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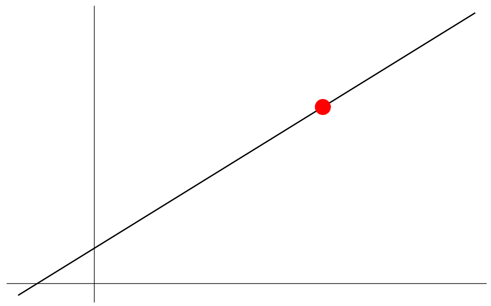

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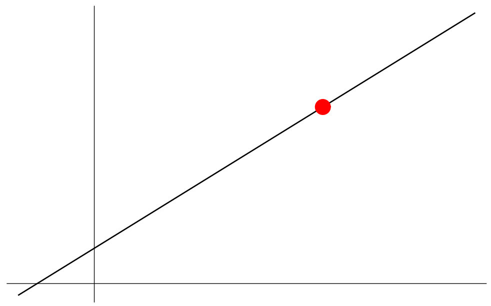

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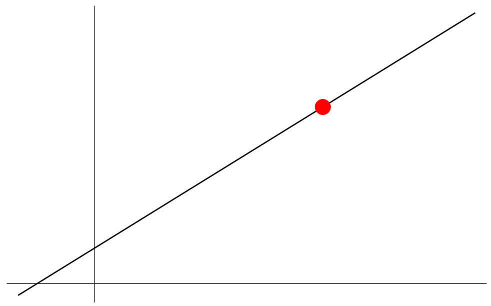

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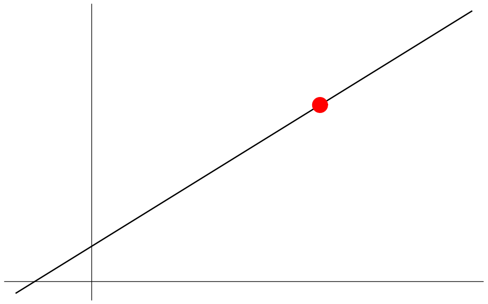

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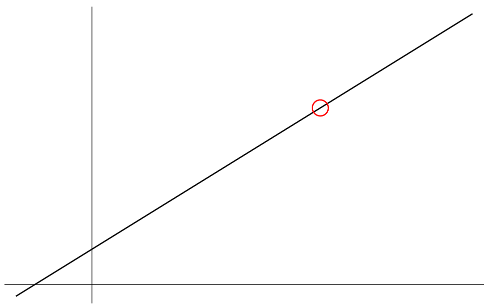

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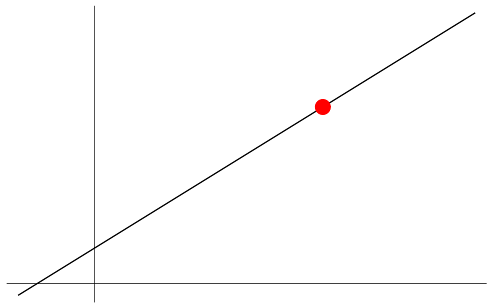

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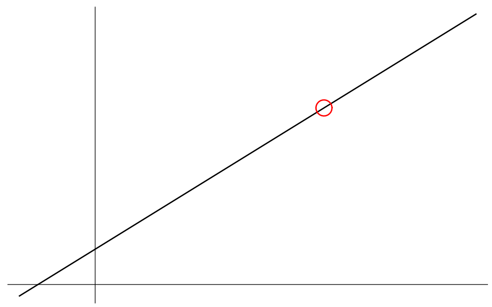
However, once you get (1), can obtain point-slope form easily.



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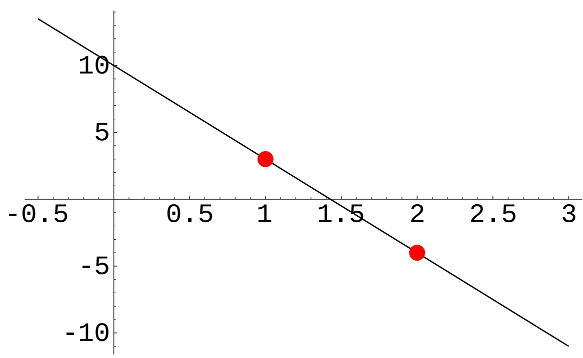

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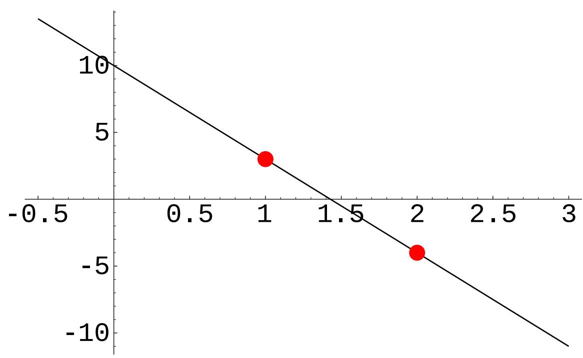

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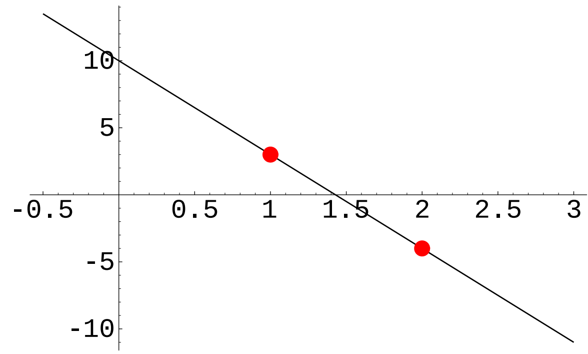

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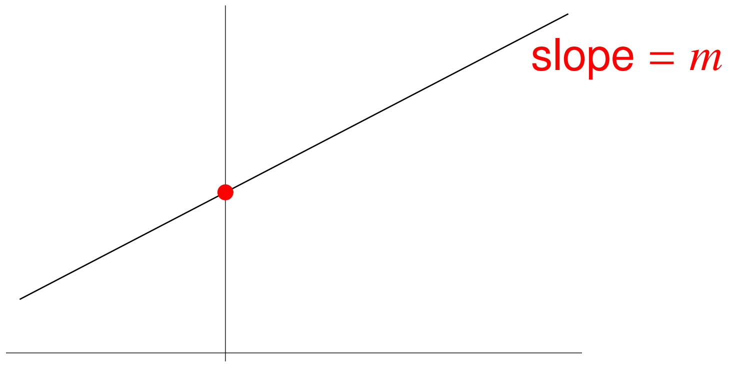

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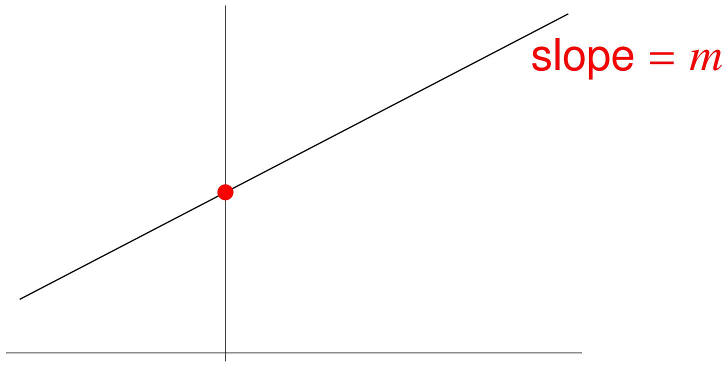

- Can be written in the form

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- The required slope is $\frac{2}{5}$.

