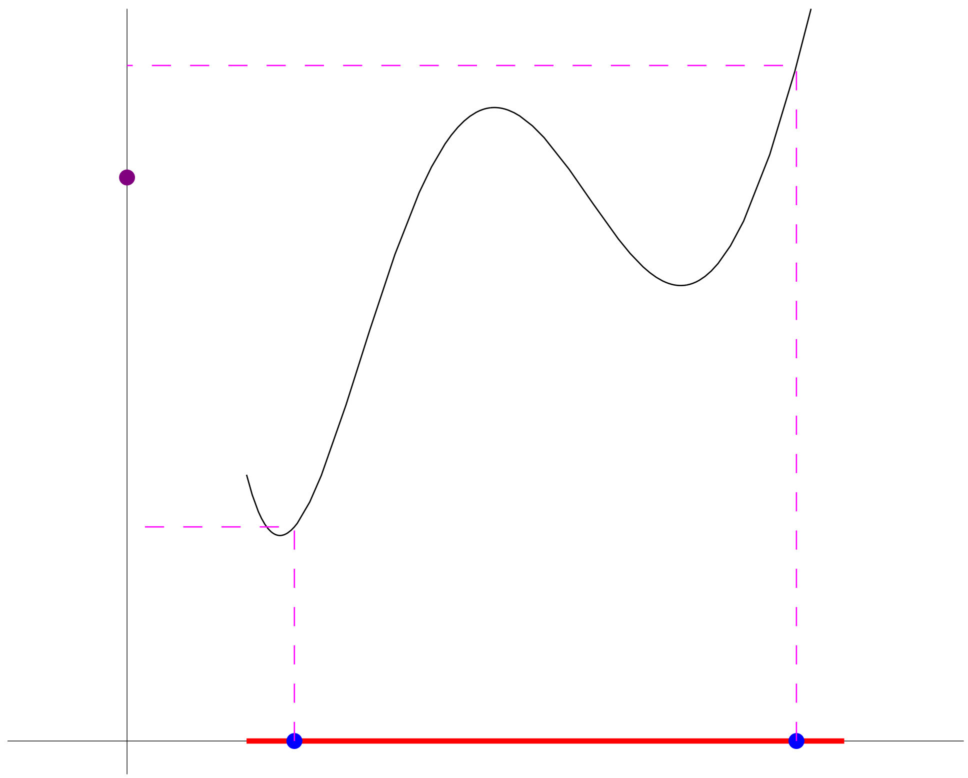

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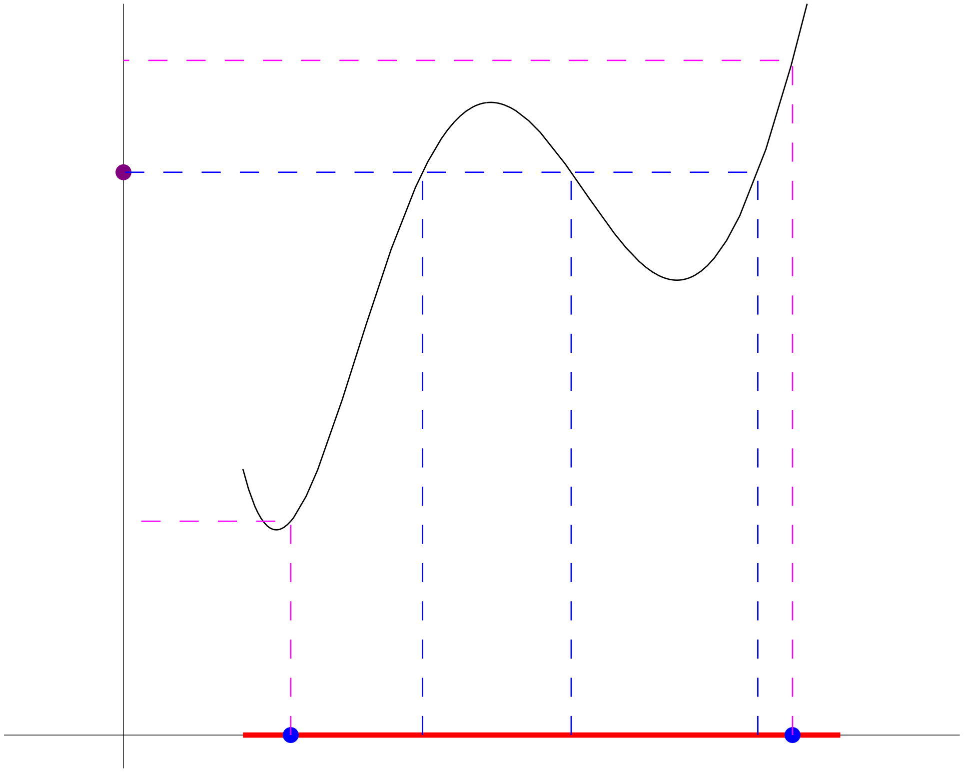

