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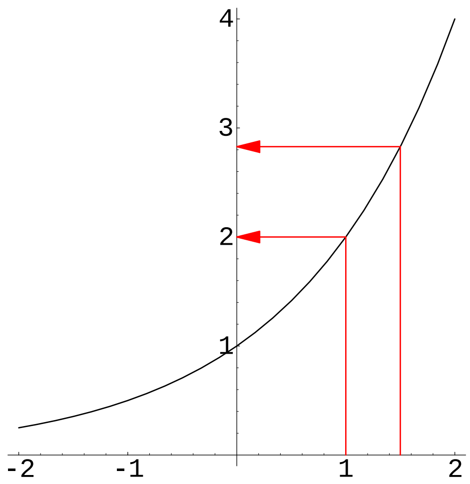

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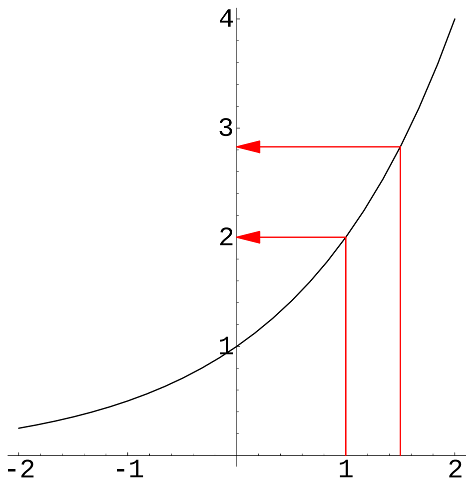

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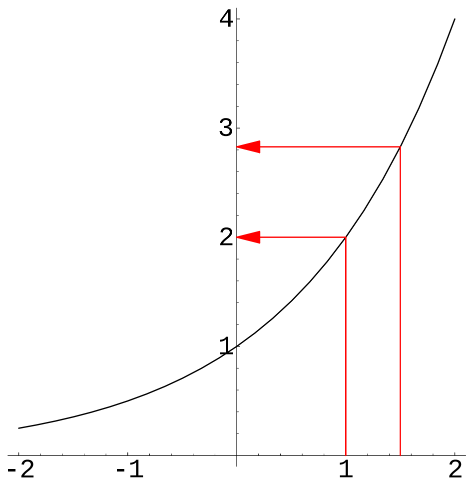

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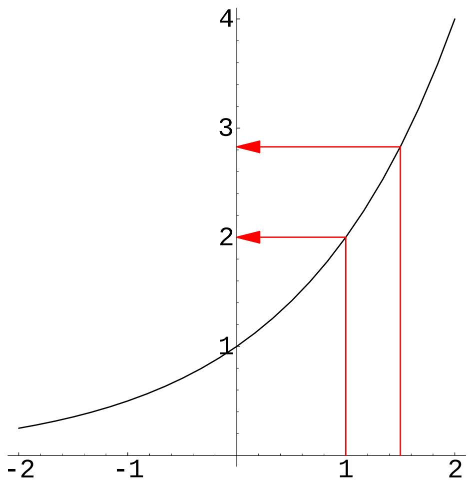

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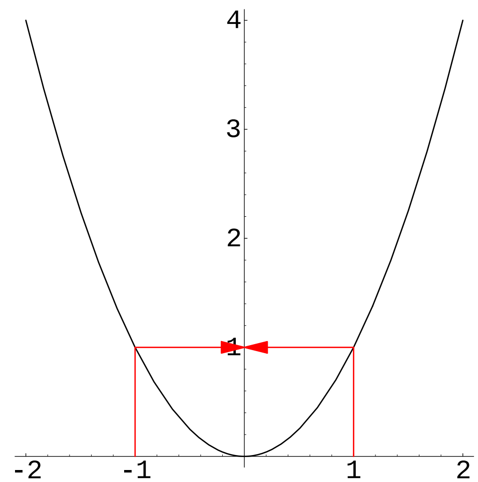

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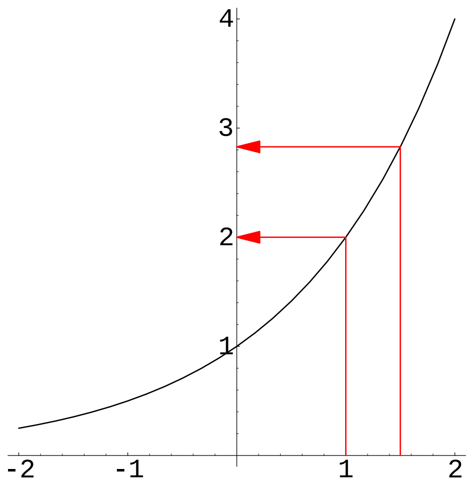

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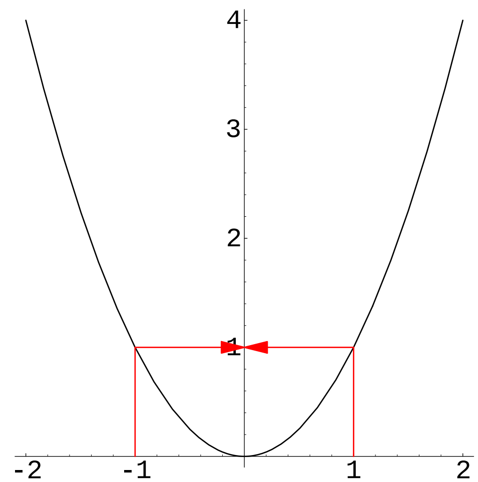

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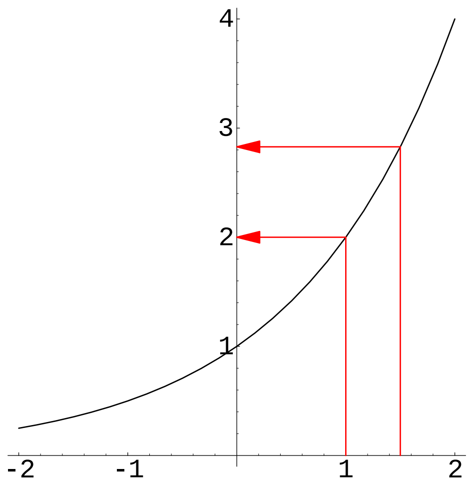

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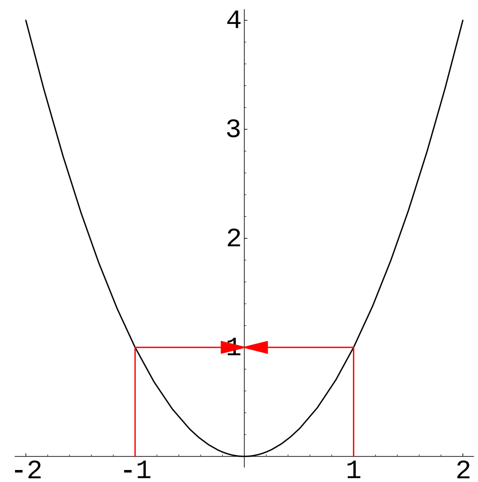

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Remark Geometrically, f is injective means that *graph of f intersects every horizontal line in at most one point*.