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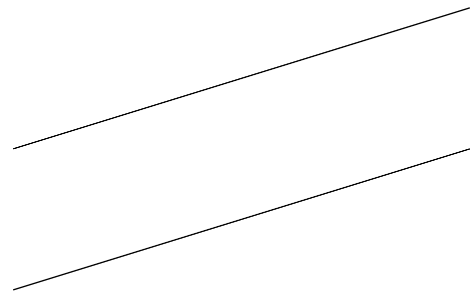

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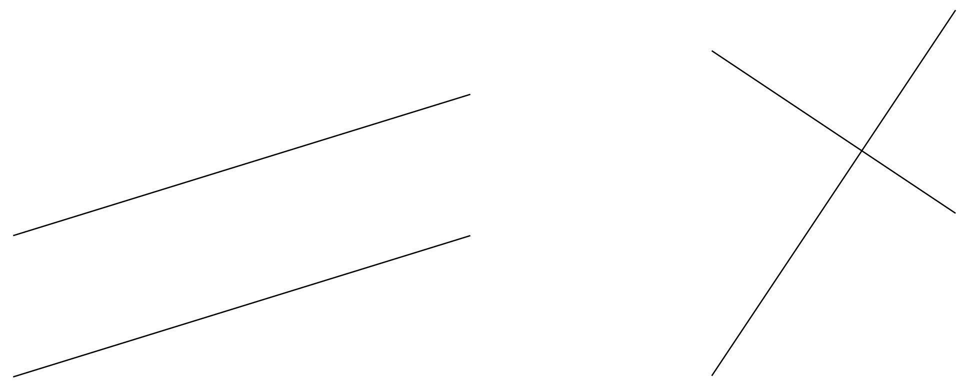

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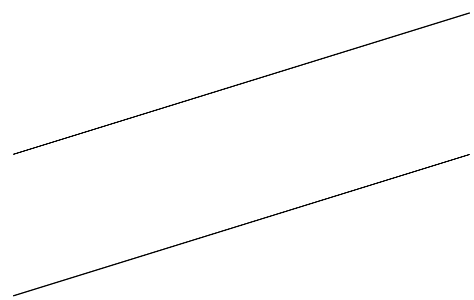

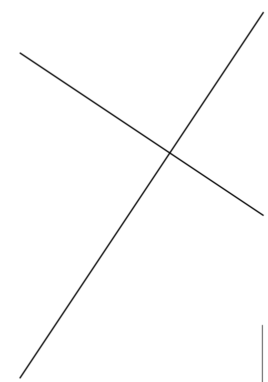

Note

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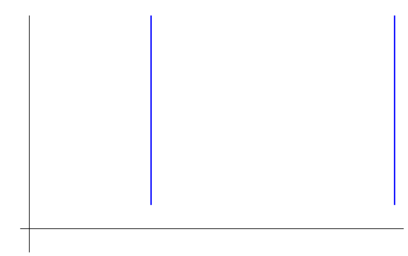

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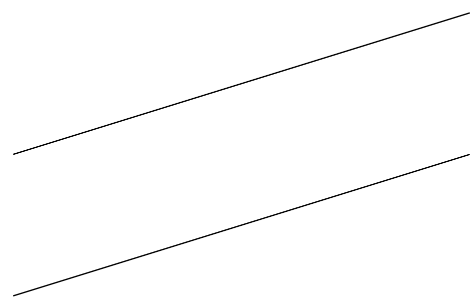

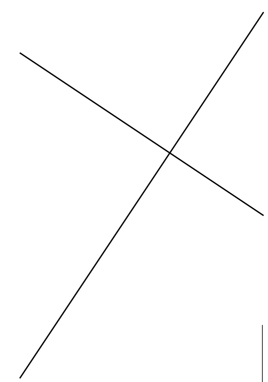

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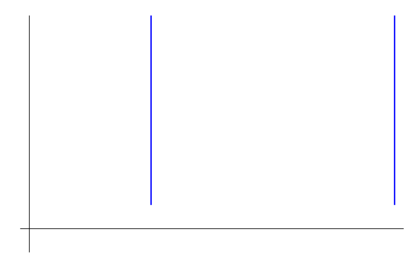

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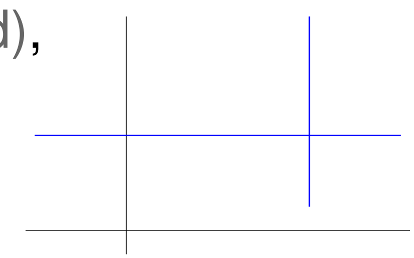

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$$3x - y - 11 = 0 \quad \text{and} \quad x + 3y + 3 = 0$$

respectively.

Pythagorus Theorem and Distance formula

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Let a, b and c be the (lengths of the) sides of a right-angled triangle where c is the hypotenus. Then

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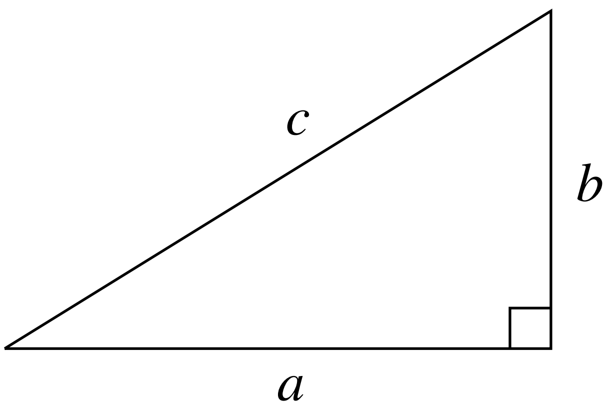

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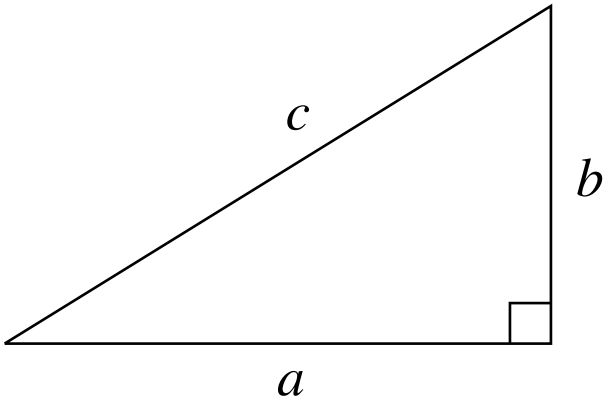

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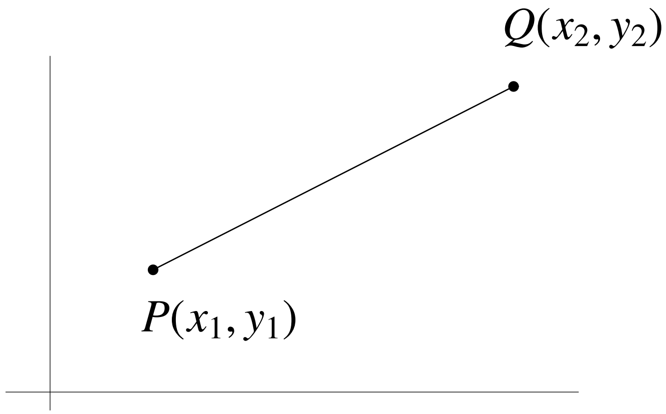

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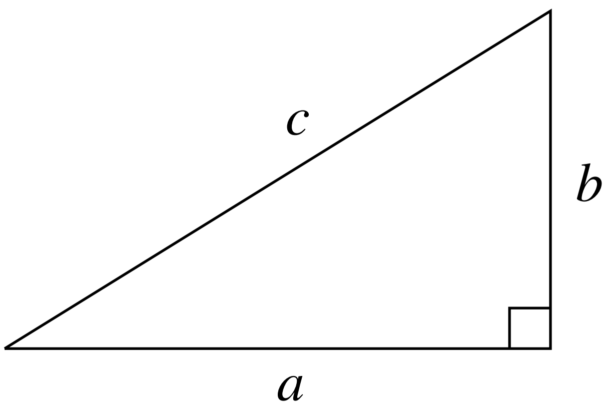