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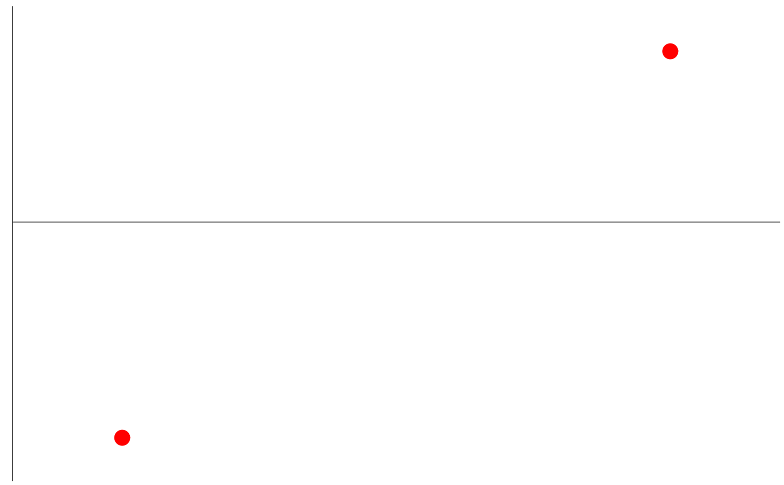

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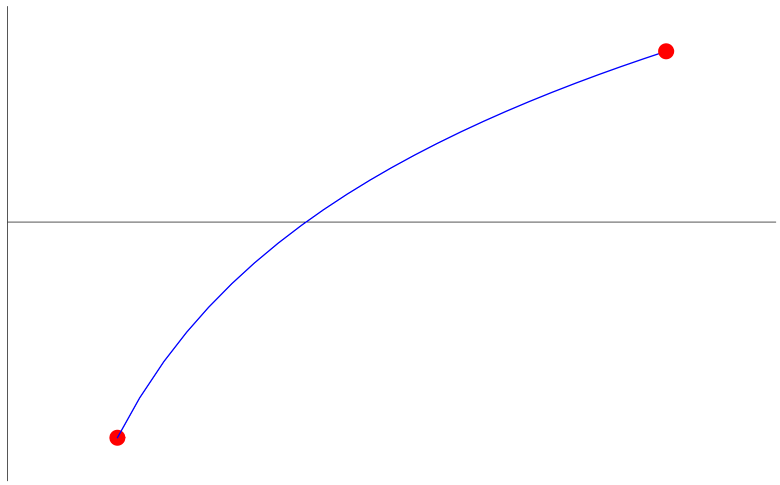

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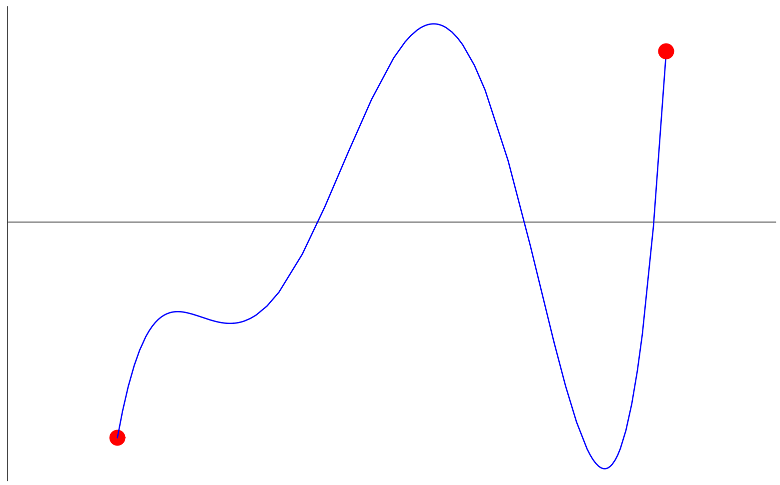

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Answer $x \le -3$ or $-2 \le x \le 2$

Chapter 4: Differentiation

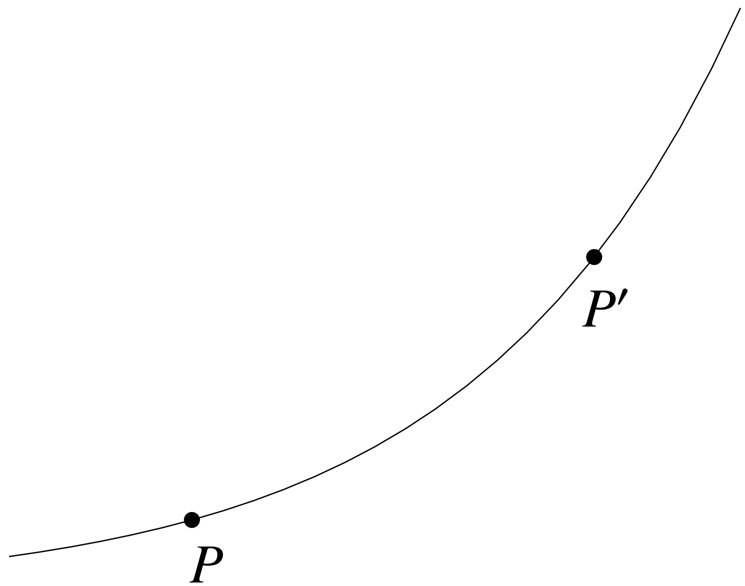
- Derivatives
- Rules for Differentiation
- Product Rules and Quotient Rules
- Higher-Order Derivatives

Objectives

- Introduce the definition and geometric meaning of derivatives.
- Introduce some rules for differentiation.
- Introduce the definition and notation of higher-order derivatives.

