Example Find the following limits, if exist.

(1) $\lim_{x\to 4}(1 + x^2)$

(2) $\lim_{x\to 2}\dfrac{x-1}{x^2+x-2}$

(3) $\lim_{x\to 1}\dfrac{x-1}{x^2+x-2}$

(4) $\lim_{x\to 1}\dfrac{x+1}{x^2+x-2}$

Example Find the following limits, if exist.

(1) $\lim_{x \to 4} (1 + x^2)$

(2) $\lim_{x \to 2} \frac{x - 1}{x^2 + x - 2}$

(3) $\lim_{x \to 1} \frac{x - 1}{x^2 + x - 2}$

(4) $\lim_{x \to 1} \frac{x + 1}{x^2 + x - 2}$

Solution (1) Use properties of limits: $\lim_{x \to 4} (1 + x^2) = \lim_{x \to 4} 1 + \lim_{x \to 4} x^2$

Example Find the following limits, if exist.

(1) $\lim_{x\to 4}(1 + x^2)$

(2) $\lim_{x\to 2}\dfrac{x-1}{x^2+x-2}$

(3) $\lim_{x\to 1}\dfrac{x-1}{x^2+x-2}$

(4) $\lim_{x\to 1}\dfrac{x+1}{x^2+x-2}$

Solution (1) Use properties of limits:

$$\lim_{x\to 4}(1 + x^2) = \lim_{x\to 4} 1 + \lim_{x\to 4} x^2$$

$$= 1 + 4^2$$

Example Find the following limits, if exist.

(1) $\lim_{x \to 4}(1 + x^2)$

(2) $\lim_{x \to 2} \frac{x - 1}{x^2 + x - 2}$

(3) $\lim_{x \to 1} \frac{x - 1}{x^2 + x - 2}$

(4) $\lim_{x \to 1} \frac{x + 1}{x^2 + x - 2}$

Solution (1) Use properties of limits:

$$\begin{aligned} \lim_{x \to 4}(1 + x^2) &= \lim_{x \to 4} 1 + \lim_{x \to 4} x^2 \\ &= 1 + 4^2 \\ &= 17 \end{aligned}$$

Example Find the following limits, if exist.

(1) $\lim_{x\to 4}(1+x^2)$

(2) $\lim_{x\to 2}\frac{x-1}{x^2+x-2}$

(3) $\lim_{x\to 1}\frac{x-1}{x^2+x-2}$

(4) $\lim_{x\to 1}\frac{x+1}{x^2+x-2}$

Solution (1) Use properties of limits:

$$\begin{aligned}\lim_{x\to 4}(1+x^2) &= \lim_{x\to 4}1+\lim_{x\to 4}x^2\\ &= 1+4^2\\ &= 17\end{aligned}$$

Remark Can use substitution $\because$ *polynomial functions are continuous*

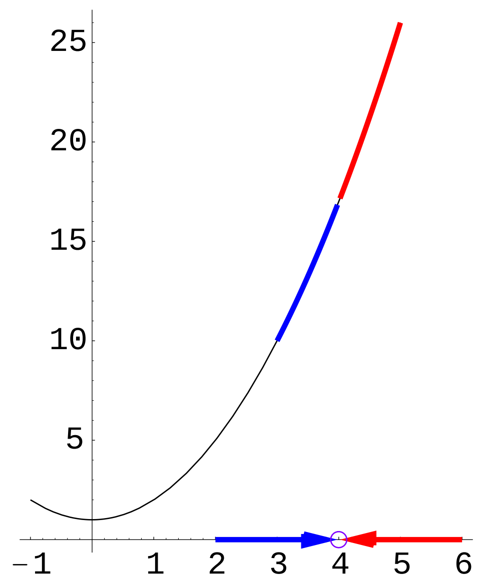
Example Find the following limits, if exist.

(1) $\lim_{x\to 4}(1+x^2)$

(2) $\lim_{x\to 2}\dfrac{x-1}{x^2+x-2}$

(3) $\lim_{x\to 1}\dfrac{x-1}{x^2+x-2}$

(4) $\lim_{x\to 1}\dfrac{x+1}{x^2+x-2}$



Solution (1) Use properties of limits:

$$\lim_{x\to 4}(1+x^2) = \lim_{x\to 4} 1 + \lim_{x\to 4} x^2 = 1 + 4^2 = 17$$

Remark Can use substitution $\because$ *polynomial functions are continuous*

Example Find $\lim_{x \to 2} \dfrac{x-1}{x^2+x-2}$, if exist.

Example Find $\lim\limits_{x \to 2} \dfrac{x-1}{x^2+x-2}$, if exist.

Solution $$\lim_{x \to 2} \frac{x-1}{x^2+x-2} = \frac{\lim\limits_{x \to 2}(x-1)}{\lim\limits_{x \to 2}(x^2+x-2)}$$

Example Find $\lim_{x\to 2} \dfrac{x-1}{x^2+x-2}$, if exist.

Solution

$$\lim_{x\to 2} \frac{x-1}{x^2+x-2} = \frac{\lim_{x\to 2}(x-1)}{\lim_{x\to 2}(x^2+x-2)}$$

$$= \frac{2-1}{2^2+2-2}$$

Example Find $\lim_{x \to 2} \frac{x-1}{x^2+x-2}$, if exist.

Solution

$$\lim_{x \to 2} \frac{x-1}{x^2+x-2} = \frac{\lim_{x \to 2}(x-1)}{\lim_{x \to 2}(x^2+x-2)}$$

$$= \frac{2-1}{2^2+2-2}$$

$$= \frac{1}{4}$$

Example Find $\lim_{x \to 2} \frac{x-1}{x^2+x-2}$, if exist.

Solution

$$\lim_{x \to 2} \frac{x-1}{x^2+x-2} = \frac{\lim_{x \to 2}(x-1)}{\lim_{x \to 2}(x^2+x-2)}$$

$$= \frac{2-1}{2^2+2-2}$$

$$= \frac{1}{4}$$

Remark Can use substitution $\because$ *rational functions are* continuous *in their domain.*

Example Find $\lim_{x \to 1} \frac{x - 1}{x^2 + x - 2}$, if exist.

Example Find $\lim_{x \to 1} \frac{x-1}{x^2+x-2}$, if exist.

Solution *Substitution gives* $\frac{0}{0}$, *doesn't work*.

Example Find $\lim_{x\to 1} \dfrac{x-1}{x^2+x-2}$, if exist.

Solution *Substitution gives* $\dfrac{0}{0}$, *doesn't work.*

$$\lim_{x\to 1} \frac{x-1}{x^2+x-2} \quad =$$

Example Find $\lim_{x \to 1} \frac{x - 1}{x^2 + x - 2}$, if exist.

Solution *Substitution gives* $\frac{0}{0}$, *doesn't work*.

$$\lim_{x \to 1} \frac{x - 1}{x^2 + x - 2} = \lim_{x \to 1} \frac{x - 1}{(x - 1)(x + 2)}$$

Example Find $\lim_{x\to 1} \frac{x-1}{x^2+x-2}$, if exist.

Solution *Substitution gives* $\frac{0}{0}$, *doesn't work.*

$$\lim_{x\to 1} \frac{x-1}{x^2+x-2} = \lim_{x\to 1} \frac{x-1}{(x-1)(x+2)}$$

$$= \lim_{x\to 1} \frac{1}{x+2}$$

Example Find $\lim_{x \to 1} \frac{x-1}{x^2+x-2}$, if exist.

Solution *Substitution gives* $\frac{0}{0}$, *doesn't work*.

$$\lim_{x \to 1} \frac{x-1}{x^2+x-2} = \lim_{x \to 1} \frac{x-1}{(x-1)(x+2)}$$

$$= \lim_{x \to 1} \frac{1}{x+2}$$

$$= \frac{1}{1+2}$$

Example Find $\lim_{x\to 1}\frac{x-1}{x^2+x-2}$, if exist.

Solution *Substitution gives* $\frac{0}{0}$, *doesn't work.*

$$\lim_{x\to 1}\frac{x-1}{x^2+x-2} = \lim_{x\to 1}\frac{x-1}{(x-1)(x+2)} = \lim_{x\to 1}\frac{1}{x+2} = \frac{1}{1+2} = \frac{1}{3}$$

Example Find $\lim_{x \to 1} \dfrac{x+1}{x^2+x-2}$, if exist.

Example Find $\lim_{x \to 1} \frac{x + 1}{x^2 + x - 2}$, if exist.

Solution $\frac{x + 1}{x^2 + x - 2} = \frac{x + 1}{(x - 1)(x + 2)}$.

Example Find $\lim_{x \to 1} \frac{x+1}{x^2+x-2}$, if exist.

Solution $\frac{x+1}{x^2+x-2} = \frac{x+1}{(x-1)(x+2)}$.

- If x is close to 1,
  - ◇ *numerator is close to* 2 whereas *denominator is close to* 0,

Example Find $\lim_{x \to 1} \dfrac{x+1}{x^2+x-2}$, if exist.

Solution $\dfrac{x+1}{x^2+x-2} = \dfrac{x+1}{(x-1)(x+2)}$.

- If x is close to 1,
  - *numerator is close to* 2 whereas *denominator is close to* 0,
  - therefore the fraction is very large in magnitude *(may be positive or negative)*.

Example Find $\lim_{x \to 1} \frac{x+1}{x^2+x-2}$, if exist.

Solution $\frac{x+1}{x^2+x-2} = \frac{x+1}{(x-1)(x+2)}$.

- If x is close to 1,
  - ◇ *numerator is close to* 2 whereas *denominator is close to* 0,
  - ◇ therefore the fraction is very large in magnitude *(may be positive or negative)*.
- Therefore $\lim_{x \to 1} \frac{x+1}{x^2+x-2} = \lim_{x \to 1} \frac{x+1}{(x-1)(x+2)}$ *does not exist*.

Example Find $\lim_{x\to 1} \dfrac{x+1}{x^2+x-2}$, if exist.

Solution $\dfrac{x+1}{x^2+x-2} = \dfrac{x+1}{(x-1)(x+2)}$.

- If x is close to 1,
  - *numerator is close to* 2 whereas *denominator is close to* 0,
  - therefore the fraction is very large in magnitude *(may be positive or negative)*.
- Therefore $\lim_{x\to 1} \dfrac{x+1}{x^2+x-2} = \lim_{x\to 1} \dfrac{x+1}{(x-1)(x+2)}$ *does not exist*.