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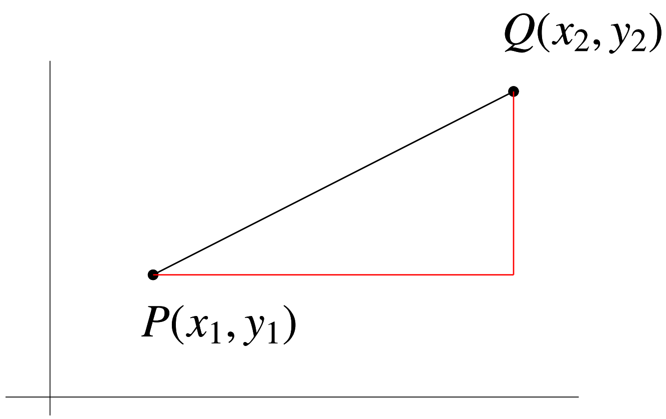
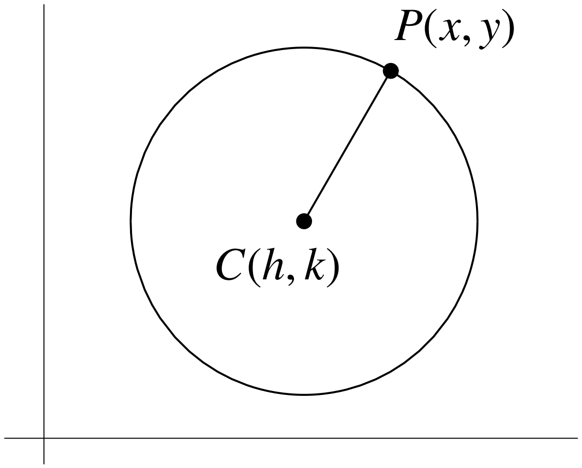


Equations of Circles

Consider the circle with center at $C(h, k)$ and radius r.



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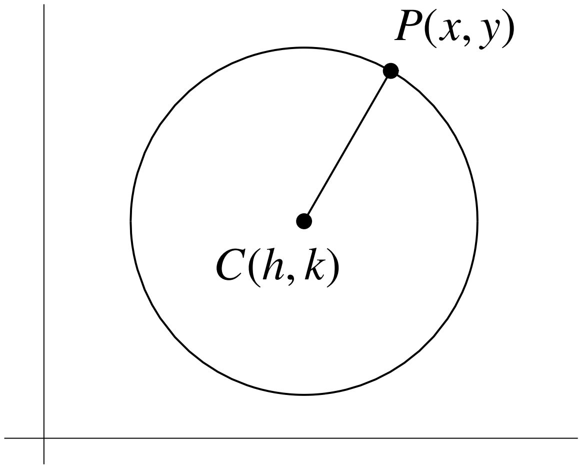

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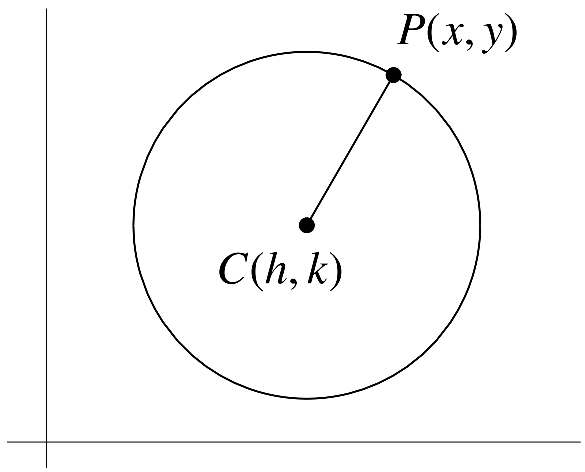

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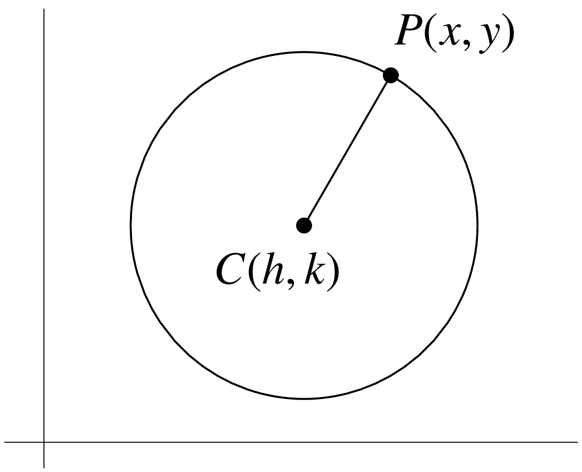

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- Squarring both sides ...

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$$x^2 - 4x + y^2 + 6y - 12 = 0$$

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(x-2)^2 + (y-(-3))^2 &= 5^2
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The center is $(2, -3)$ and the radius is 5.

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Summary a = *square of (half of the coefficient of y)*

Graphs of functions

Let f be a function *(with domain and range contained in $\mathbb{R}$)*. The set of points

$$\{(x, y) \in \mathbb{R}^2 : x \in \mathrm{dom}\,(f),\ y = f(x)\}$$

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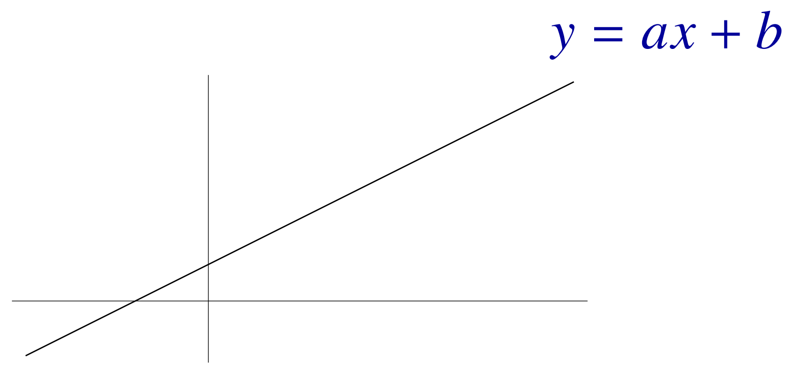

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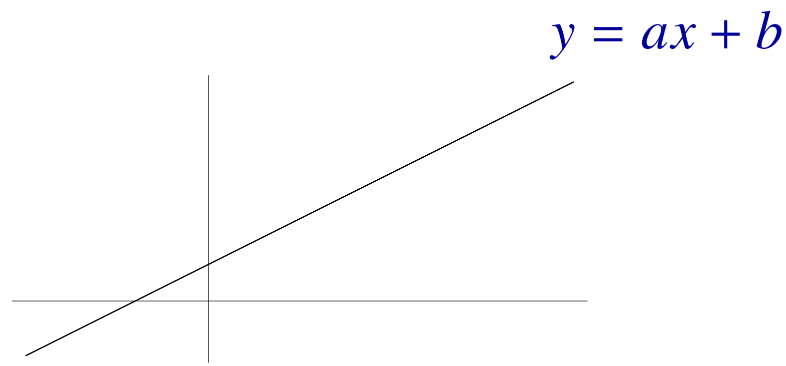

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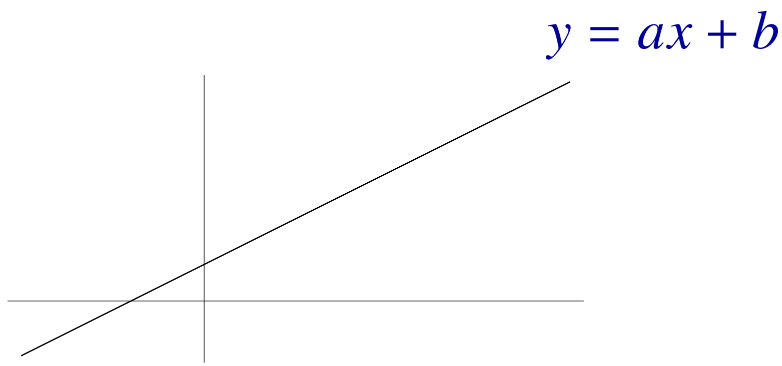