Derivatives

Consider the curve shown below.



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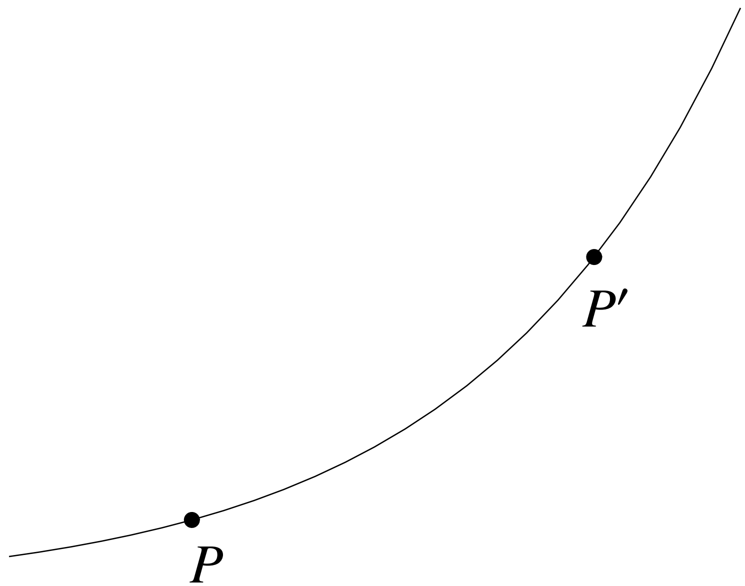

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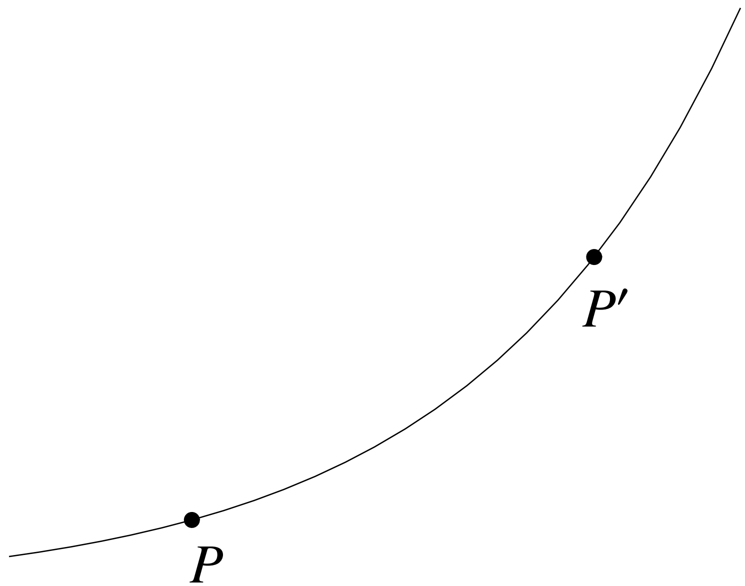
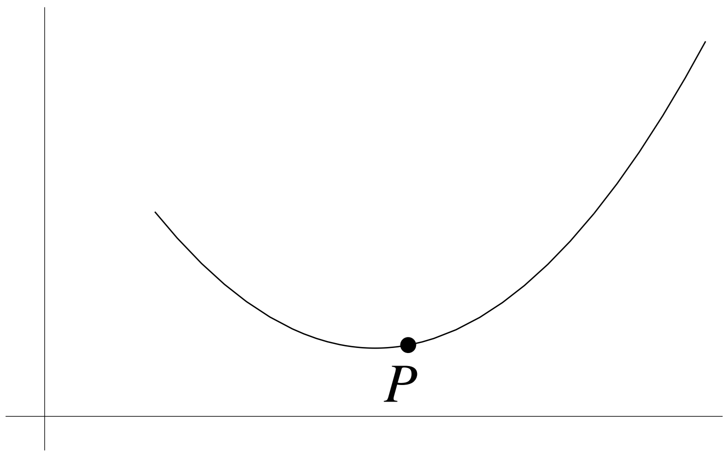


- "Slope" changes as we move along the curve.
- *Slope at the point P' is larger (which means "steeper") than that at P.*