Remark To be more precise, $\lim_{x\to 1+} \dfrac{x+1}{x^2+x-2} = \infty$ Show graph

$$\lim_{x\to 1-} \dfrac{x+1}{x^2+x-2} = -\infty$$

Example Let $f(x) = x^2 + 3$. Find $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

Example Let $f(x) = x^2 + 3$. Find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

Solution *Treat x as a constant because h, not x, is changing.*

Example Let $f(x) = x^2 + 3$. Find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

Solution *Treat x as a constant because h, not x, is changing*.

The expression $\frac{f(x+h) - f(x)}{h}$ is called a *difference quotient*.

Example Let $f(x) = x^2 + 3$. Find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

Solution *Treat x as a constant because h, not x, is changing.*

The expression $\frac{f(x+h) - f(x)}{h}$ is called a *difference quotient*.

Can't use substitution (indeterminate form $\frac{0}{0}$).

Example Let $f(x) = x^2 + 3$. Find $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

Solution *Treat x as a constant because h, not x, is changing.*

The expression $\dfrac{f(x+h) - f(x)}{h}$ is called a *difference quotient*.

Can't use substitution (indeterminate form $\frac{0}{0}$).

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 + 3 - (x^2 + 3)}{h}$$

Example Let $f(x) = x^2 + 3$. Find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

Solution *Treat x as a constant because h, not x, is changing*.

The expression $\frac{f(x+h) - f(x)}{h}$ is called a *difference quotient*.

Can't use substitution (indeterminate form $\frac{0}{0}$).

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 + 3 - (x^2 + 3)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3 - x^2 - 3}{h}$$

Example Let $f(x) = x^2 + 3$. Find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

Solution *Treat x as a constant because h, not x, is changing.*

The expression $\frac{f(x+h) - f(x)}{h}$ is called a *difference quotient*.

Can't use substitution (indeterminate form $\frac{0}{0}$).

$$\begin{aligned}
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \to 0} \frac{(x+h)^2 + 3 - (x^2 + 3)}{h} \\
&= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3 - x^2 - 3}{h} \\
&= \lim_{h \to 0} \frac{2xh + h^2}{h}
\end{aligned}$$

Example Let $f(x) = x^2 + 3$. Find $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$.

Solution *Treat x as a constant because h, not x, is changing.*

The expression $\dfrac{f(x+h) - f(x)}{h}$ is called a *difference quotient*.

Can't use substitution (indeterminate form $\frac{0}{0}$).

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 + 3 - (x^2+3)}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 3 - x^2 - 3}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h\to 0} \frac{h(2x+h)}{h}$$

$$= \lim_{h\to 0} (2x + h)$$

Example Let $f(x) = x^2 + 3$. Find $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$.

Solution *Treat x as a constant because h, not x, is changing.*

The expression $\frac{f(x+h) - f(x)}{h}$ is called a *difference quotient*.

Can't use substitution (indeterminate form $\frac{0}{0}$).

$$\begin{aligned}
\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h\to 0} \frac{(x+h)^2 + 3 - (x^2 + 3)}{h} \\
&= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 3 - x^2 - 3}{h} \\
&= \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h\to 0} \frac{h(2x+h)}{h} \\
&= \lim_{h\to 0} (2x + h) \\
&= 2x
\end{aligned}$$

Two Important Limits

(1) $\lim_{x \to 0} \frac{\sin x}{x} = 1$ (*where x is in radian*) *(will be used in Chapter 7)*

Two Important Limits

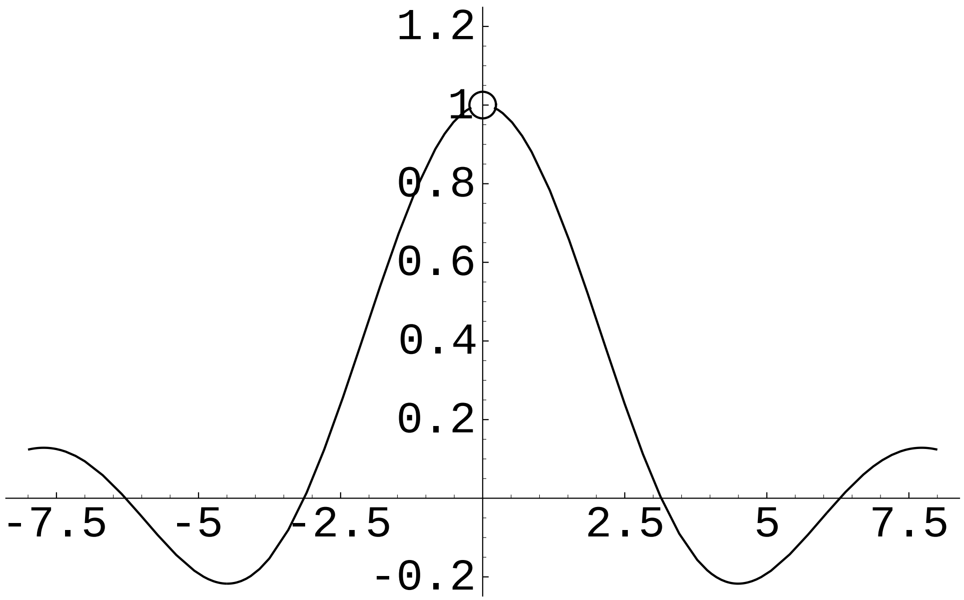
(1) $\lim_{x \to 0} \frac{\sin x}{x} = 1$ (*where x is in radian*) *(will be used in Chapter 7)*

x	$\frac{\sin x}{x}$
0.5	0.9589
0.2	0.9933
0.1	0.9983
0.05	0.9996
0.02	0.9999
⋮	Show sin_x.xls

Two Important Limits

(1) $\lim_{x \to 0} \frac{\sin x}{x} = 1$ (*where x is in radian*) *(will be used in Chapter 7)*

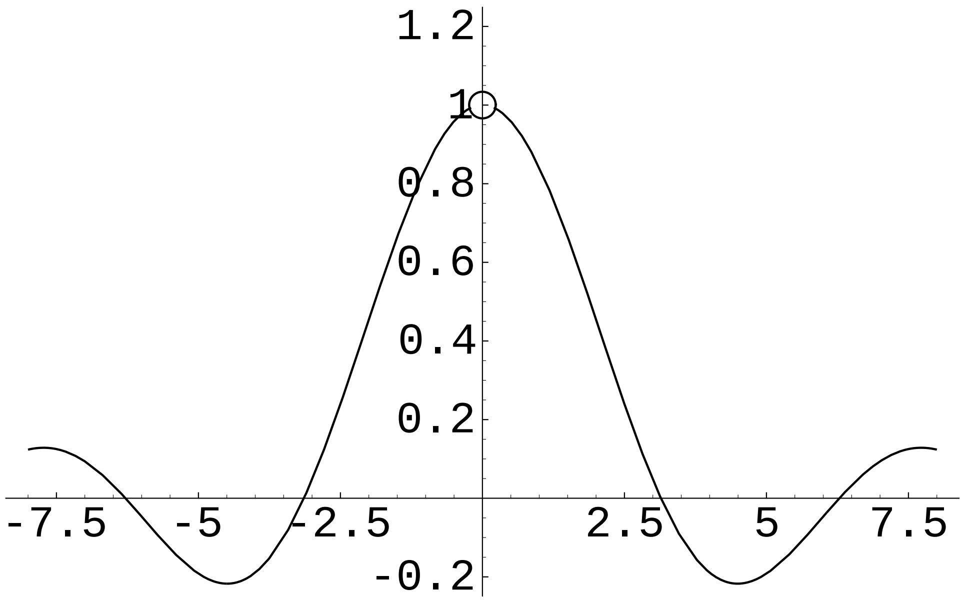
x	$\frac{\sin x}{x}$
0.5	0.9589
0.2	0.9933
0.1	0.9983
0.05	0.9996
0.02	0.9999
⋮	Show sin_x.xls



Two Important Limits

(1) $\lim_{x\to 0} \frac{\sin x}{x} = 1$ (*where x is in radian*) (*will be used in Chapter 7)*

x	$\frac{\sin x}{x}$
0.5	0.9589
0.2	0.9933
0.1	0.9983
0.05	0.9996
0.02	0.9999
$\vdots$	Show sin_x.xls

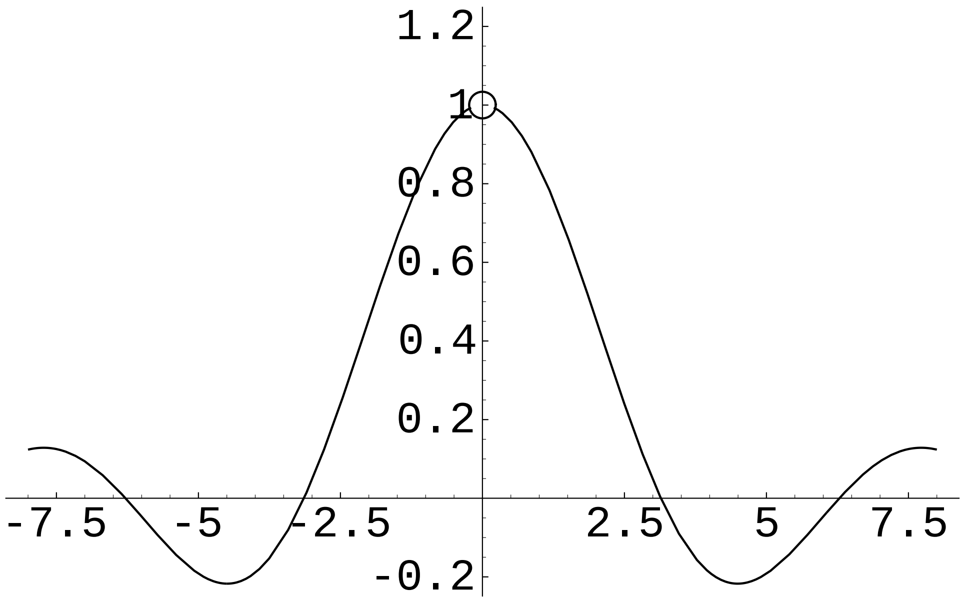


(2) $\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x$ *exists*, denoted by $e \approx 2.7182818$ Show e.xls