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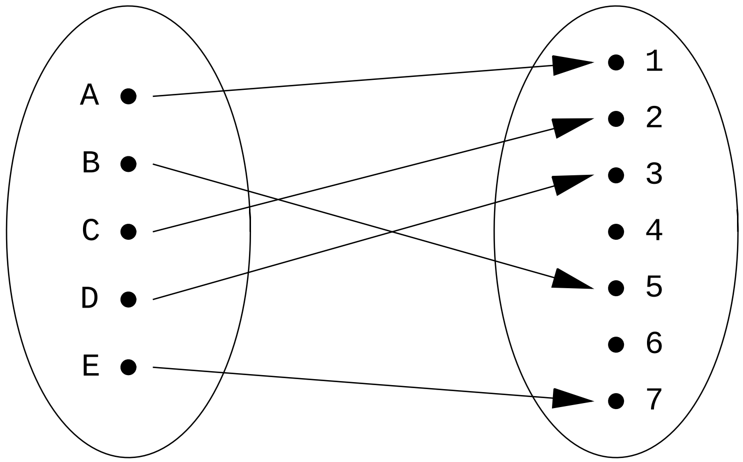

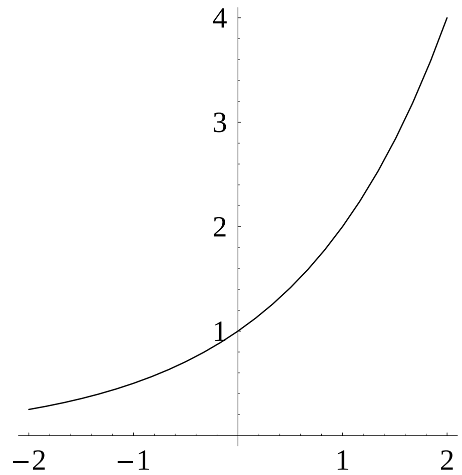

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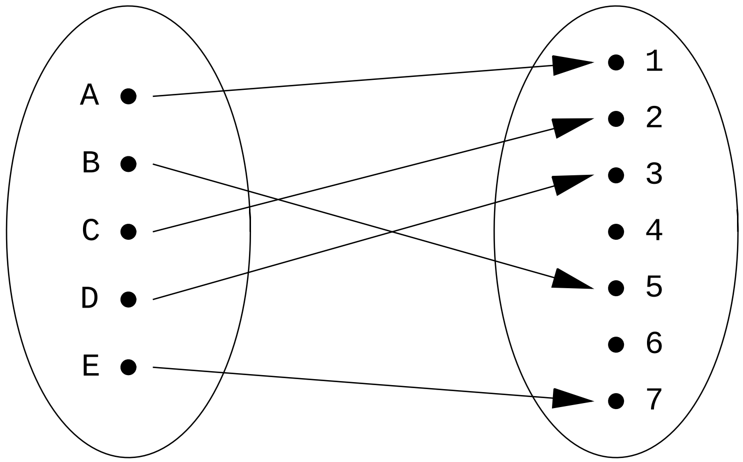

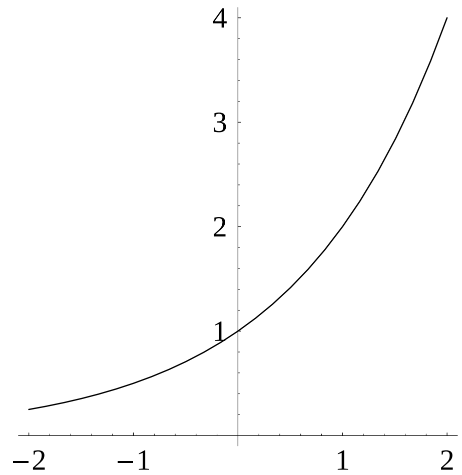

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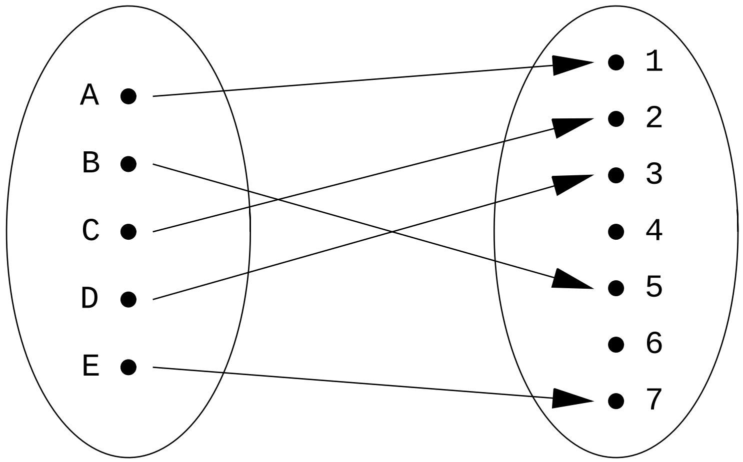