Definition The slope at the point P is defined as follows



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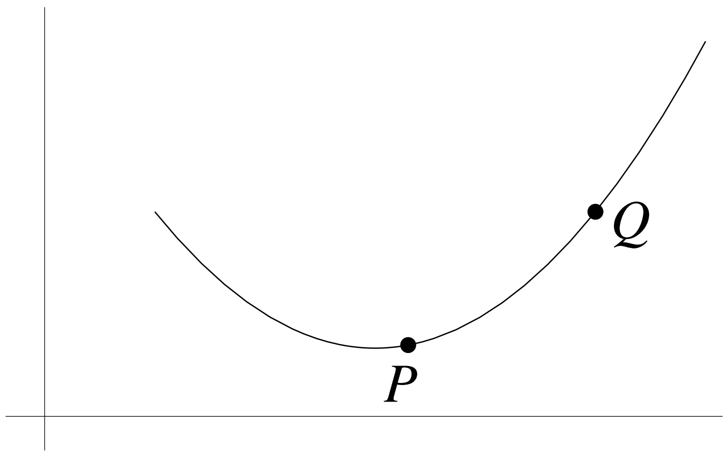

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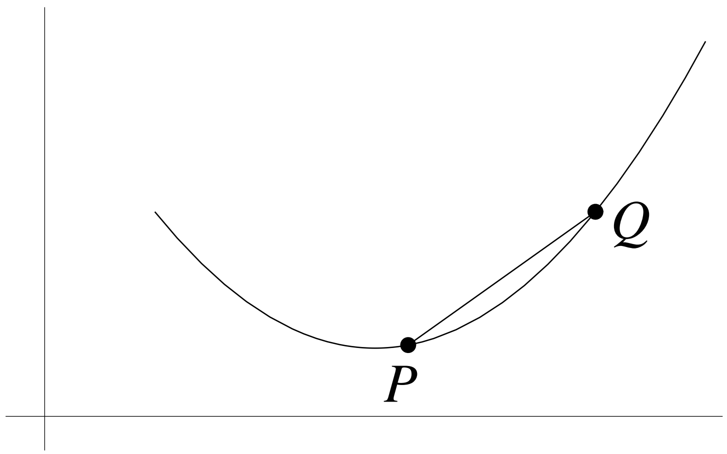
- Take a point Q on the curve different from P.
- The slope m_{PQ} of the line PQ (called a *secant line*) can be found by using the two points P and Q.



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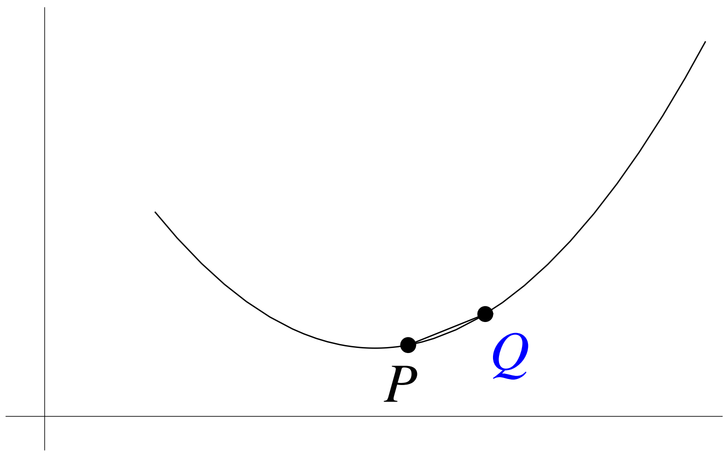

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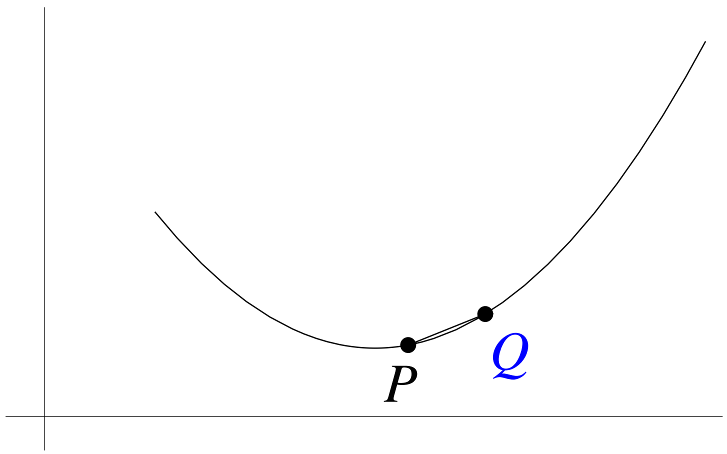