Two Important Limits

(1) $\lim_{x \to 0} \frac{\sin x}{x} = 1$ (*where x is in radian*) *(will be used in Chapter 7)*

x	$\frac{\sin x}{x}$
0.5	0.9589
0.2	0.9933
0.1	0.9983
0.05	0.9996
0.02	0.9999
⋮	Show sin_x.xls



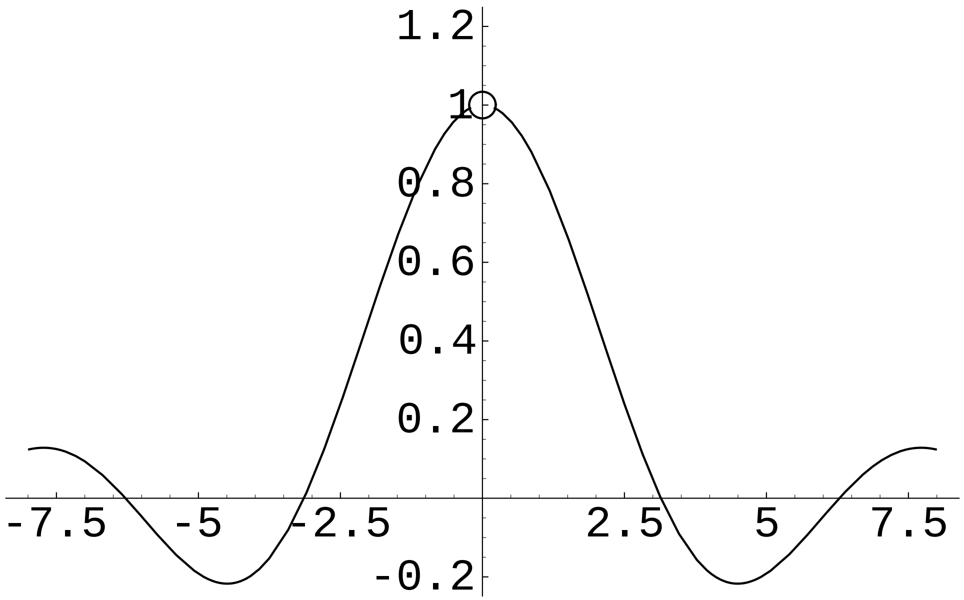
(2) $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$ *exists*, denoted by $e \approx 2.7182818$ Show e.xls

Appear in various discipline: *Radioactive decay* $A = A_0 e^{-\lambda t}$

Two Important Limits

(1) $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$ (*where x is in radian*) *(will be used in Chapter 7)*

x	$\dfrac{\sin x}{x}$
0.5	0.9589
0.2	0.9933
0.1	0.9983
0.05	0.9996
0.02	0.9999
⋮	Show sin_x.xls



(2) $\lim_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x$ *exists*, denoted by $\mathrm{e} \approx 2.7182818$ Show e.xls

Appear in various discipline: *Radioactive decay* $A = A_0 \mathrm{e}^{-\lambda t}$

Interest compounded continuously $A = P\mathrm{e}^{rt}$

Continuous Functions

For *“nice”* functions, can use *substitution* to find limits, that is,

$$\lim_{x \to a} f(x) = f(a)$$

Continuous Functions

For *“nice”* functions, can use *substitution* to find limits, that is,

$$\lim_{x \to a} f(x) = f(a)$$

Functions with this property are called continuous functions.

Continuous Functions

For *"nice"* functions, can use *substitution* to find limits, that is,

$$\lim_{x \to a} f(x) = f(a)$$

Functions with this property are called continuous functions.

Definition Let $a \in \mathbb{R}$ and let f be a function defined on an open interval containing a. We say that f is *continuous* at a if

$$(*) \quad \lim_{x \to a} f(x) = f(a)$$

Continuous Functions

For *"nice"* functions, can use *substitution* to find limits, that is,

$$\lim_{x \to a} f(x) = f(a)$$

Functions with this property are called continuous functions.

Definition Let $a \in \mathbb{R}$ and let f be a function defined on an open interval containing a. We say that f is *continuous* at a if

$$(*) \quad \lim_{x \to a} f(x) = f(a)$$

Meaning $f(x)$ *is close to* $f(a)$ *if* x *is close to* a.

Continuous Functions

For *"nice"* functions, can use *substitution* to find limits, that is,

$$\lim_{x \to a} f(x) = f(a)$$

Functions with this property are called continuous functions.

Definition Let $a \in \mathbb{R}$ and let f be a function defined on an open interval containing a. We say that f is *continuous* at a if

$(*)$ $\lim_{x \to a} f(x) = f(a)$ *Meaning* $f(x)$ *is close to* $f(a)$ *if* x *is close to* a.

Assumption

(0) $f(a)$ is defined.

Continuous Functions

For *"nice"* functions, can use *substitution* to find limits, that is,

$$\lim_{x \to a} f(x) = f(a)$$

Functions with this property are called continuous functions.

Definition Let $a \in \mathbb{R}$ and let f be a function defined on an open interval containing a. We say that f is *continuous* at a if

$(*)$ $\lim_{x \to a} f(x) = f(a)$ *Meaning* $f(x)$ *is close to* $f(a)$ *if* x *is close to* a.

Assumption

(0) $f(a)$ is defined.

Condition $(*)$ requires that

(1) $\lim_{x \to a} f(x)$ exists;

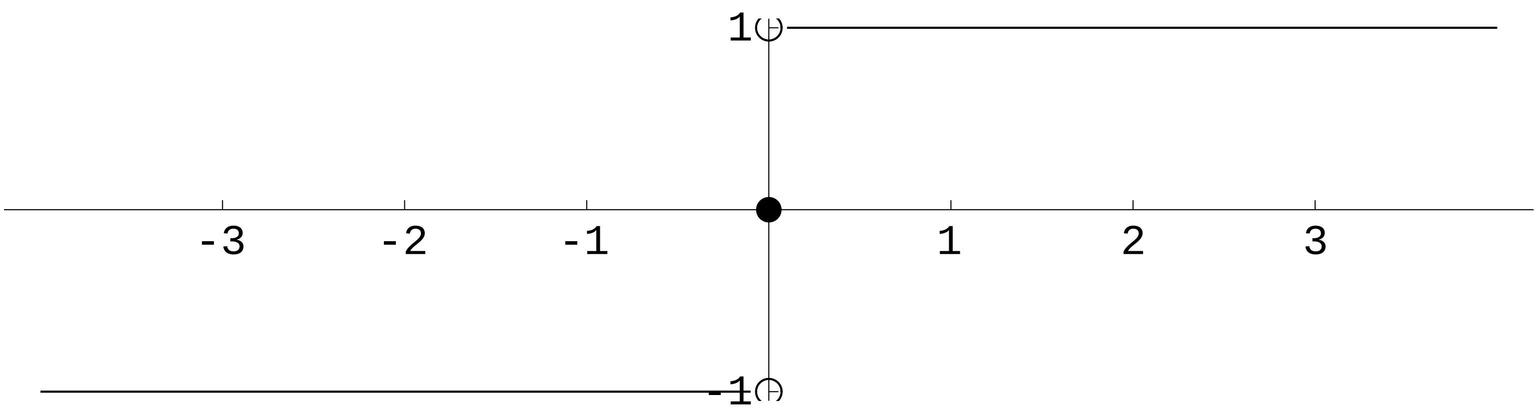
(2) the two values in (0) and (1) are equal.

Example

(1) Let $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$

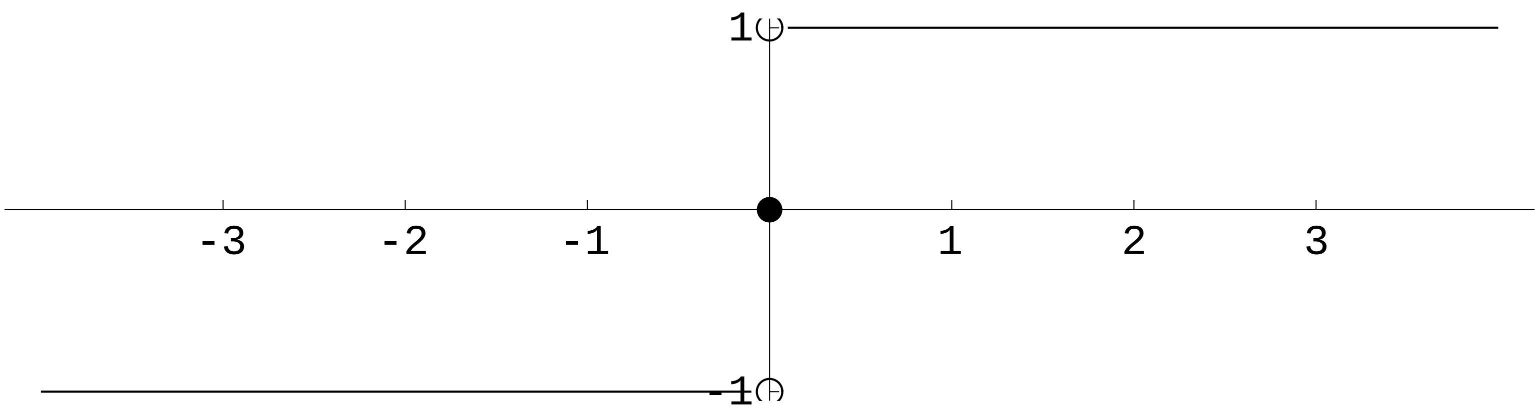
Example

(1) Let $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$



Example

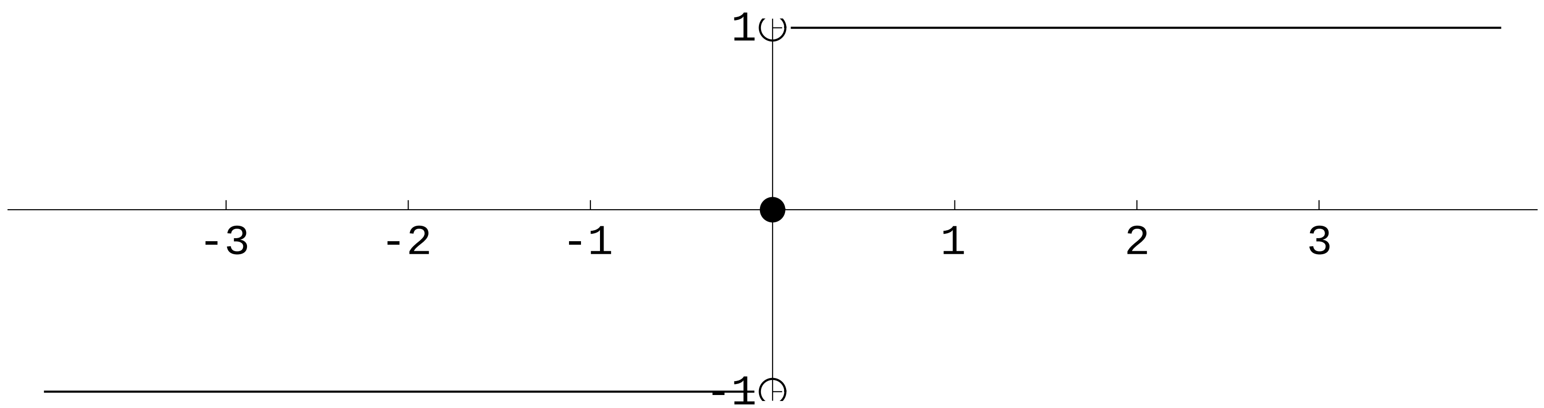
(1) Let $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$