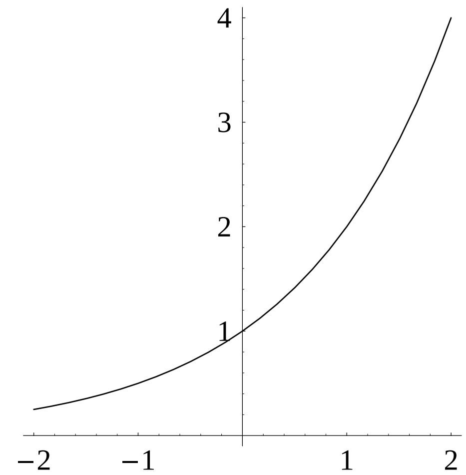

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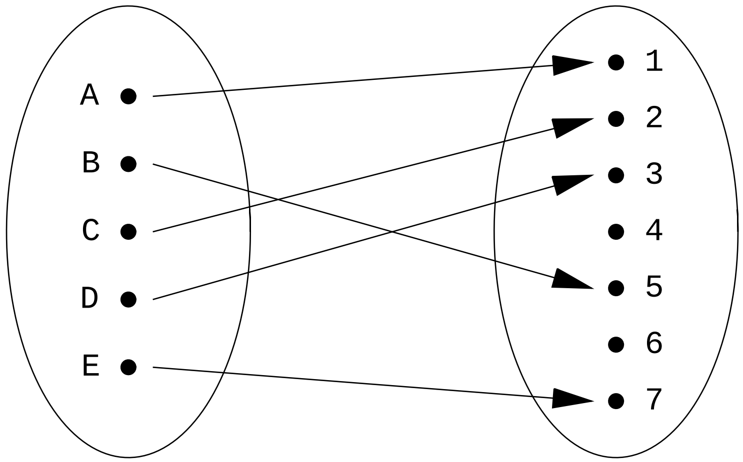

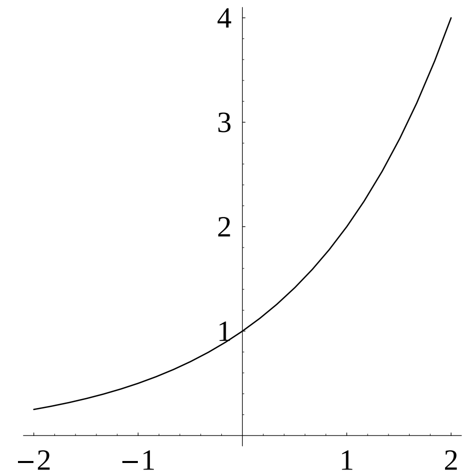

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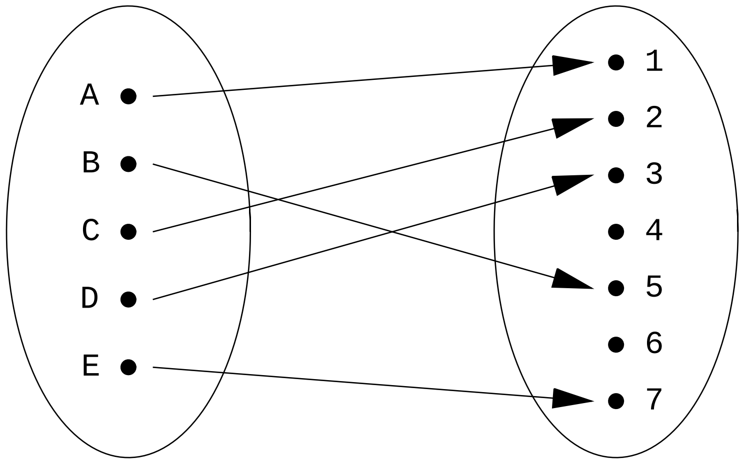

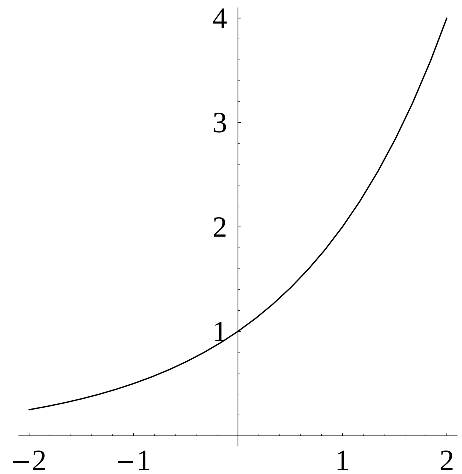

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- Get a function, called the **inverse function** of f, denoted by $f^{-1} : Y_1 \longrightarrow X$, where

$$f^{-1}(y) = x \qquad \text{means} \qquad f(x) = y$$

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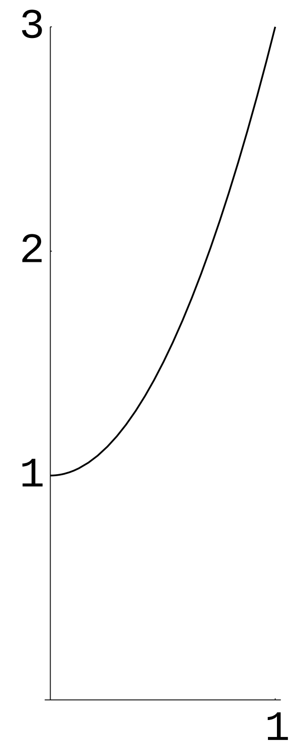

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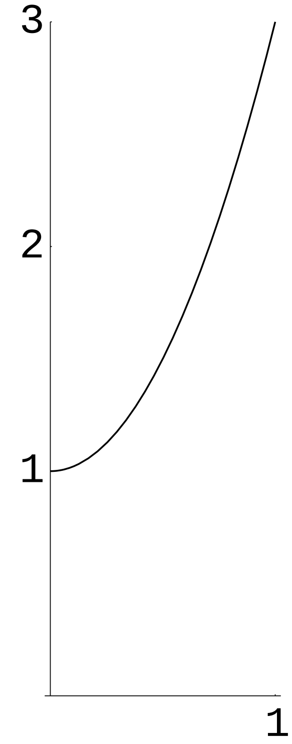

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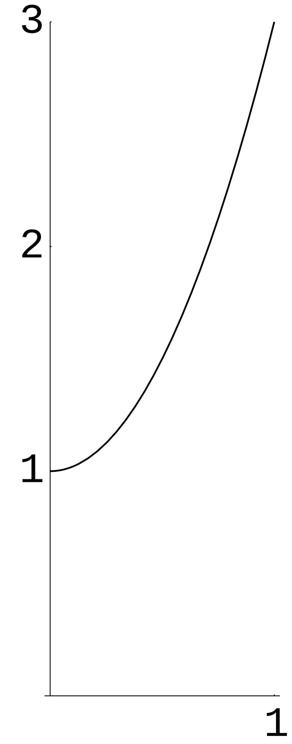