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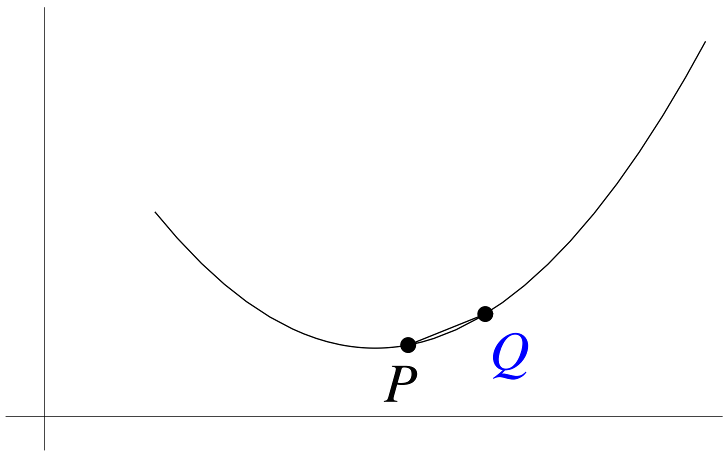
- If m_{PQ} approaches a fixed value, that is, if

$$\lim_{\substack{Q \to P \\ \text{along the curve}}} m_{PQ}$$

exists, this limit is called the *slope of the curve at P*, denoted by $m_{\tan}$.



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$$\lim_{\substack{Q \to P \\ \text{along the curve}}} m_{PQ}$$

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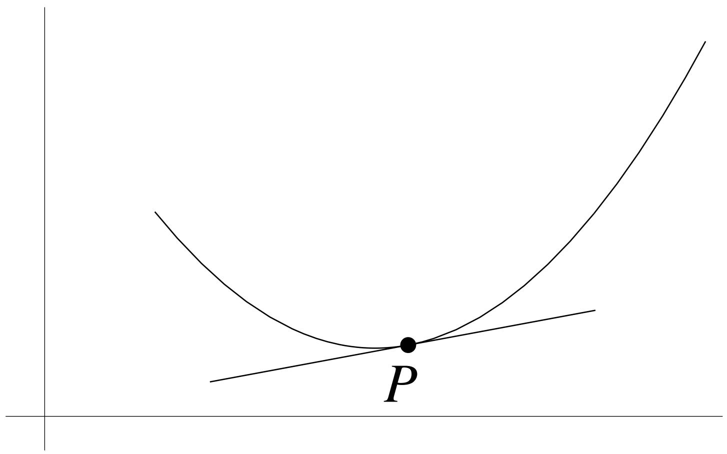

Definition The slope at the point P is defined as follows

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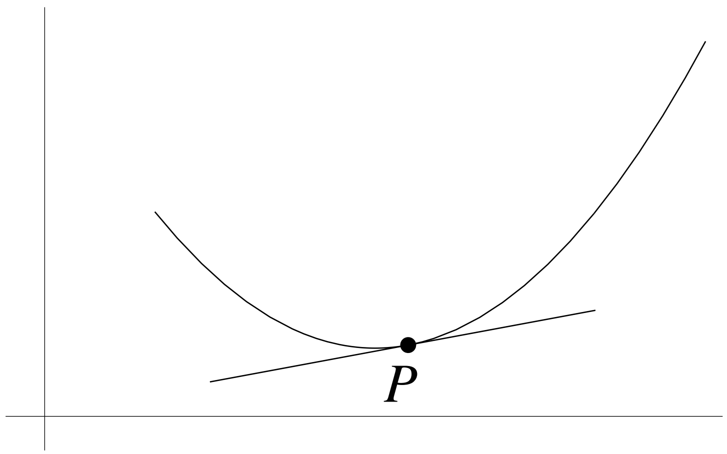


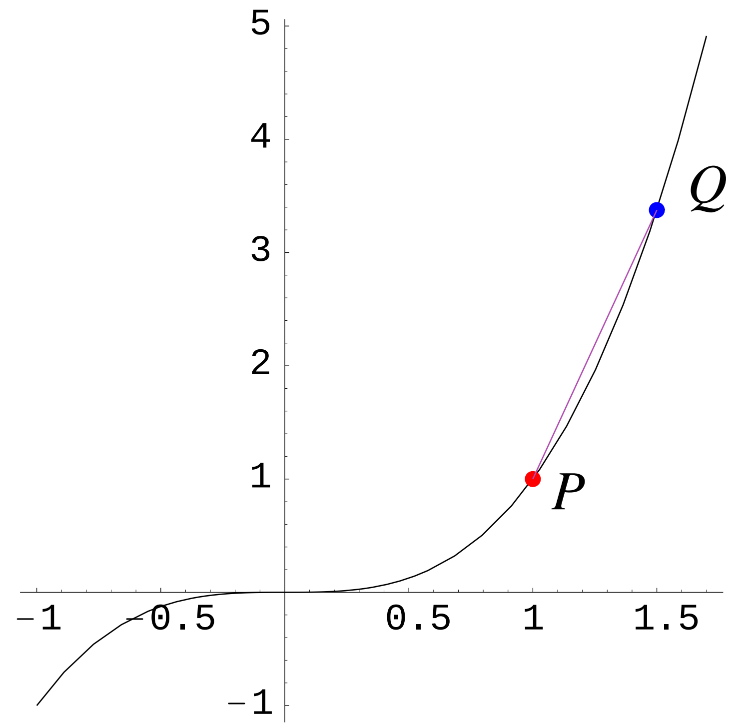
Show slope.html

Show slope1.html

Example Consider the curve given by

$$y = x^3$$

What is the slope at the point $P = (1, 1)$?