$\lim_{x \to 0} f(x)$ does not exist

Example

(1) Let $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$

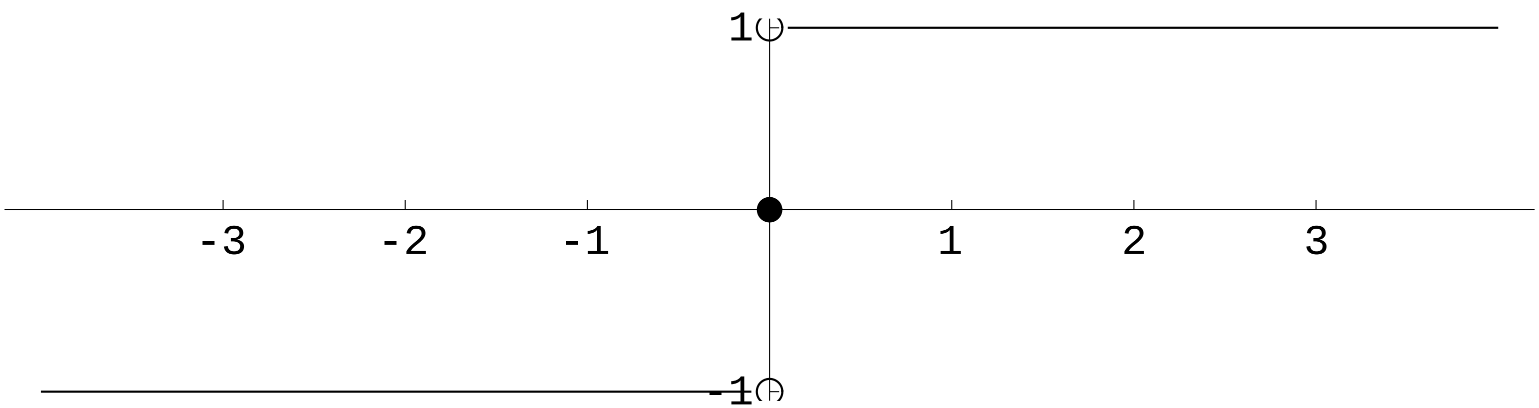


$\lim_{x \to 0} f(x)$ does not exist *because* $\lim_{x \to 0-} f(x) = -1$

$\lim_{x \to 0+} f(x) = 1$

Example

(1) Let $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$



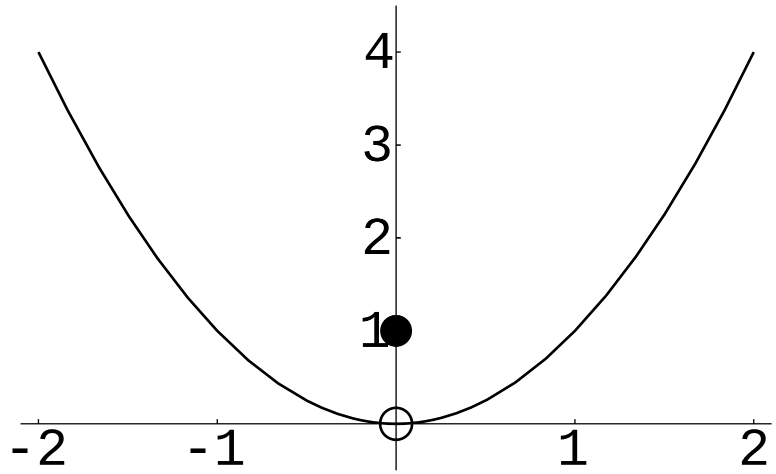
$\lim_{x \to 0} f(x)$ does not exist *because* $\lim_{x \to 0-} f(x) = -1$

$\lim_{x \to 0+} f(x) = 1$

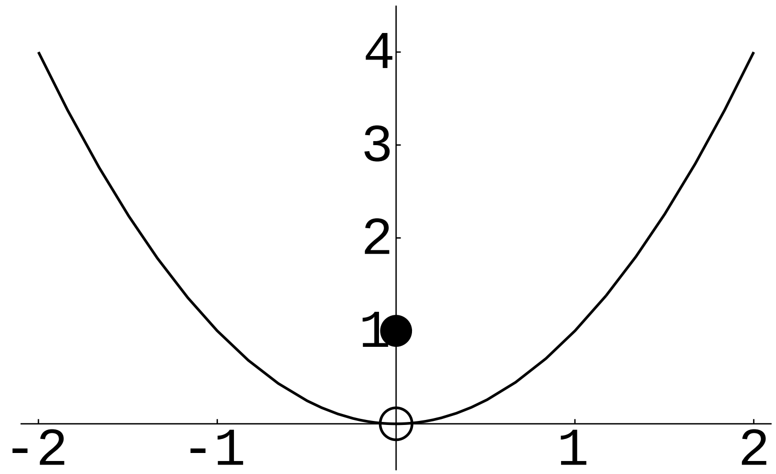
Therefore f is *not continuous at* 0.

(2) Let $f(x) = \begin{cases} x^2 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

(2) Let $f(x) = \begin{cases} x^2 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

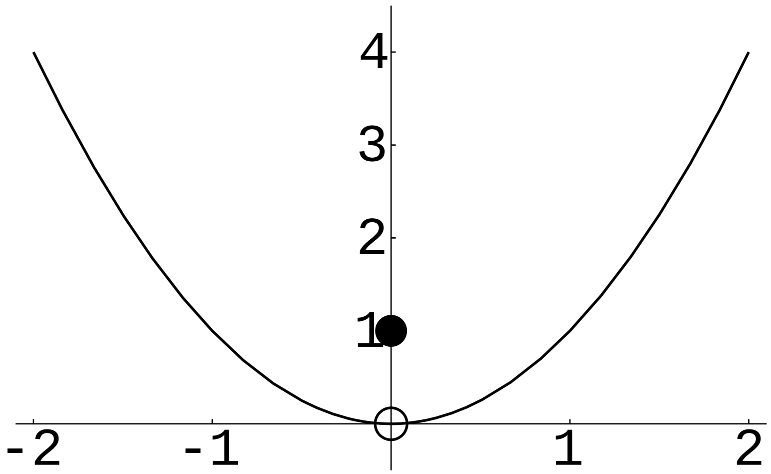


(2) Let $f(x) = \begin{cases} x^2 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$



$\lim_{x \to 0} f(x) = 0$ *but* $f(0) = 1$.

(2) Let $f(x) = \begin{cases} x^2 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

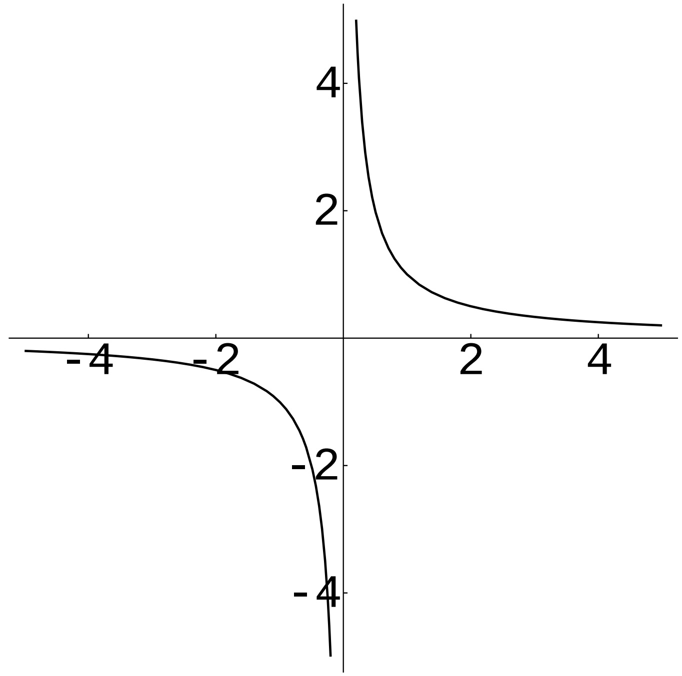


$\lim_{x \to 0} f(x) = 0$ *but* $f(0) = 1$.

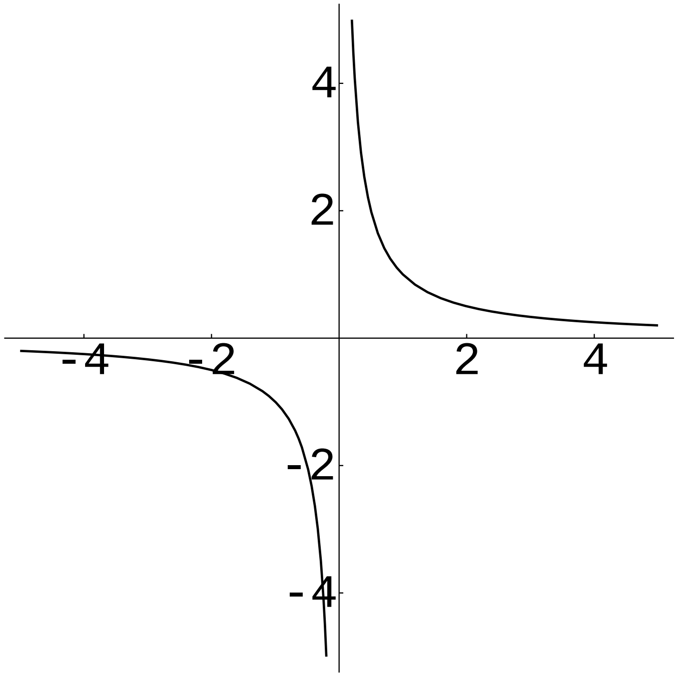
Therefore f is *not continuous at* 0.