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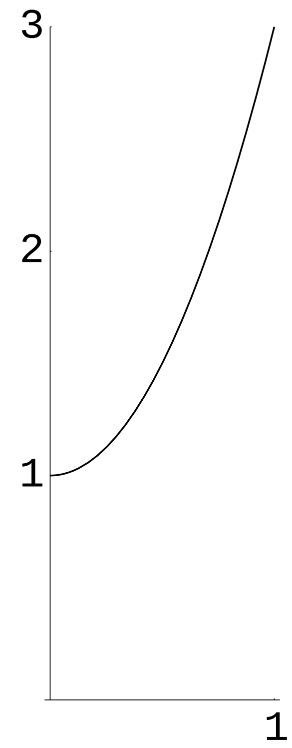

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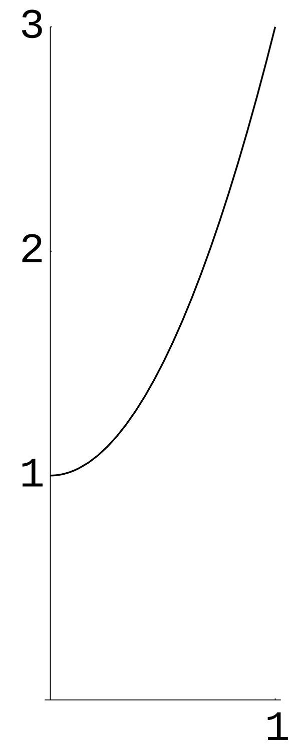

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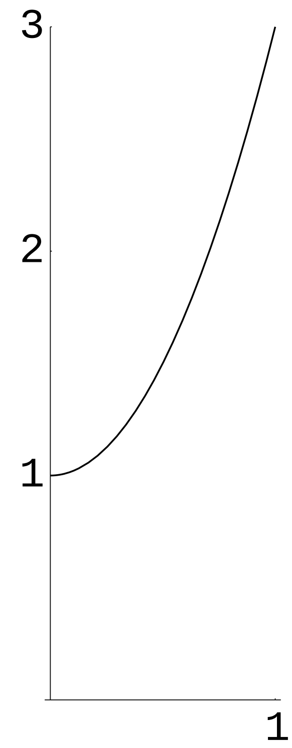

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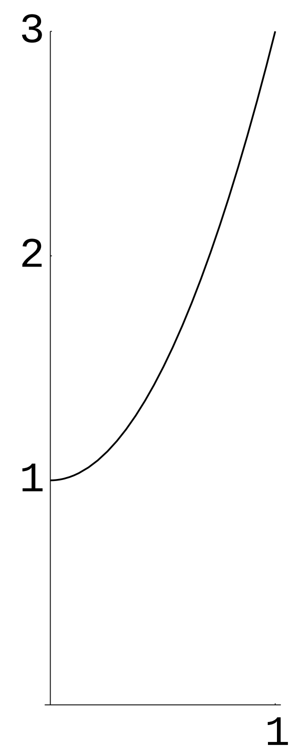

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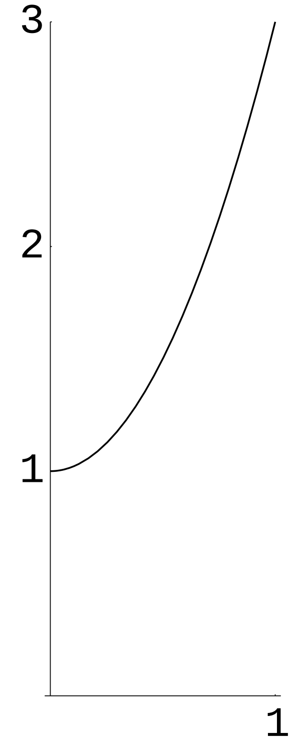

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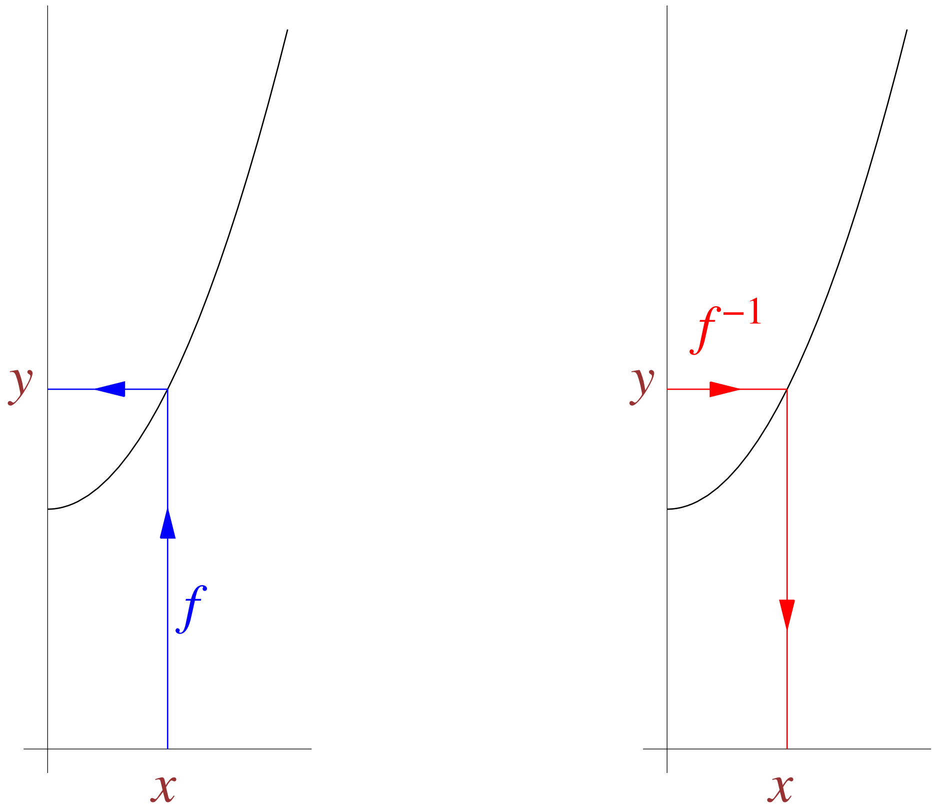