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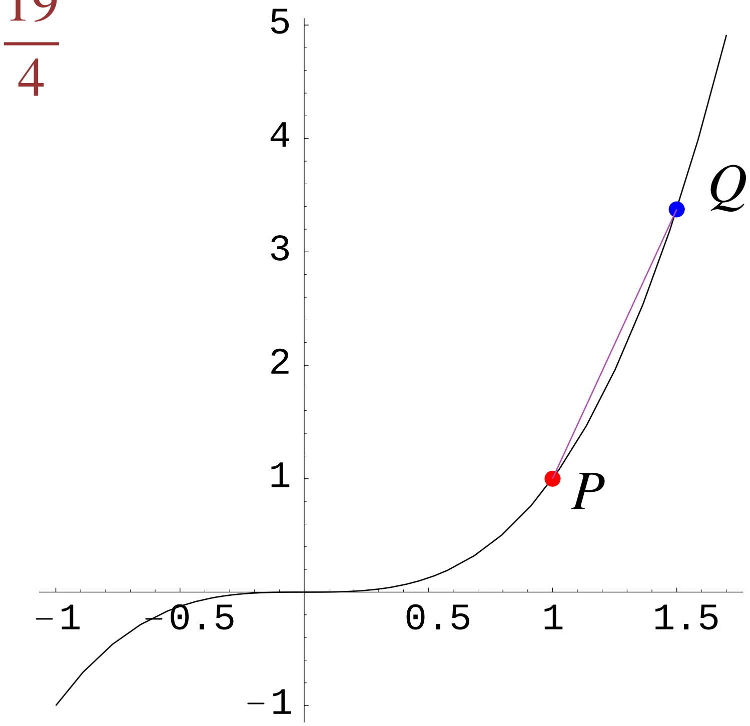

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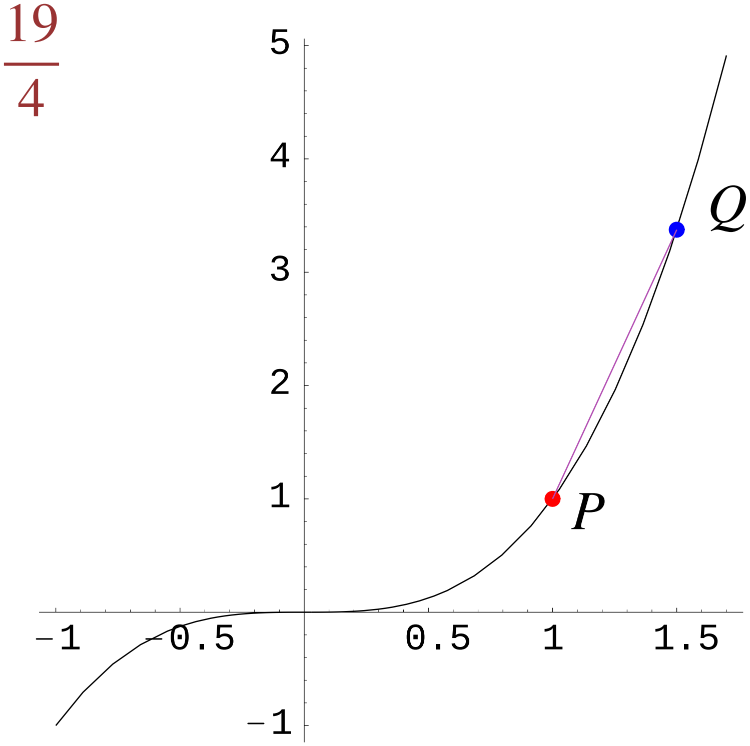
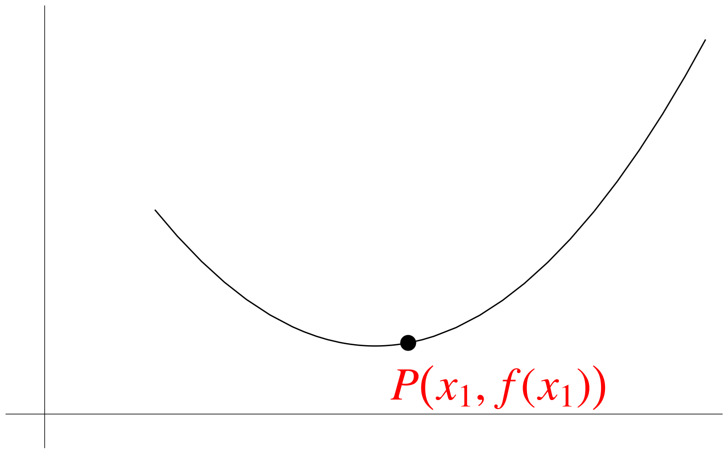


-

Q	slope of PQ
(1.5, 3.375)	4.75
(1.3, 2.197)	3.99
(1.1, 1.331)	3.31
(1.05, 1.157625)	3.1525
(1.01, 1.030301)	3.0301
⋮	show secant_slope.xls

- Let $P = (x_1, f(x_1))$.