(3) Let $f(x) = \dfrac{1}{x}$.

(3) Let $f(x) = \dfrac{1}{x}$.

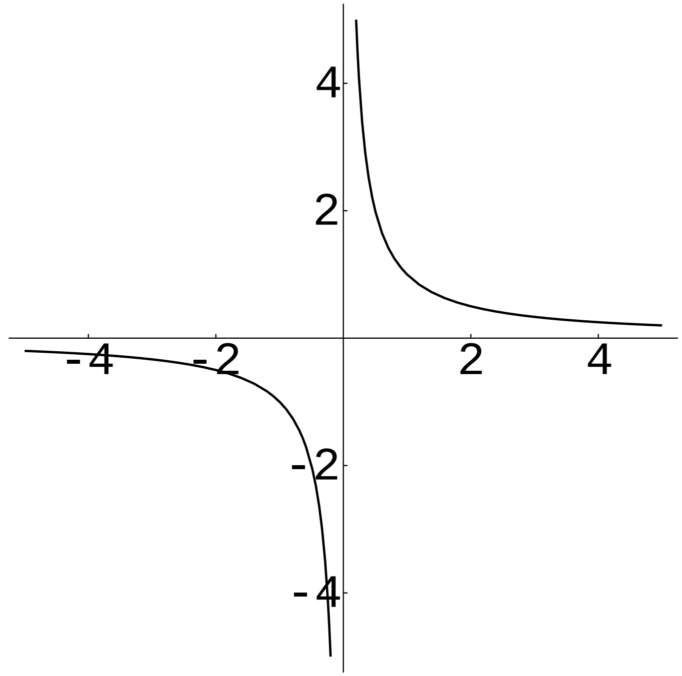


(3) Let $f(x) = \dfrac{1}{x}$.



- f is *undefined at* 0, meaningless to talk about continuity of f at 0.

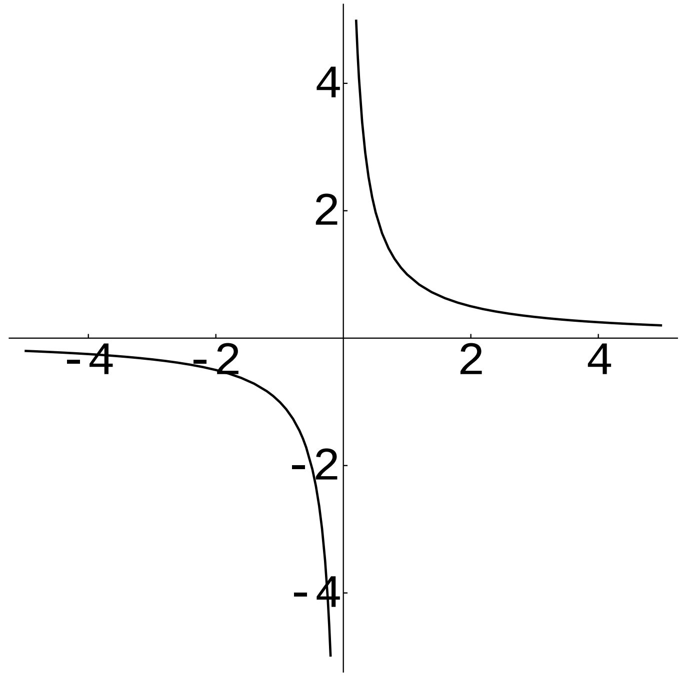
(3) Let $f(x) = \frac{1}{x}$.



- f is *undefined at* 0, meaningless to talk about continuity of f at 0.
- For $a \neq 0$,

$$\lim_{x \to a} f(x) = \lim_{x \to a} \frac{1}{x}$$

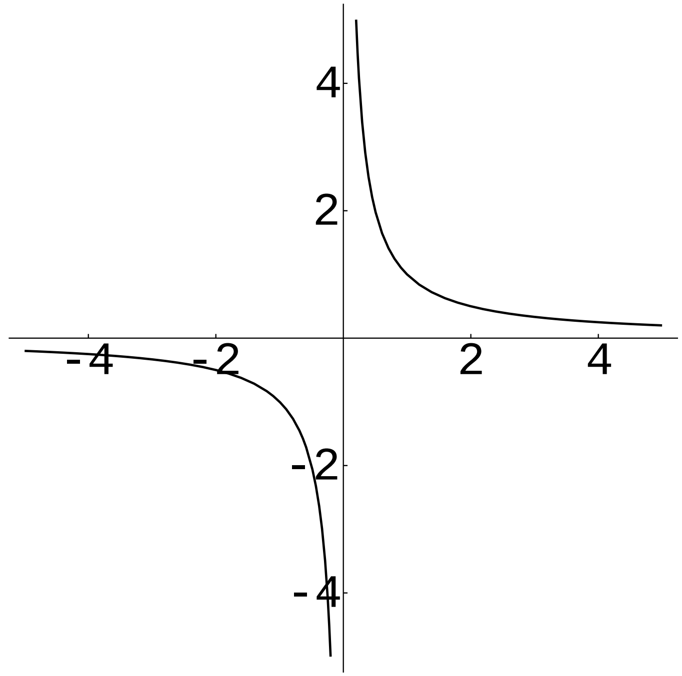
(3) Let $f(x) = \dfrac{1}{x}$.



- f is *undefined at* 0, meaningless to talk about continuity of f at 0.
- For $a \neq 0$,

$$\lim_{x \to a} f(x) = \lim_{x \to a} \frac{1}{x} = \frac{1}{a}$$

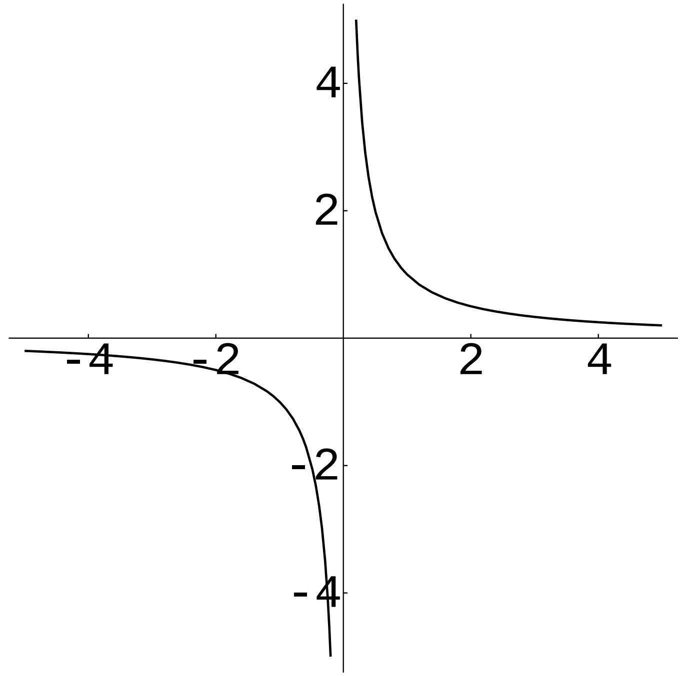
(3) Let $f(x) = \frac{1}{x}$.



- f is *undefined at* 0, meaningless to talk about continuity of f at 0.
- For $a \neq 0$,

$$\lim_{x \to a} f(x) = \lim_{x \to a} \frac{1}{x} = \frac{1}{a} = f(a).$$

(3) Let $f(x) = \dfrac{1}{x}$.



- f is *undefined at* 0, meaningless to talk about continuity of f at 0.
- For $a \neq 0$,

$$\lim_{x\to a} f(x) = \lim_{x\to a} \frac{1}{x} = \frac{1}{a} = f(a).$$

f is *continuous at every non-zero* a.

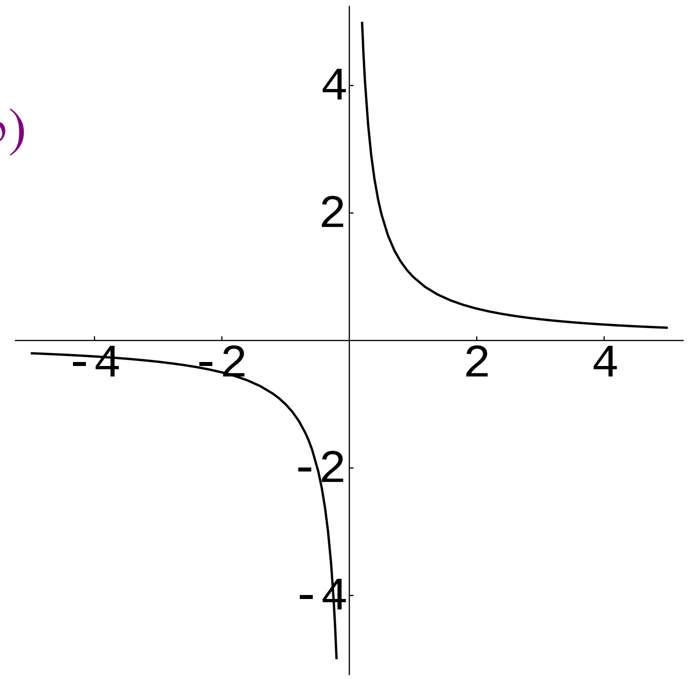
Definition A function f is said to be *continuous* on an open interval I if *it is continuous at every point in* I.

Definition A function f is said to be *continuous* on an open interval I if *it is continuous at every point in* I.

Example Let $f(x) = \frac{1}{x}$.

Then f is *continuous on* $(0, \infty)$

as well as *on* $(-\infty, 0)$.