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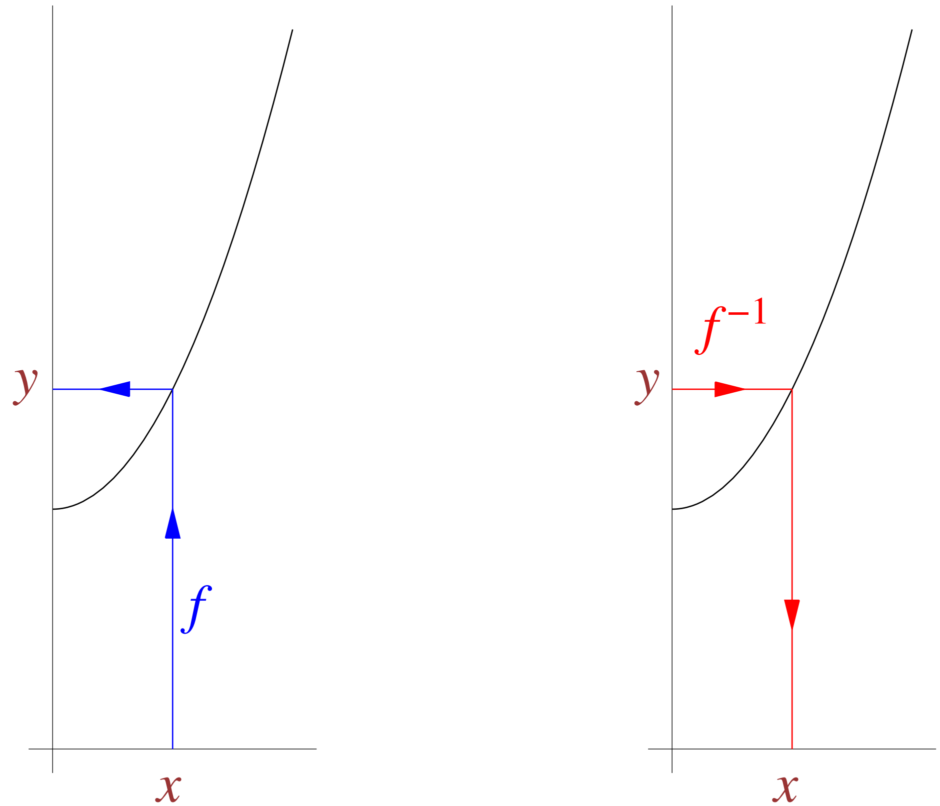

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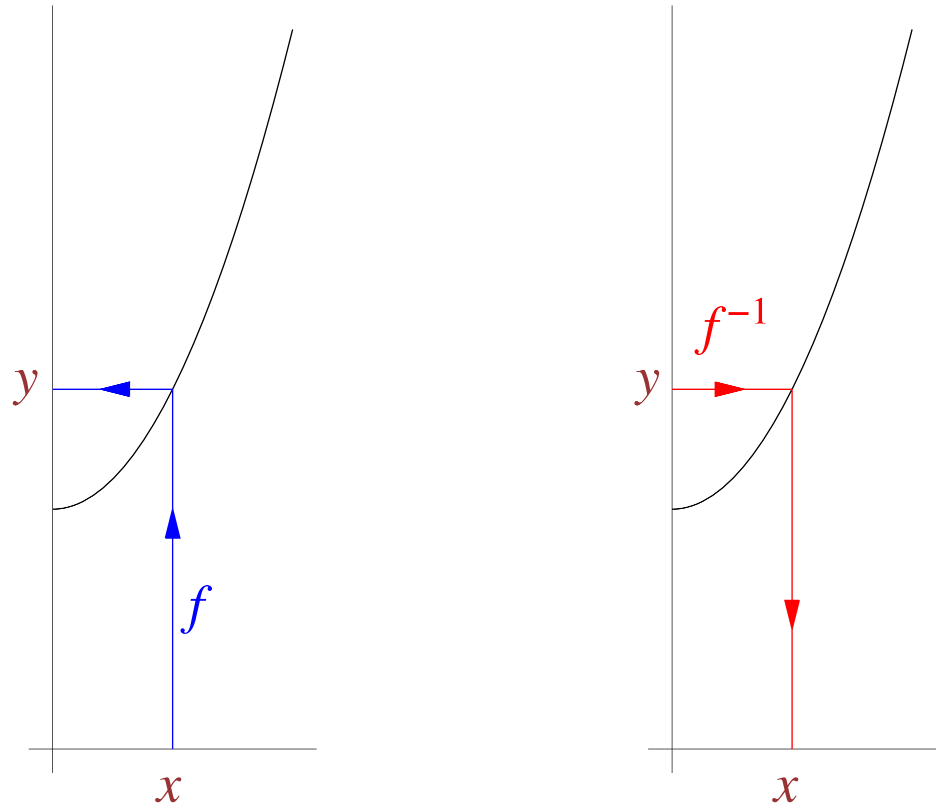

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Warning $f^{-1}(x) \neq \dfrac{1}{f(x)}$

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(4) To get formula for $f^{-1}(x)$, replace y by x in $f^{-1}(y) =$ the expression in y

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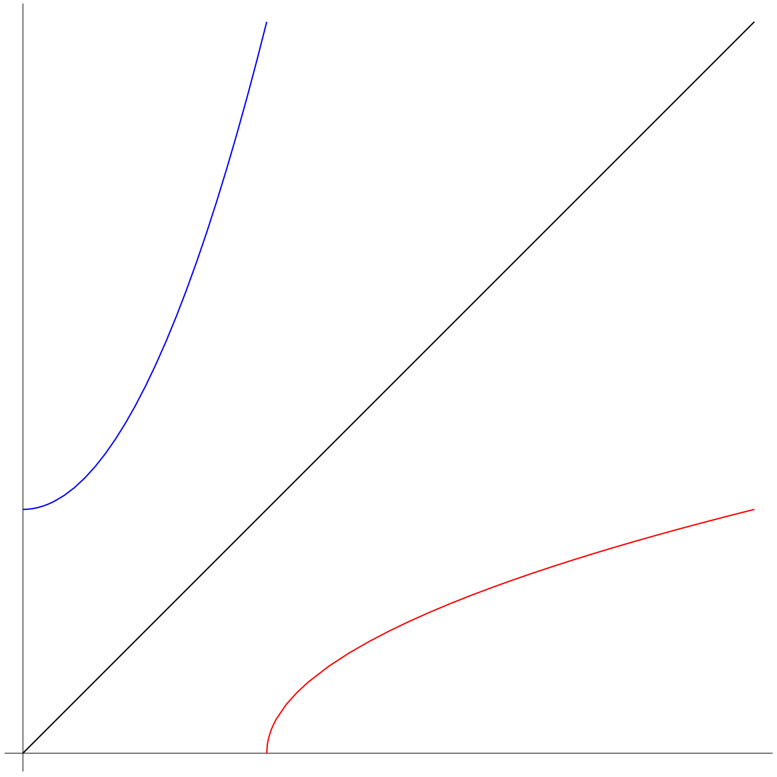