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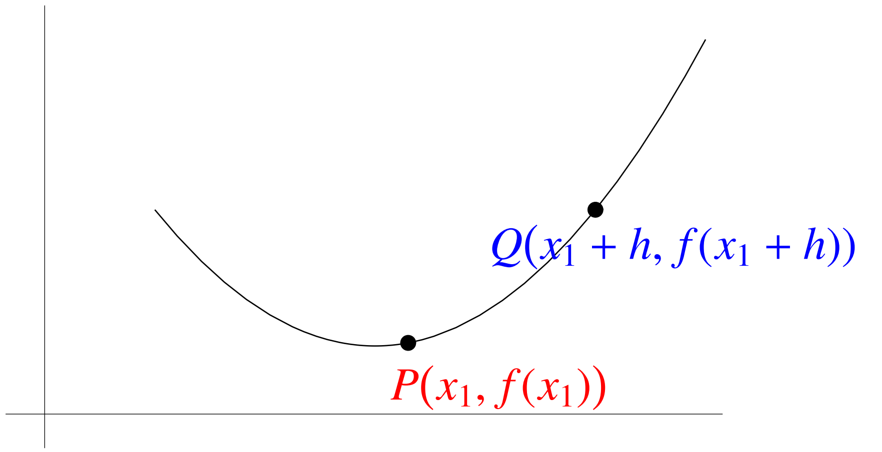

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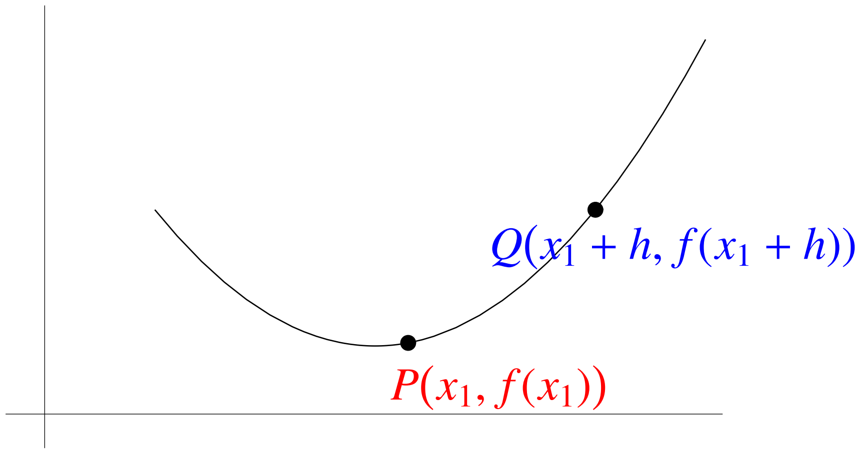

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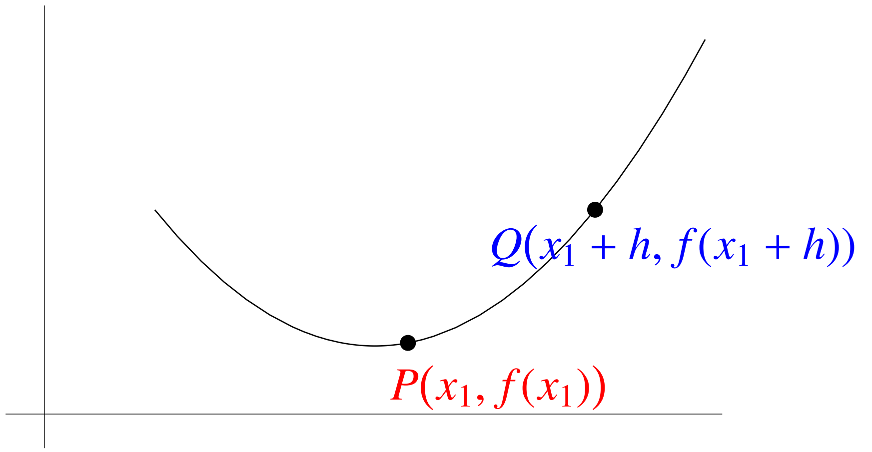