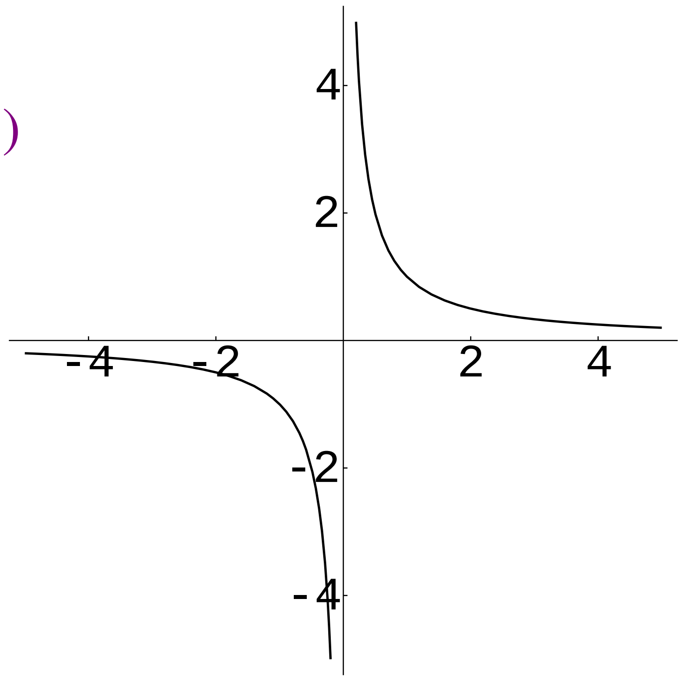


Definition A function f is said to be *continuous* on an open interval I if *it is continuous at every point in* I.

Example Let $f(x) = \dfrac{1}{x}$.

Then f is *continuous on* $(0, \infty)$

as well as *on* $(-\infty, 0)$.



Remark A function f is continuous on an open interval I means that the graph of f on I has *no "break" (can draw the graph continuously)*.

Remark

- Can be shown that *polynomial functions* are *continuous on* $\mathbb{R}$.

Remark

- Can be shown that *polynomial functions* are *continuous on* $\mathbb{R}$.

Example $\lim_{x \to 5}(x^2 - 2x + 3) =$

Remark

- Can be shown that *polynomial functions* are *continuous on* $\mathbb{R}$.

  Example $\lim_{x \to 5}(x^2 - 2x + 3) = 5^2 - 2(5) + 3 = 18$

Remark

- Can be shown that *polynomial functions* are *continuous on* $\mathbb{R}$.

  Example $\lim_{x \to 5}(x^2 - 2x + 3) = 5^2 - 2(5) + 3 = 18$

- Can also be shown that *rational functions* are *continuous <u>on any open interval contained in their domains</u>*, that is, *continuous at every $a \in$ domain*.

Remark

- Can be shown that *polynomial functions* are *continuous on* $\mathbb{R}$.

  Example $\lim_{x\to 5}(x^2 - 2x + 3) = 5^2 - 2(5) + 3 = 18$

- Can also be shown that *rational functions* are *continuous on any open interval contained in their domains*, that is, *continuous at every* $a \in$ *domain*.

  Example $\lim_{x\to 1} \dfrac{x^2 + 1}{x^3 - 2x^2 + x + 3} =$

Remark

- Can be shown that *polynomial functions* are *continuous on* $\mathbb{R}$.

  Example $\lim_{x\to 5}(x^2 - 2x + 3) = 5^2 - 2(5) + 3 = 18$

- Can also be shown that *rational functions* are *continuous on any open interval contained in their domains*, that is, *continuous at every* $a \in$ *domain*.

  Example $\lim_{x\to 1}\dfrac{x^2+1}{x^3-2x^2+x+3} = \dfrac{1^2+1}{1^3-2(1^2)+1+3} = \dfrac{2}{3}$

Remark

- Can be shown that *polynomial functions* are *continuous on* $\mathbb{R}$.

  Example $\lim_{x\to 5}(x^2 - 2x + 3) = 5^2 - 2(5) + 3 = 18$

- Can also be shown that *rational functions* are *continuous on any open interval contained in their domains*, that is, *continuous at every* $a \in$ *domain*.

  Example $\lim_{x\to 1}\dfrac{x^2+1}{x^3-2x^2+x+3} = \dfrac{1^2+1}{1^3-2(1^2)+1+3} = \dfrac{2}{3}$

  Example $\lim_{x\to 1}\dfrac{x^2-1}{x^3-2x^2+x}$

Remark

- Can be shown that *polynomial functions* are *continuous on* $\mathbb{R}$.

  Example $\lim_{x\to 5}(x^2 - 2x + 3) = 5^2 - 2(5) + 3 = 18$

- Can also be shown that *rational functions* are *continuous on any open interval contained in their domains*, that is, *continuous at every* $a \in$ *domain*.

  Example $\lim_{x\to 1}\dfrac{x^2+1}{x^3-2x^2+x+3} = \dfrac{1^2+1}{1^3-2(1^2)+1+3} = \dfrac{2}{3}$

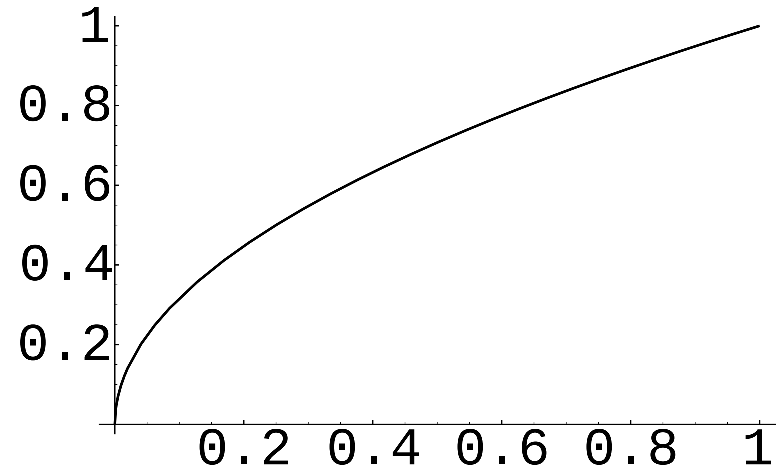
  Example $\lim_{x\to 1}\dfrac{x^2-1}{x^3-2x^2+x}$

  Can't use substitution, *because* $1 \notin$ *domain of the rational function*.

  Substitution gives $\dfrac{0}{0}$.

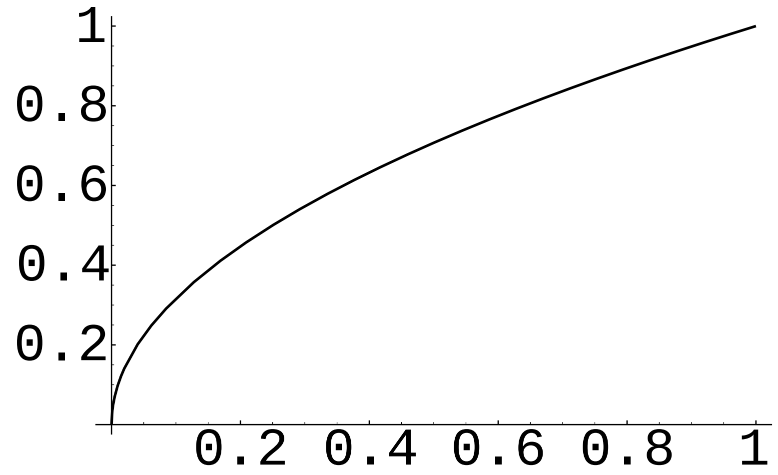
Example Consider $f : [0, \infty) \longrightarrow \mathbb{R}$ defined by

$$f(x) = \sqrt{x}$$



Example Consider $f : [0, \infty) \longrightarrow \mathbb{R}$ defined by

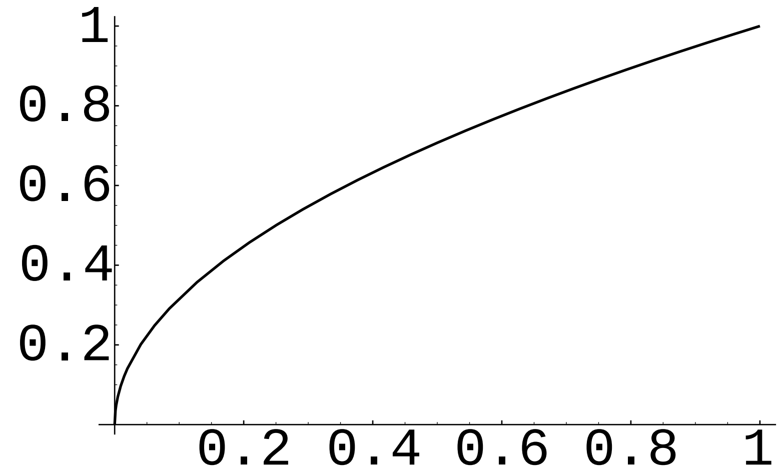
$$f(x) = \sqrt{x}$$



- f is *continuous at every point in* $(0, \infty)$.

Example Consider $f : [0, \infty) \longrightarrow \mathbb{R}$ defined by

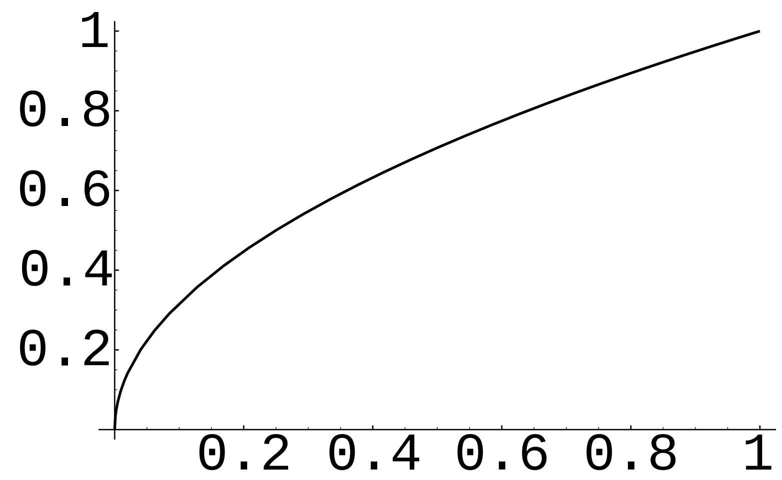
$$f(x) = \sqrt{x}$$



- f is *continuous at every point in* $(0, \infty)$.
- At $a = 0$, since $f(x)$ is undefined for $x < 0$, *can't consider* $\lim_{x \to 0-} f(x)$.