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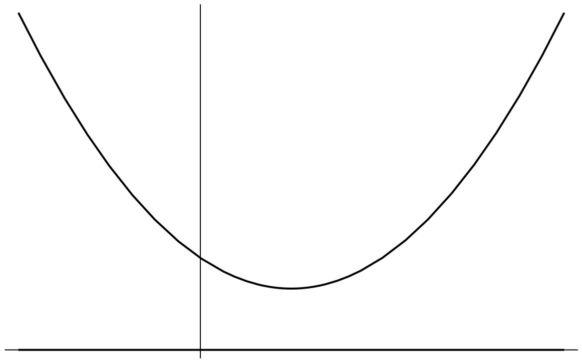

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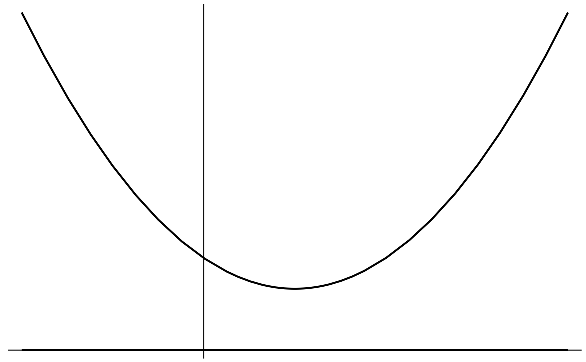

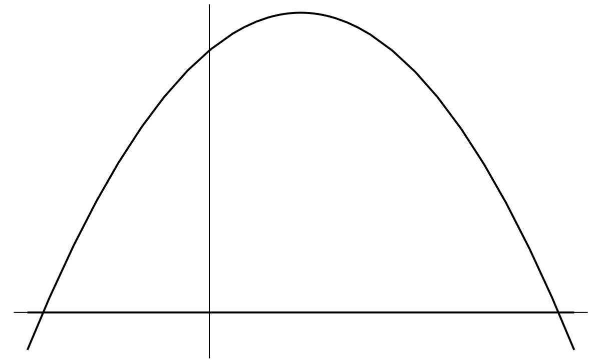

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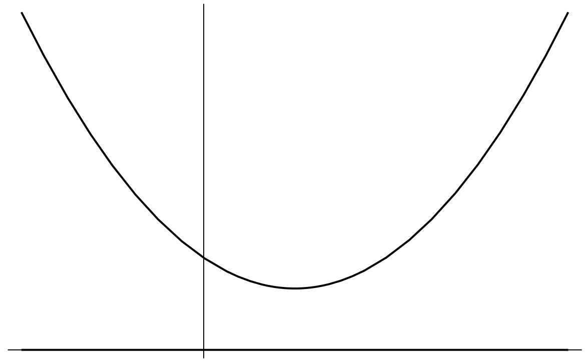

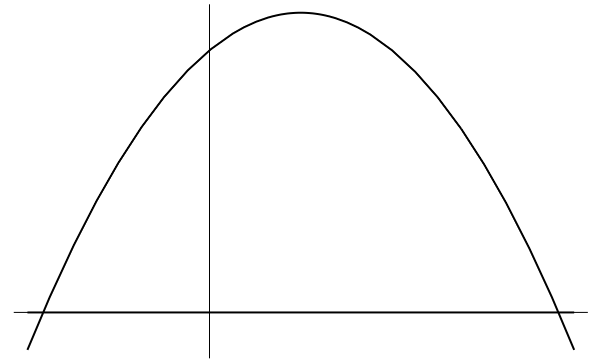

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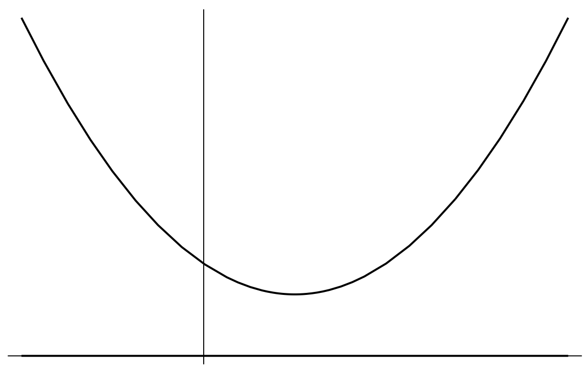

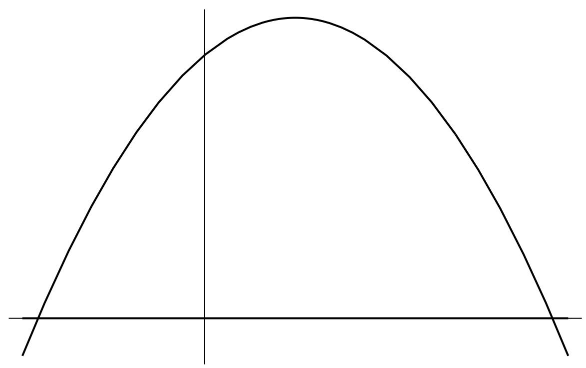

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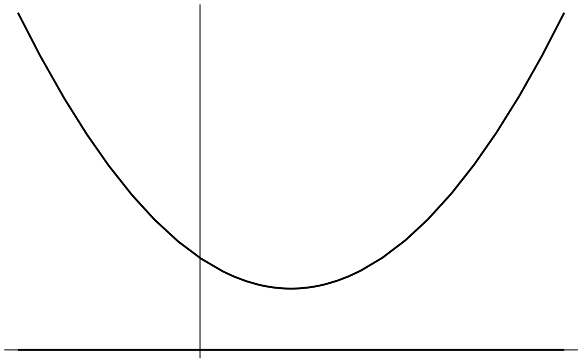

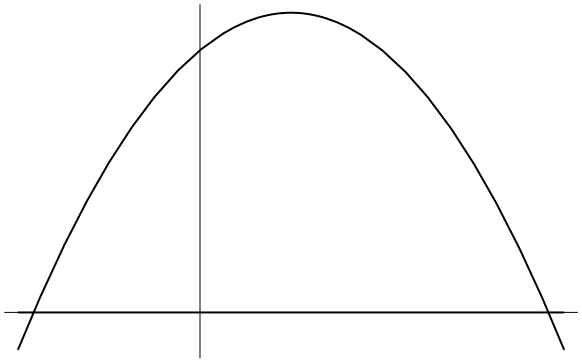

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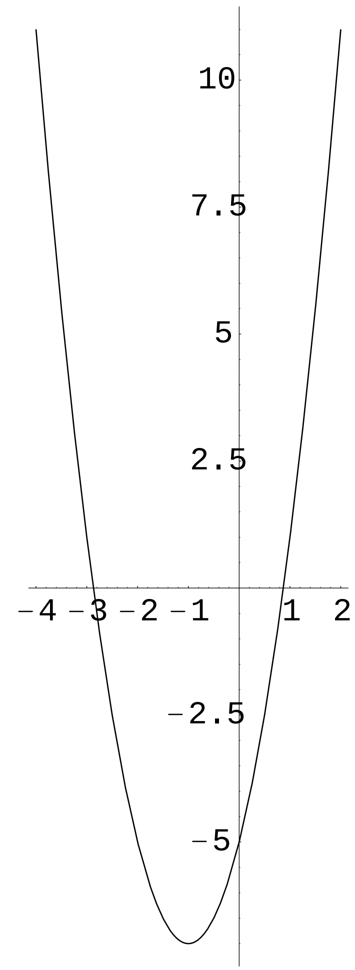