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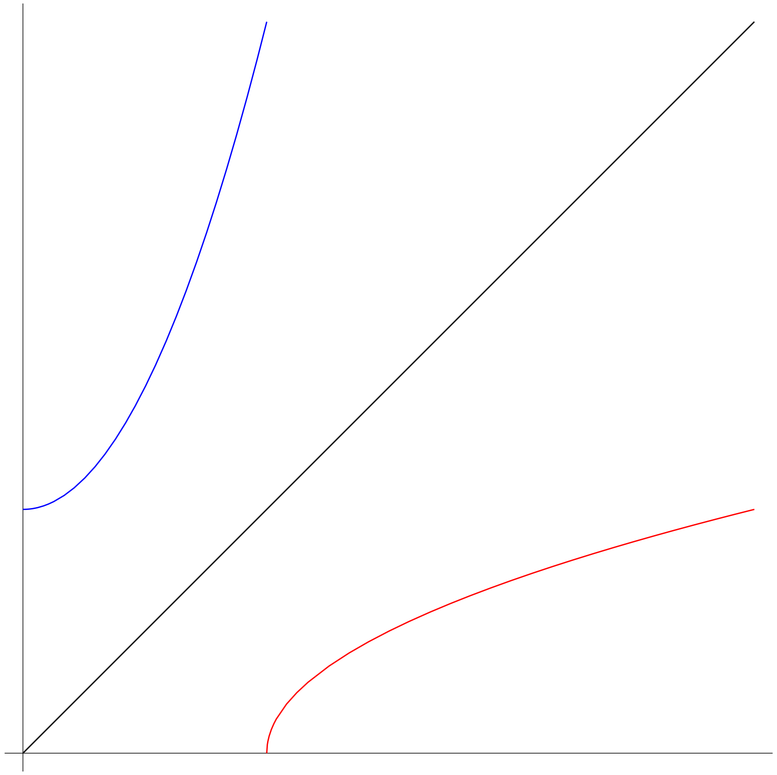

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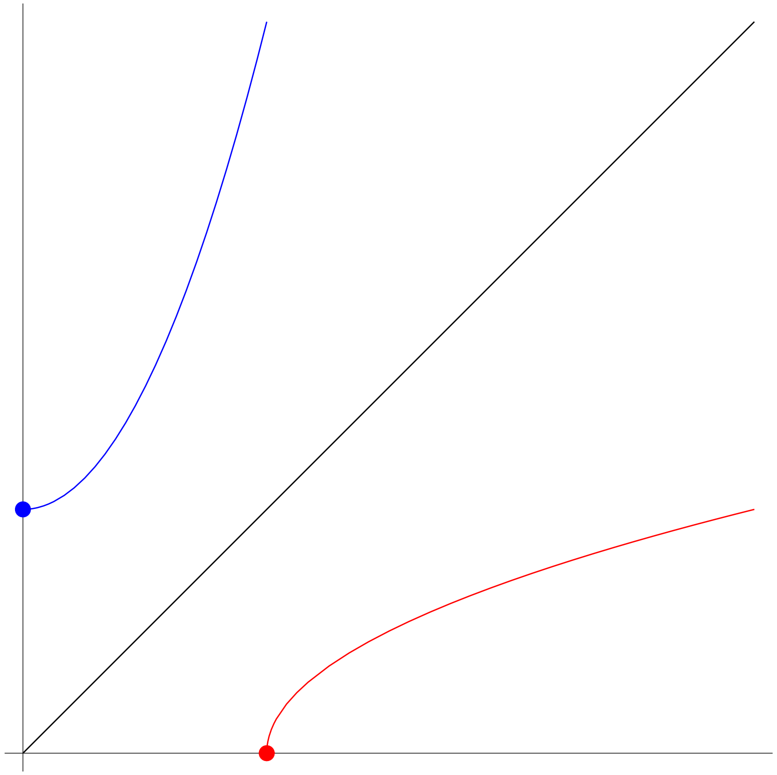

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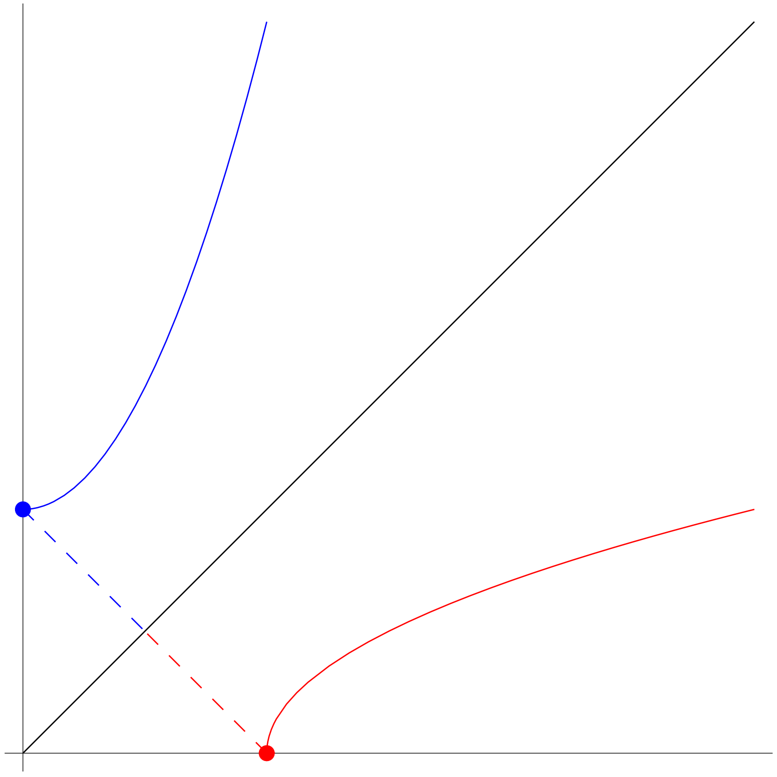