Example Consider $f : [0, \infty) \longrightarrow \mathbb{R}$ defined by

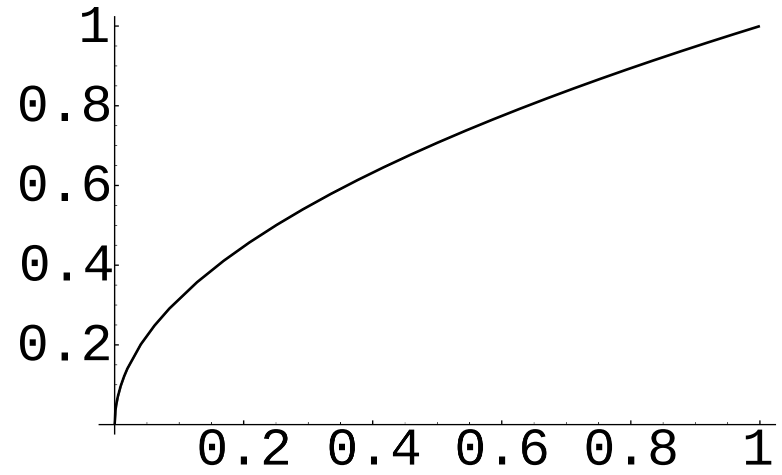
$$f(x) = \sqrt{x}$$



- f is *continuous at every point in* $(0, \infty)$.
- At $a = 0$, since $f(x)$ is undefined for $x < 0$, *can't consider* $\lim_{x \to 0-} f(x)$.
- **Note** $\lim_{x \to 0+} f(x) = 0 = f(0)$

Example Consider $f : [0, \infty) \longrightarrow \mathbb{R}$ defined by

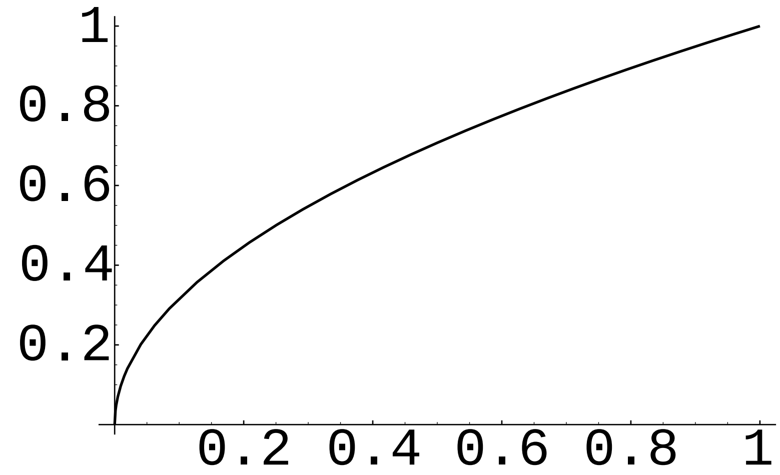
$$f(x) = \sqrt{x}$$



- f is *continuous at every point in* $(0, \infty)$.
- At $a = 0$, since $f(x)$ is undefined for $x < 0$, *can't consider* $\lim_{x \to 0-} f(x)$.
- **Note** $\lim_{x \to 0+} f(x) = 0 = f(0)$
- Say that f is *continuous at* 0. Thus f *is continuous on* $[0, \infty)$.

Example Consider $f : [0, \infty) \longrightarrow \mathbb{R}$ defined by

$$f(x) = \sqrt{x}$$



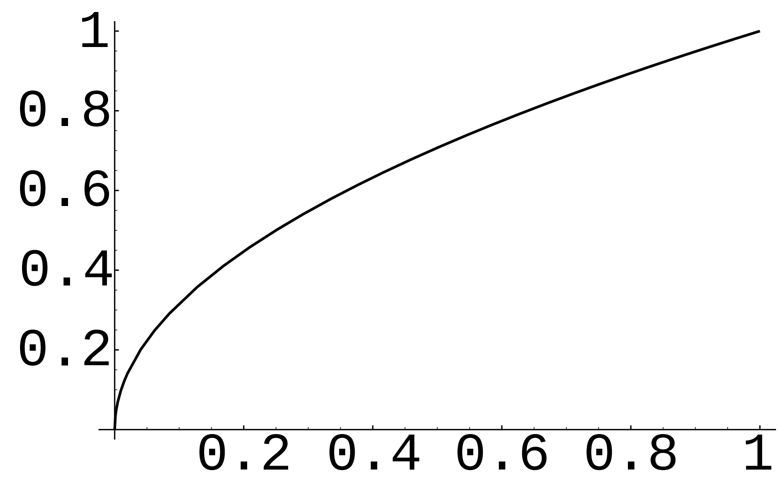
- f is *continuous at every point in* $(0, \infty)$.
- At $a = 0$, since $f(x)$ is undefined for $x < 0$, *can't consider* $\lim_{x \to 0-} f(x)$.
- **Note** $\lim_{x \to 0+} f(x) = 0 = f(0)$
- Say that f is *continuous at* 0. Thus f *is continuous on* $[0, \infty)$.

Example Let $g : [0, 1] \longrightarrow \mathbb{R}$ be defined by

$$g(x) = \sqrt{x} + 2\sqrt{1 - x}$$

Example Consider $f : [0, \infty) \longrightarrow \mathbb{R}$ defined by

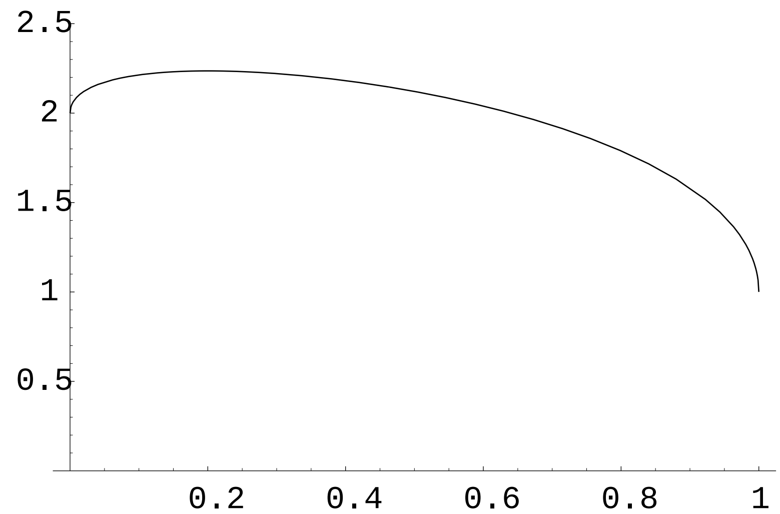
$$f(x) = \sqrt{x}$$



- f is *continuous at every point in* $(0, \infty)$.
- At $a = 0$, since $f(x)$ is undefined for $x < 0$, *can't consider* $\lim_{x \to 0-} f(x)$.
- **Note** $\lim_{x \to 0+} f(x) = 0 = f(0)$
- Say that f is *continuous at* 0. Thus f *is continuous on* $[0, \infty)$.

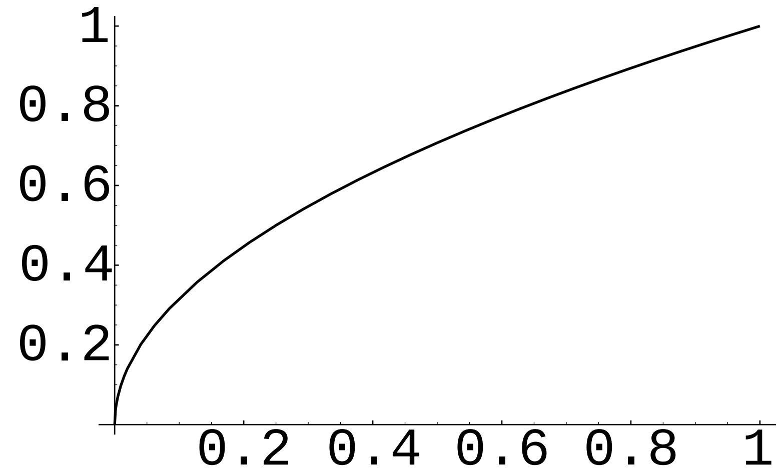
Example Let $g : [0, 1] \longrightarrow \mathbb{R}$ be defined by

$$g(x) = \sqrt{x} + 2\sqrt{1 - x}$$



Example Consider $f : [0, \infty) \longrightarrow \mathbb{R}$ defined by

$$f(x) = \sqrt{x}$$

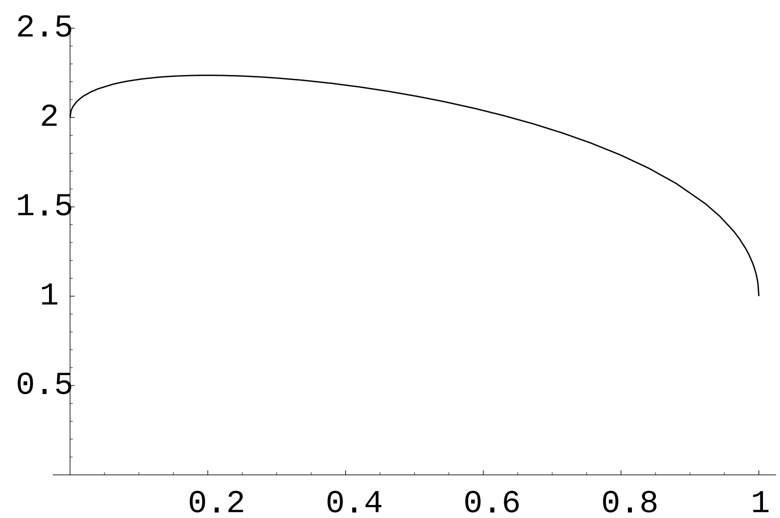


- f is *continuous at every point in* $(0, \infty)$.
- At $a = 0$, since $f(x)$ is undefined for $x < 0$, *can't consider* $\lim_{x \to 0-} f(x)$.
- **Note** $\lim_{x \to 0+} f(x) = 0 = f(0)$
- Say that f is *continuous at* 0. Thus f *is continuous on* $[0, \infty)$.