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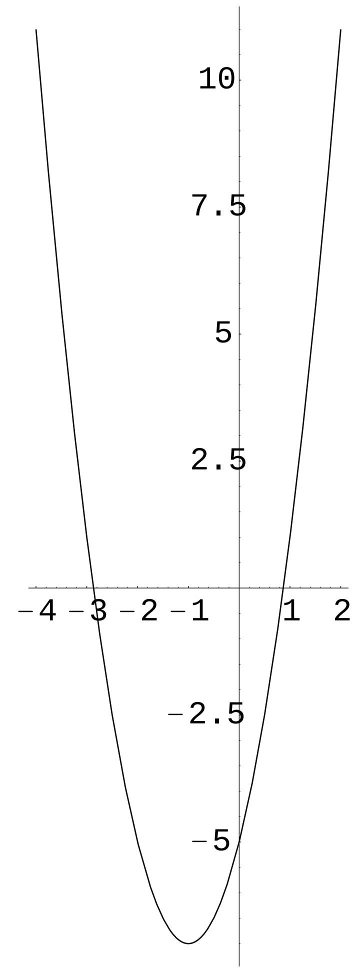

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$n = 4$ *Eg.* $g(x) = 1 - 2x + 3x^2 - 5x^4$

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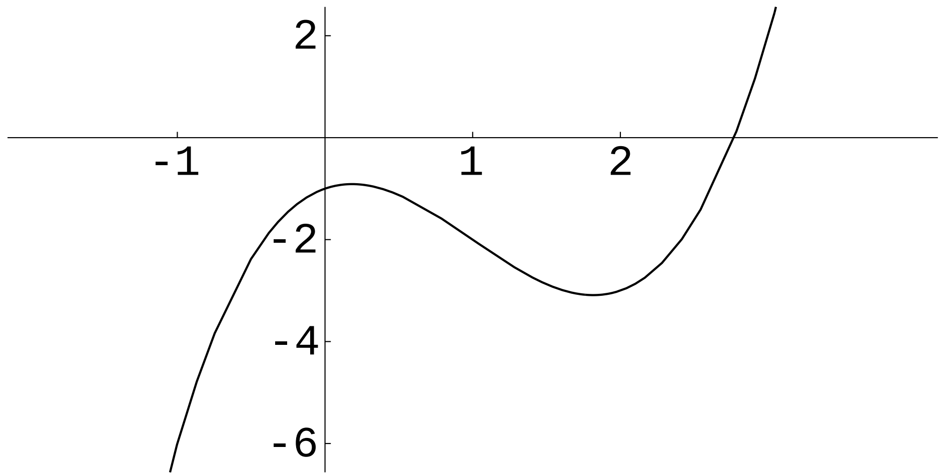

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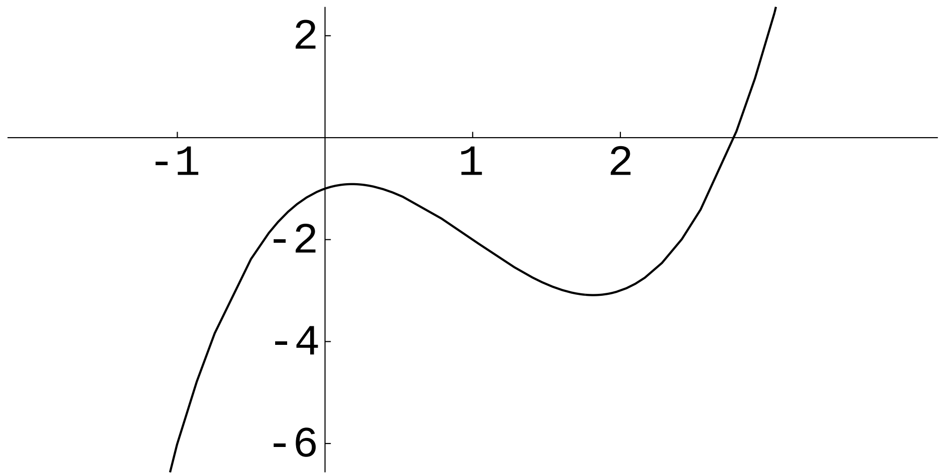

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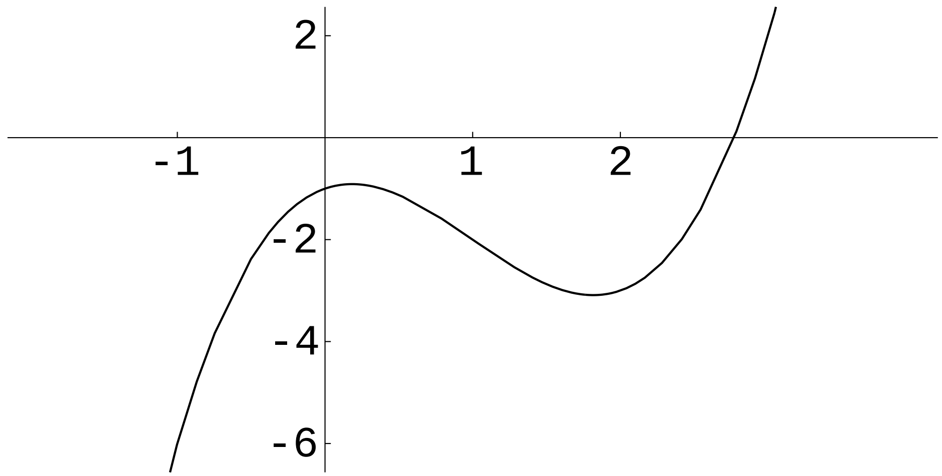

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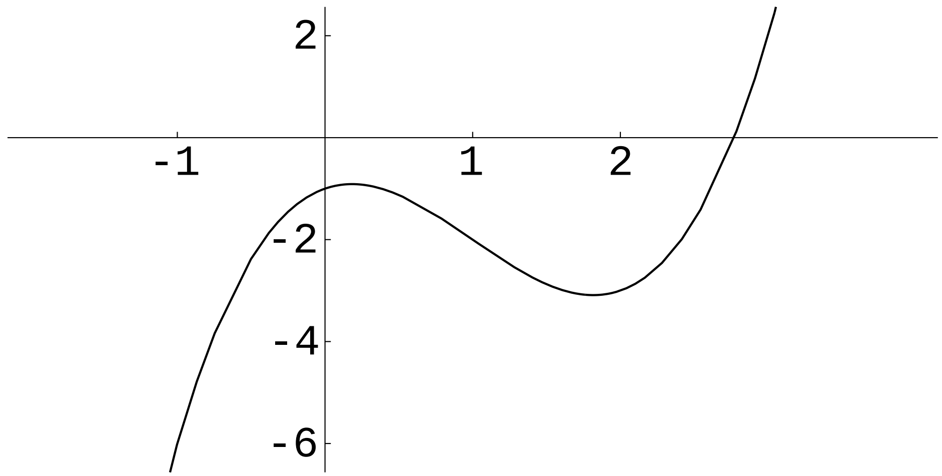

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Remark In Chapter 5, we will describe how to sketch graphs of polynomial functions.