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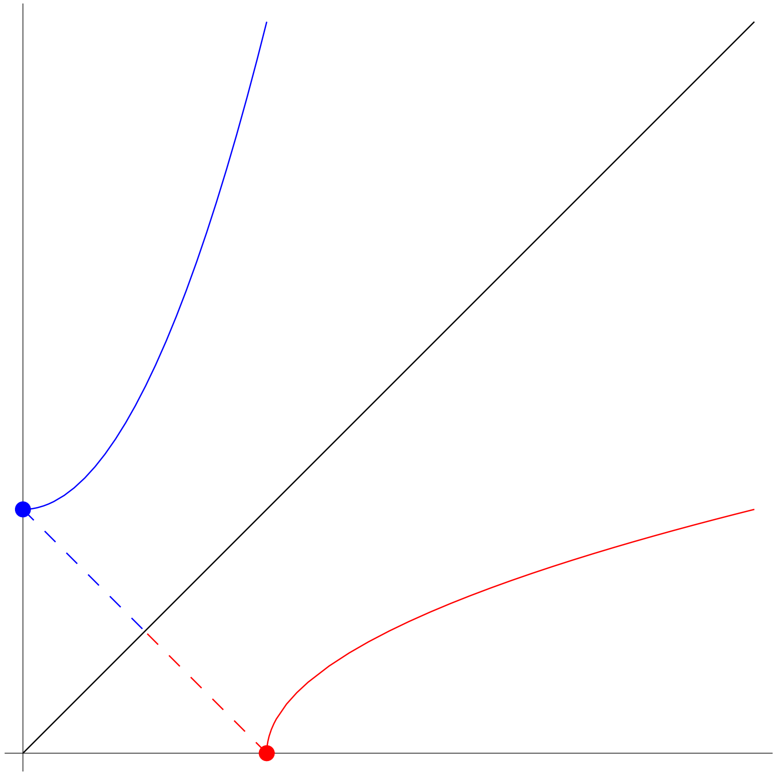
- In general, (a, b) *lies on graph of f iff (b, a) lies on graph of f^{-1}.*

  $\because \; f(a) = b \iff f^{-1}(b) = a$



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Solving we get, $x = 1$ or $x = -3$.

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&\quad \vdots \\
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Chapter 3: Limits

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(2) Limits of Sequences

(3) Limits of Functions at Infinity

(4) One-Sided Limits

(5) Two-Sided Limits

(6) Continuous Functions

Objectives

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Suppose an object moves along the x-axis and its *displacement* (in meters) s at time t (in seconds) is given by

$$s(t) = t^2, \qquad t \geq 0$$

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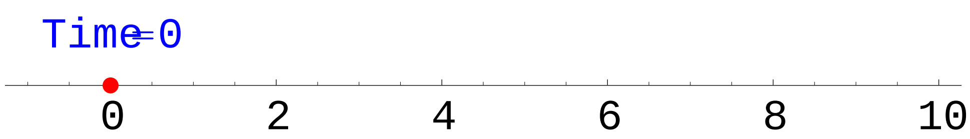

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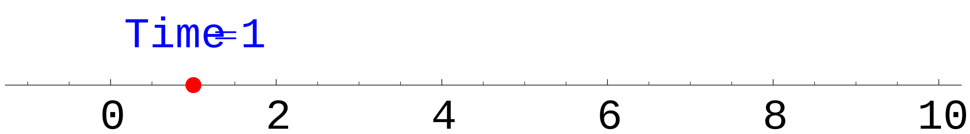

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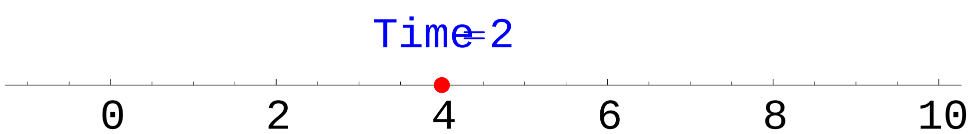

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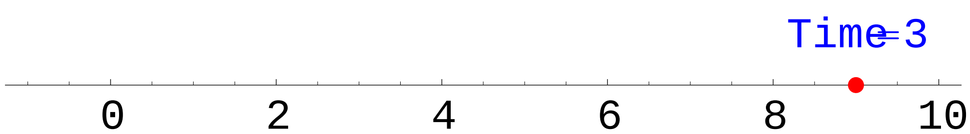

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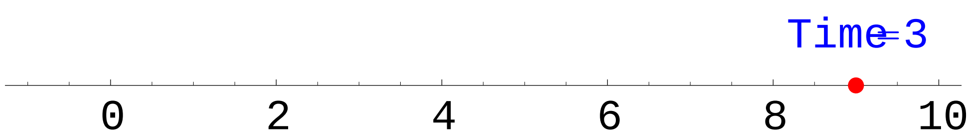

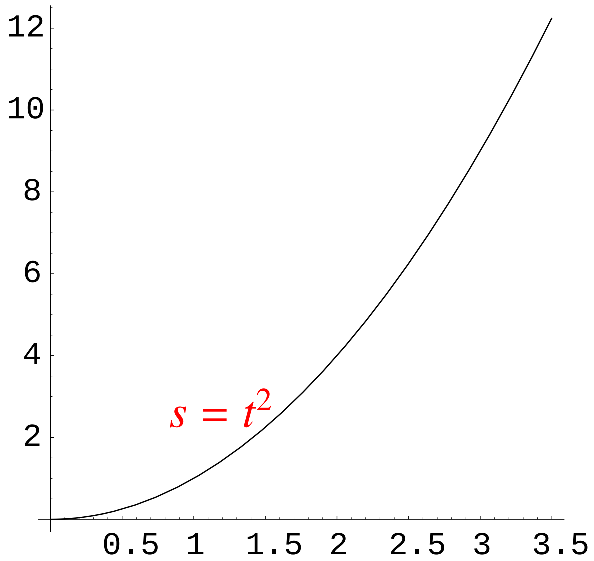

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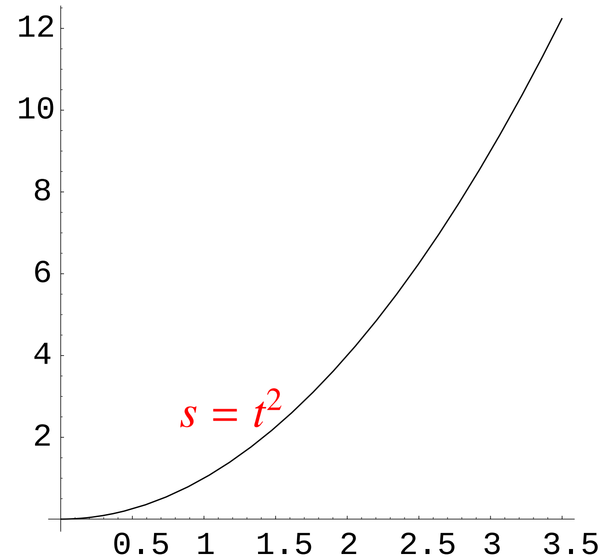