Example Let $g : [0, 1] \longrightarrow \mathbb{R}$ be defined by

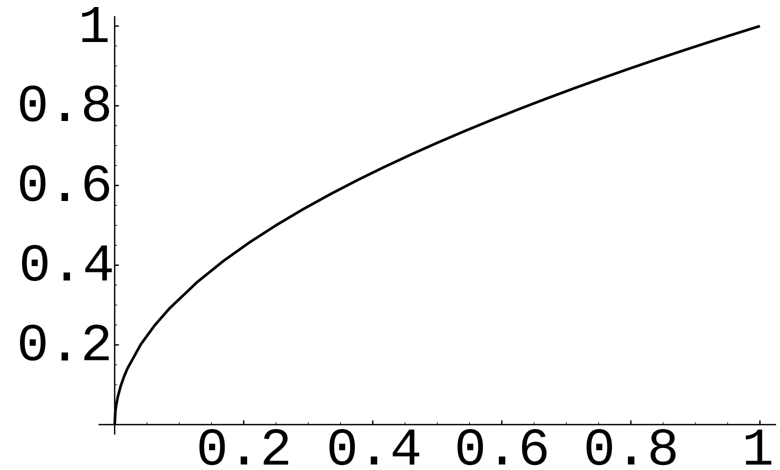
$$g(x) = \sqrt{x} + 2\sqrt{1 - x}$$

- g is *continuous at every point in* $(0, 1)$.



Example Consider $f : [0, \infty) \longrightarrow \mathbb{R}$ defined by

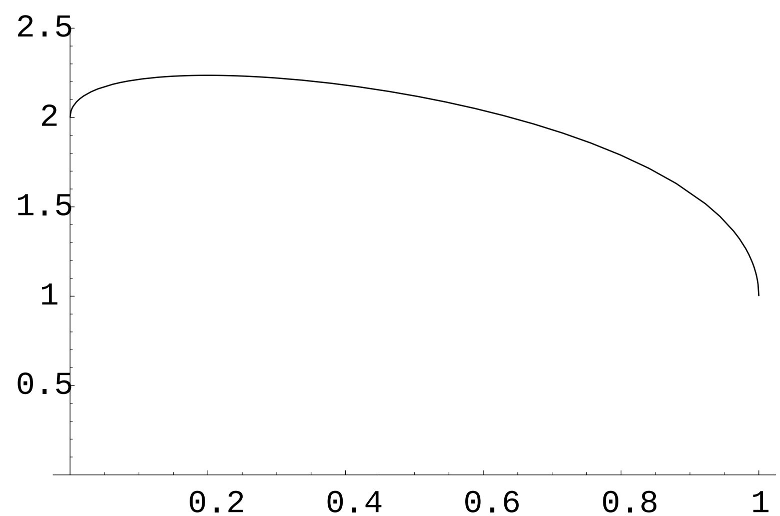
$$f(x) = \sqrt{x}$$



- f is *continuous at every point in* $(0, \infty)$.
- At $a = 0$, since $f(x)$ is undefined for $x < 0$, *can't consider* $\lim_{x \to 0-} f(x)$.
- **Note** $\lim_{x \to 0+} f(x) = 0 = f(0)$
- Say that f is *continuous at* 0. Thus f *is continuous on* $[0, \infty)$.

Example Let $g : [0, 1] \longrightarrow \mathbb{R}$ be defined by

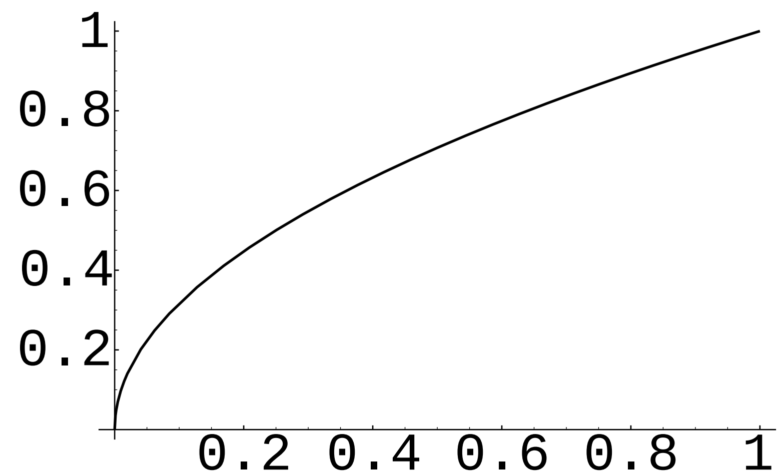
$$g(x) = \sqrt{x} + 2\sqrt{1 - x}$$



- g is *continuous at every point in* $(0, 1)$.
- **Note** $\lim_{x \to 0+} g(x) = 2 = g(0)$

Example Consider $f : [0, \infty) \longrightarrow \mathbb{R}$ defined by

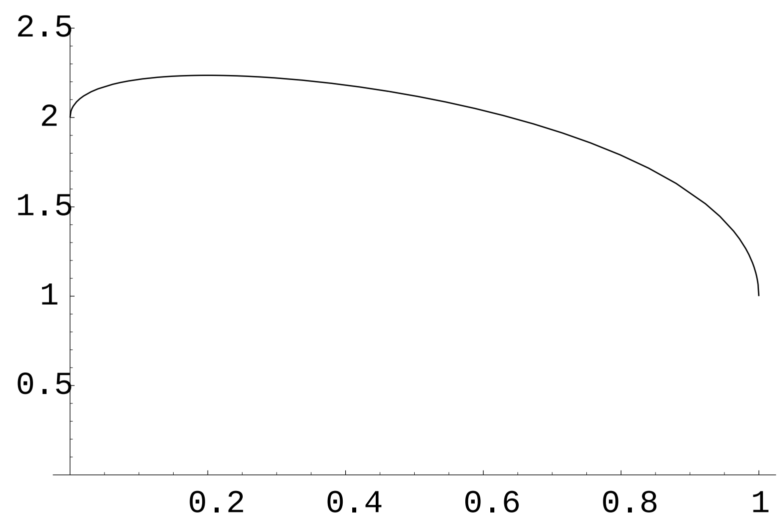
$$f(x) = \sqrt{x}$$



- f is *continuous at every point in* $(0, \infty)$.
- At $a = 0$, since $f(x)$ is undefined for $x < 0$, *can't consider* $\lim_{x \to 0-} f(x)$.
- **Note** $\lim_{x \to 0+} f(x) = 0 = f(0)$
- Say that f is *continuous at* 0. Thus f *is continuous on* $[0, \infty)$.

Example Let $g : [0, 1] \longrightarrow \mathbb{R}$ be defined by

$$g(x) = \sqrt{x} + 2\sqrt{1 - x}$$

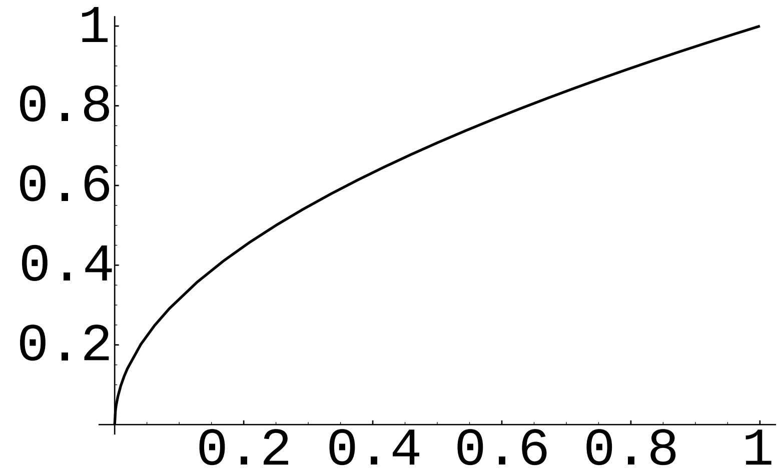


- g is *continuous at every point in* $(0, 1)$.
- **Note** $\lim_{x \to 0+} g(x) = 2 = g(0)$

  $\lim_{x \to 1-} g(x) = 1 = g(1)$

Example Consider $f : [0, \infty) \longrightarrow \mathbb{R}$ defined by

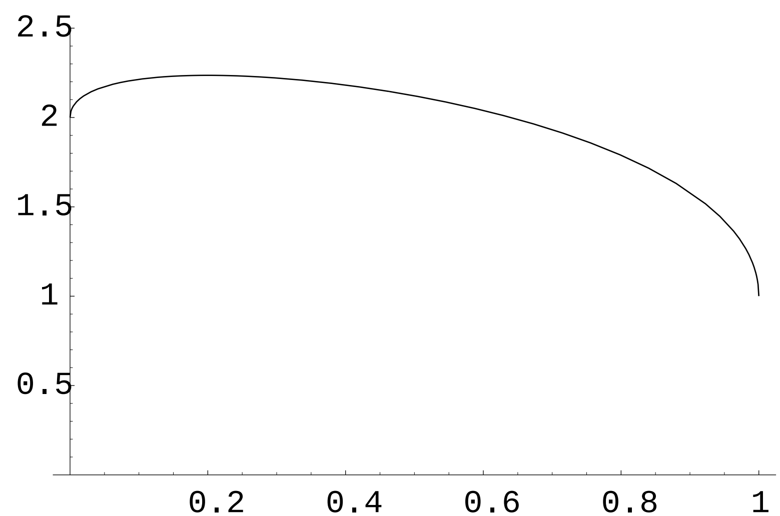
$$f(x) = \sqrt{x}$$



- f is *continuous at every point in* $(0, \infty)$.
- At $a = 0$, since $f(x)$ is undefined for $x < 0$, *can't consider* $\lim_{x \to 0-} f(x)$.
- **Note** $\lim_{x \to 0+} f(x) = 0 = f(0)$
- Say that f is *continuous at* 0. Thus f *is continuous on* $[0, \infty)$.

Example Let $g : [0, 1] \longrightarrow \mathbb{R}$ be defined by

$$g(x) = \sqrt{x} + 2\sqrt{1 - x}$$



- g is *continuous at every point in* $(0, 1)$.
- **Note** $\lim_{x \to 0+} g(x) = 2 = g(0)$

  $\lim_{x \to 1-} g(x) = 1 = g(1)$
- Say that g is *continuous on* $[0, 1]$.