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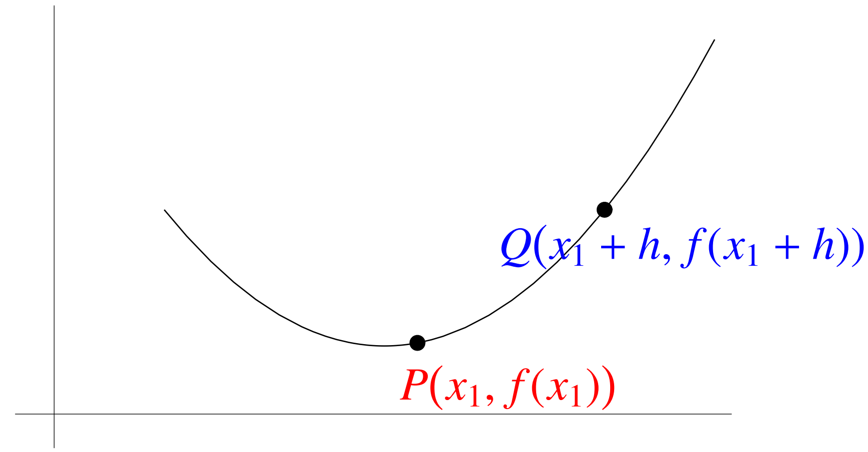

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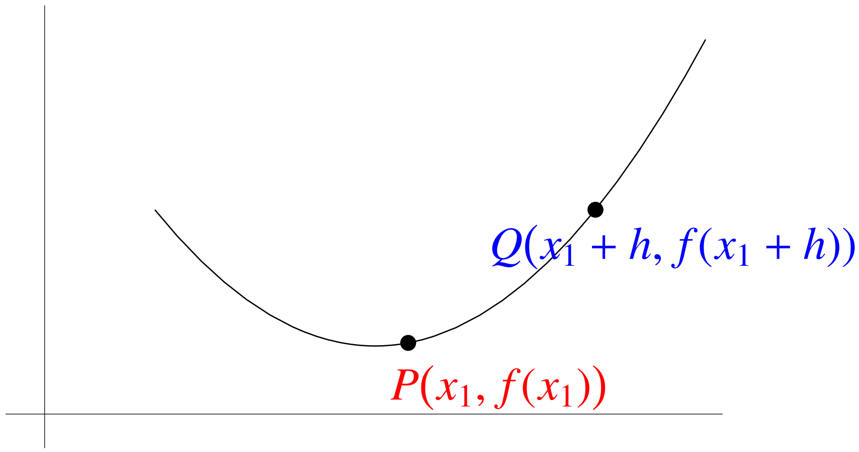

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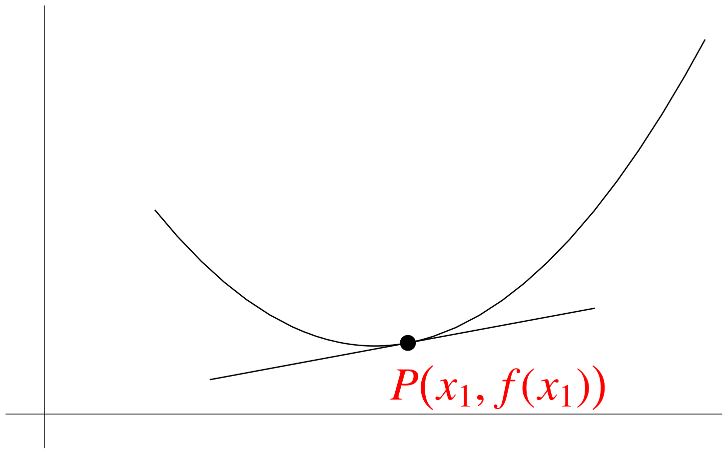

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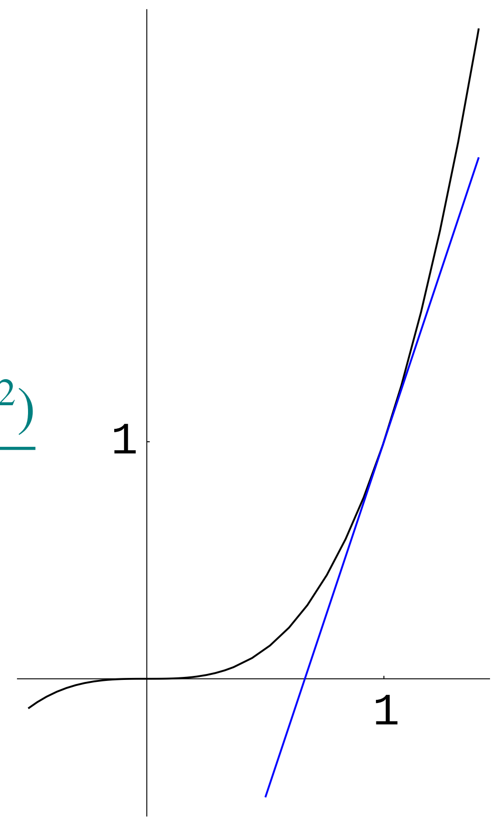

Exercise Find the slope of the curve $y = x^2$

- at the point $(3, 9)$;
- at the point $(-2, 4)$.