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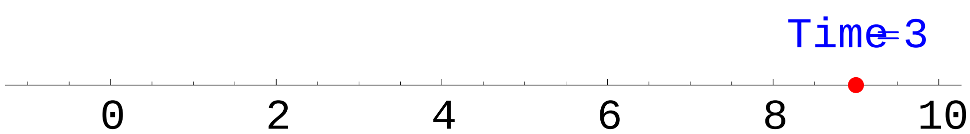

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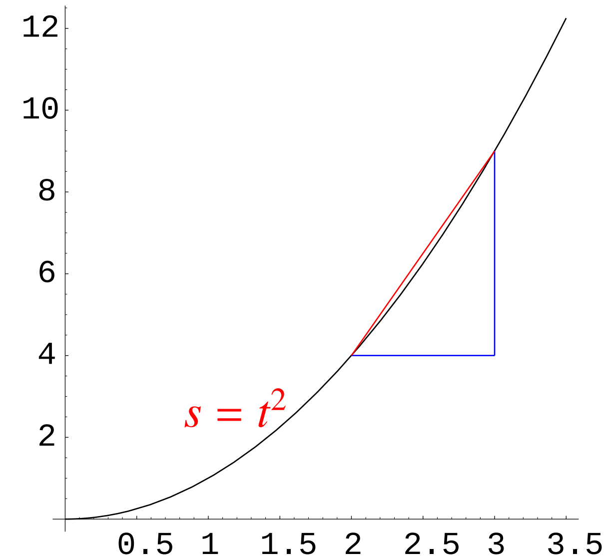

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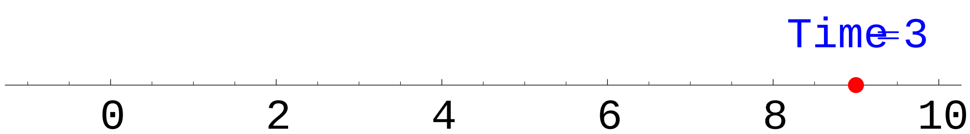

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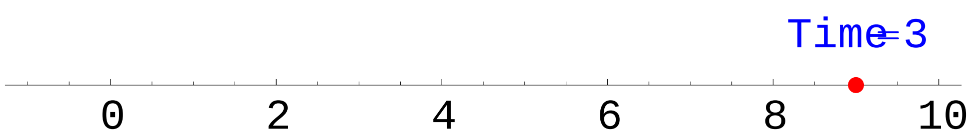

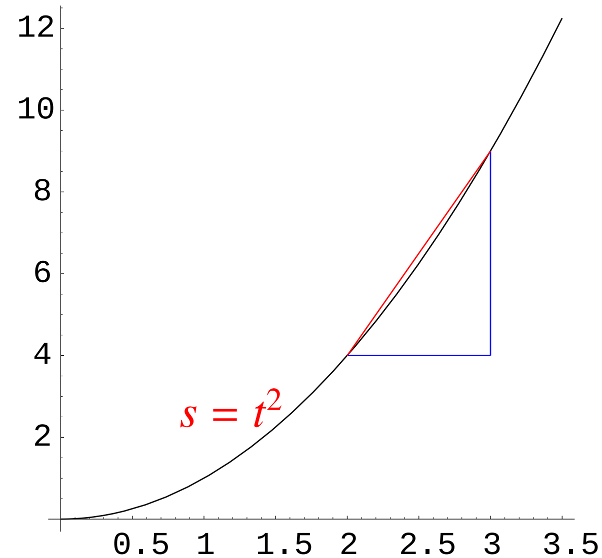

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⋮	show velocity.xls	

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- from $t = 2$ to $t = 2.5$
- from $t = 2$ to $t = 2.25$
- from $t = 2$ to $t = 2.125$

from $t = 2$ to $t = 2 + \frac{1}{2^n}$.

n	Time interval	Velocity
0	[2,3]	5 m/s
1	[2,2.5]	4.5 m/s
2	[2,2.25]	4.25 m/s
3	[2,2.125]	4.125 m/s
4	[2,2.0625]	4.0625 m/s
$\vdots$	show velocity.xls	

Average velocity over time interval $[2, 2+\frac{1}{2^n}]$ is

$$= \frac{(2+\frac{1}{2^n})^2 - 2^2}{\frac{1}{2^n}}$$

$$= \frac{4 + 2 \cdot 2 \cdot \frac{1}{2^n} + \left(\frac{1}{2^n}\right)^2 - 4}{\frac{1}{2^n}}$$

$$= 4 + \frac{1}{2^n}$$

When n is very large *(short time interval)*, aevarge velocity is very close to 4.

Consider short time intervals about $t = 2$:

- from $t = 2$ to $t = 3$
- from $t = 2$ to $t = 2.5$
- from $t = 2$ to $t = 2.25$
- from $t = 2$ to $t = 2.125$

from $t = 2$ to $t = 2 + \frac{1}{2^n}$.

n	Time interval	Velocity
0	[2,3]	5 m/s
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⋮	show velocity.xls	

Average velocity over time interval $[2, 2+\frac{1}{2^n}]$ is

$$= \frac{(2+\frac{1}{2^n})^2 - 2^2}{\frac{1}{2^n}}$$

$$= \frac{4 + 2 \cdot 2 \cdot \frac{1}{2^n} + \left(\frac{1}{2^n}\right)^2 - 4}{\frac{1}{2^n}}$$

$$= 4 + \frac{1}{2^n}$$

When n is very large *(short time interval)*, aveerge velocity is very close to 4.

The velocity at $t = 2$ (called *instantaneous velocity*) is 4 m/s.