In general, for a polynomial function

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

(where n is a positive integer and $a_n \neq 0$)

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- **Rational Functions**

A function of the form

$$f(x) = \frac{p(x)}{q(x)}$$

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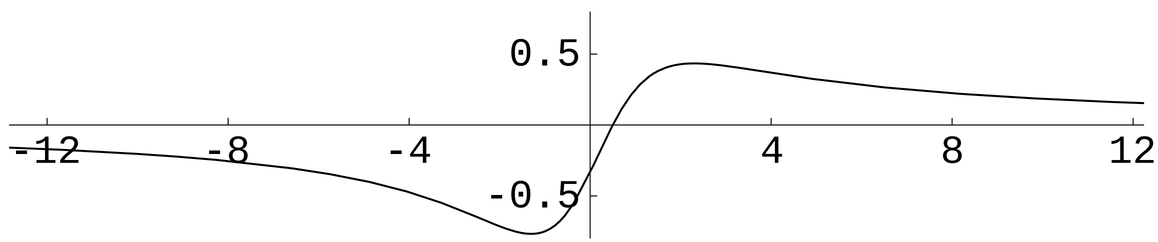

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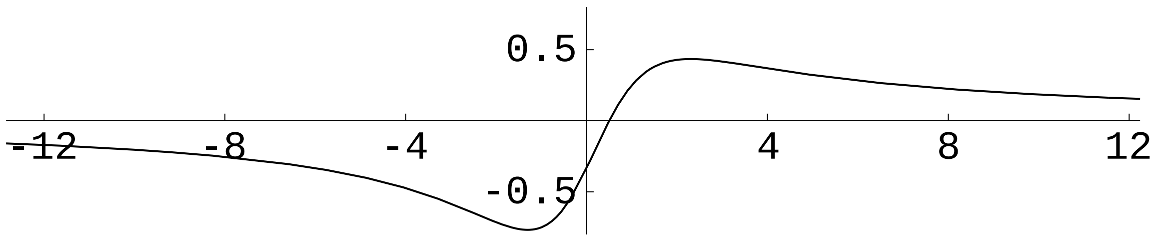

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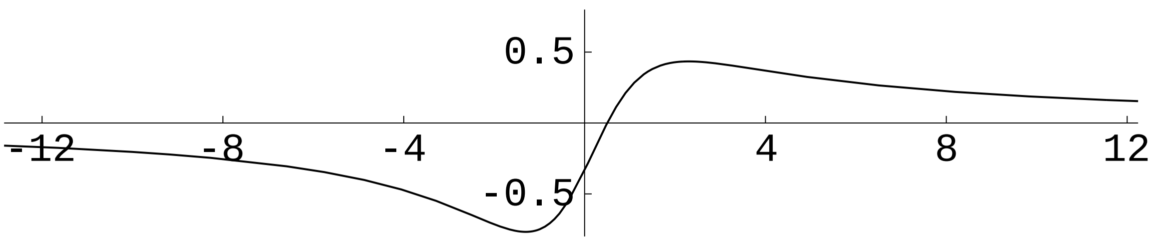

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Note When x is very large in magnitude, $f(x)$ is very small.

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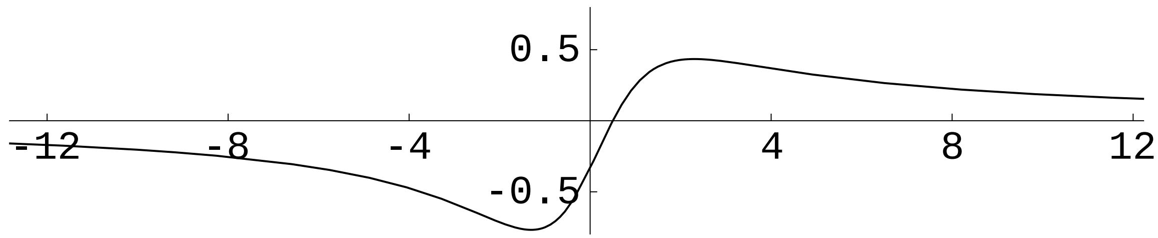

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Reason Degree of the numerator is smaller than that of the denominator *(discuss in Chapter 3)*.

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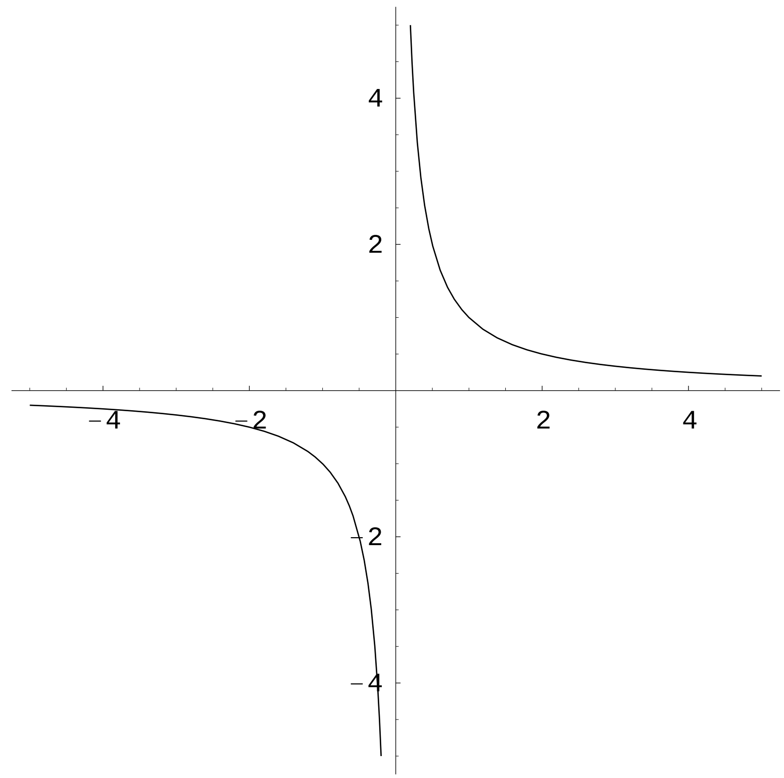

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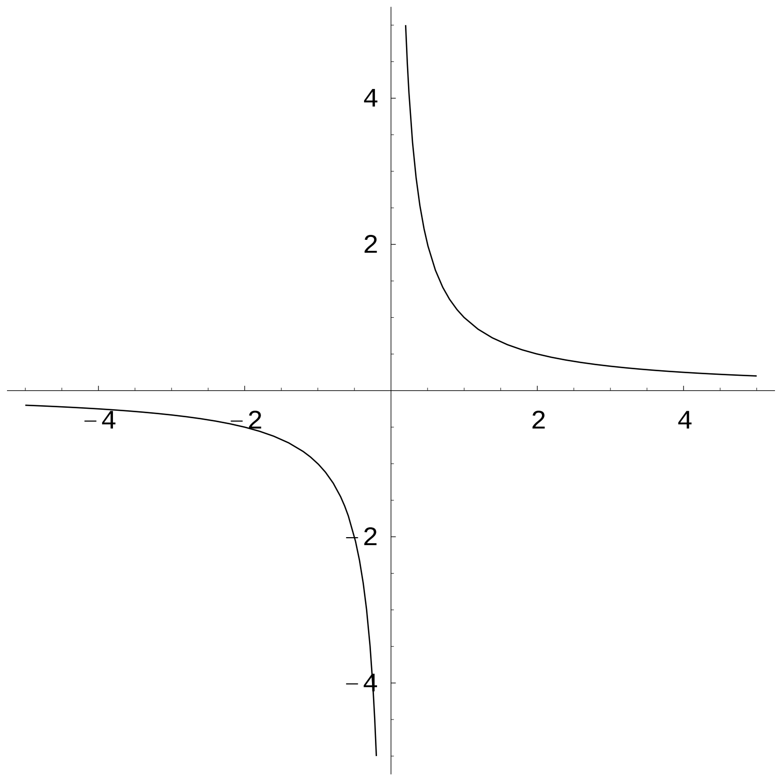

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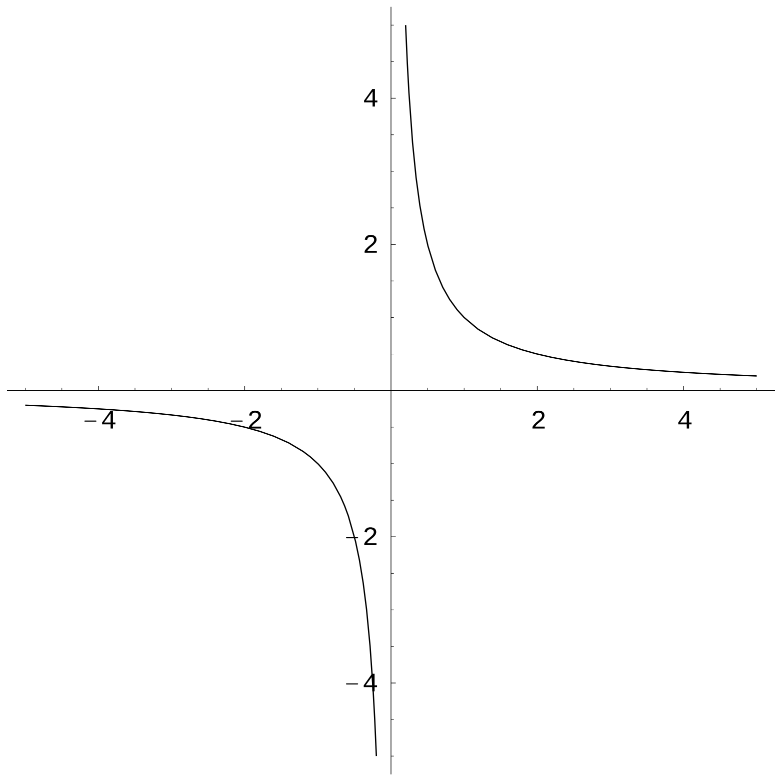

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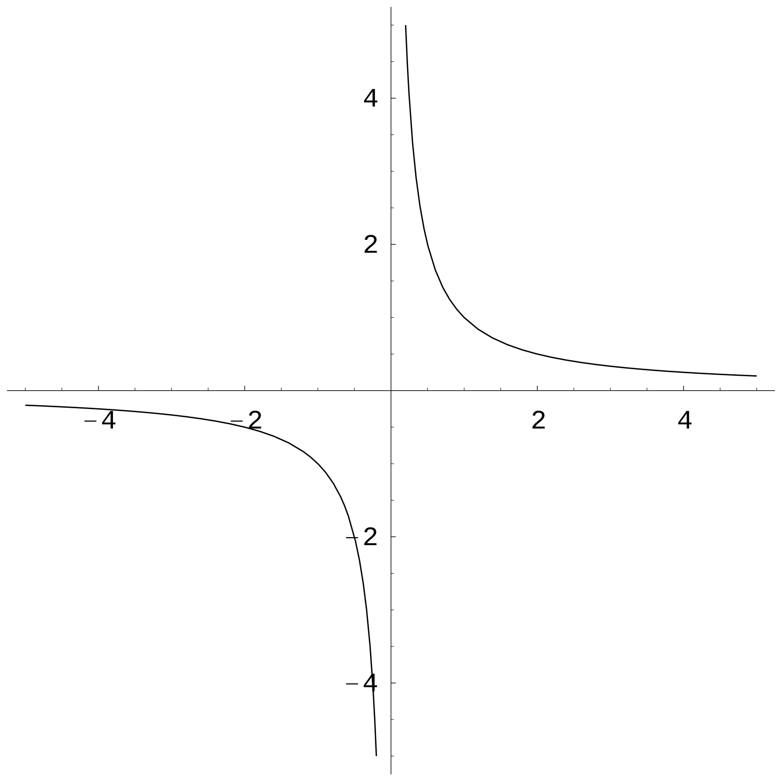

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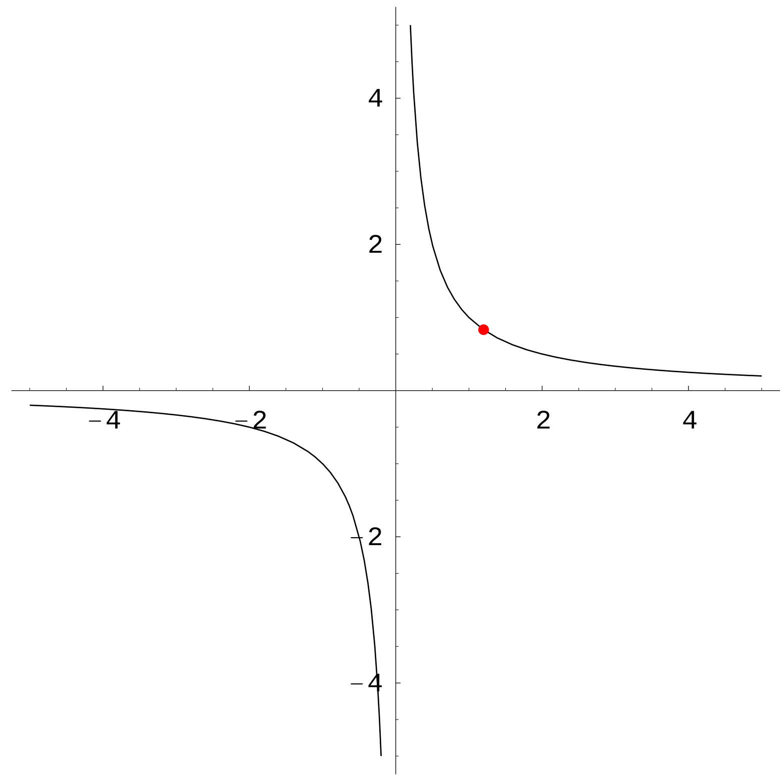

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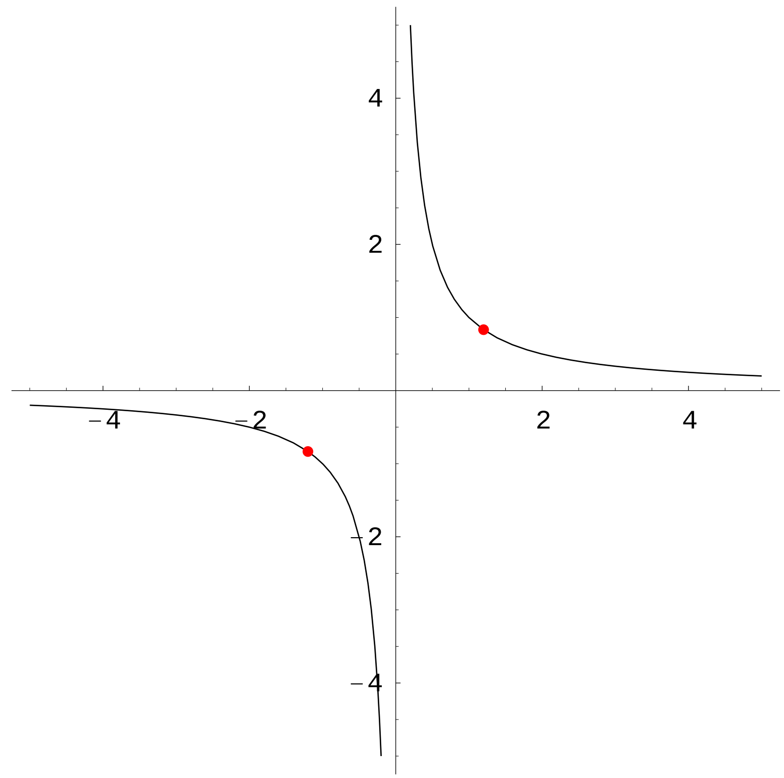

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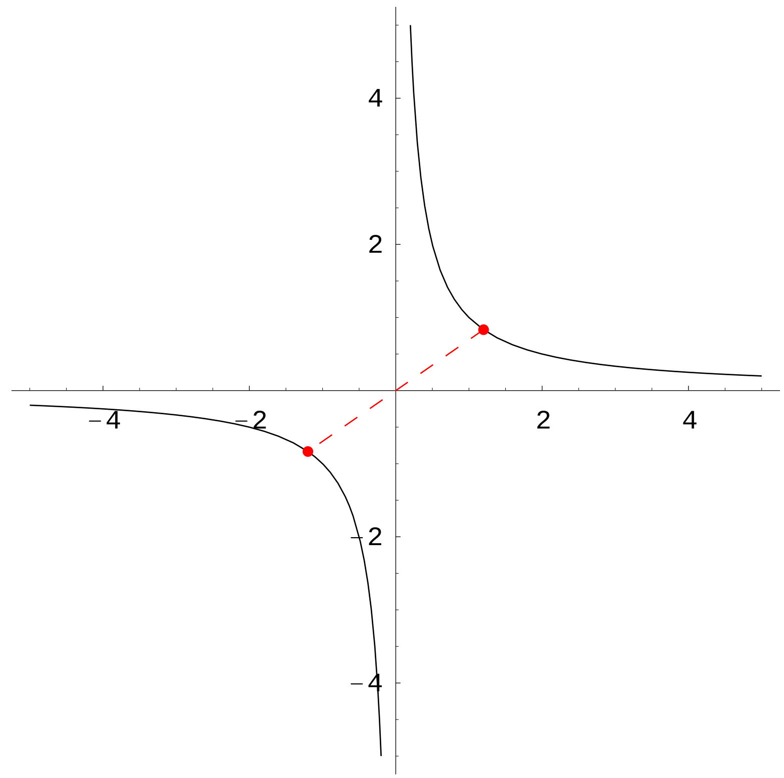

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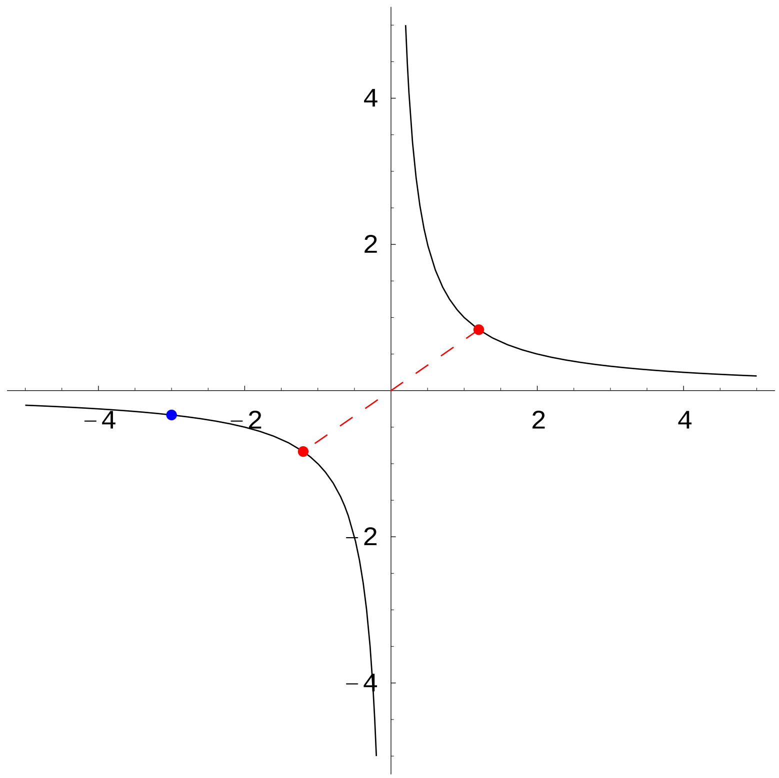

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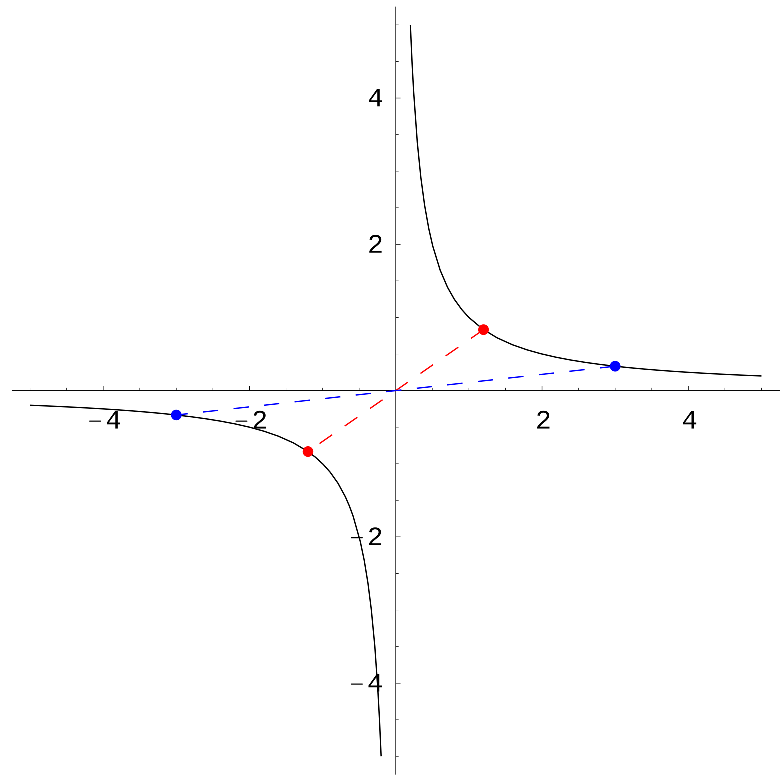

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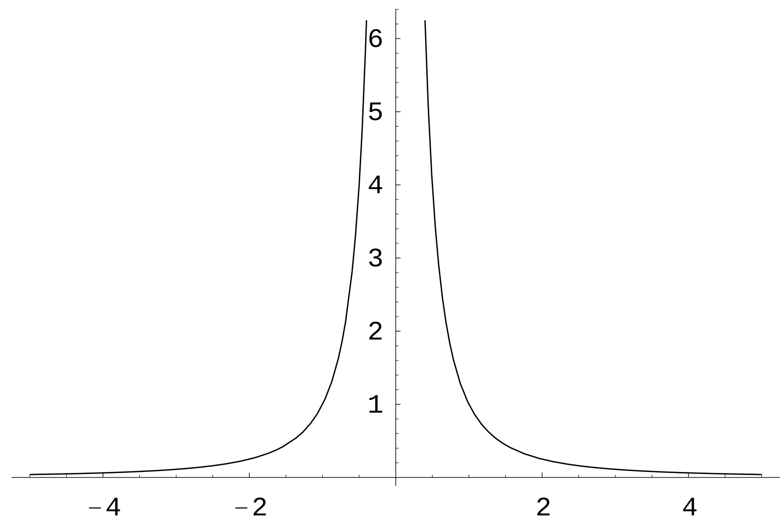

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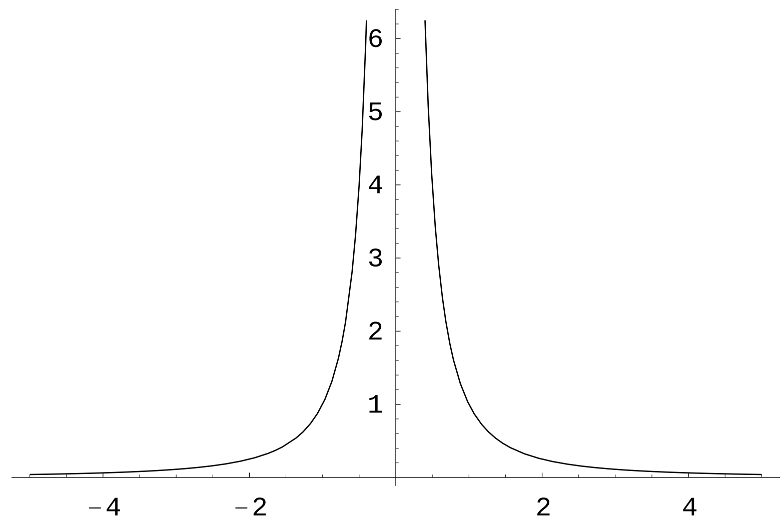

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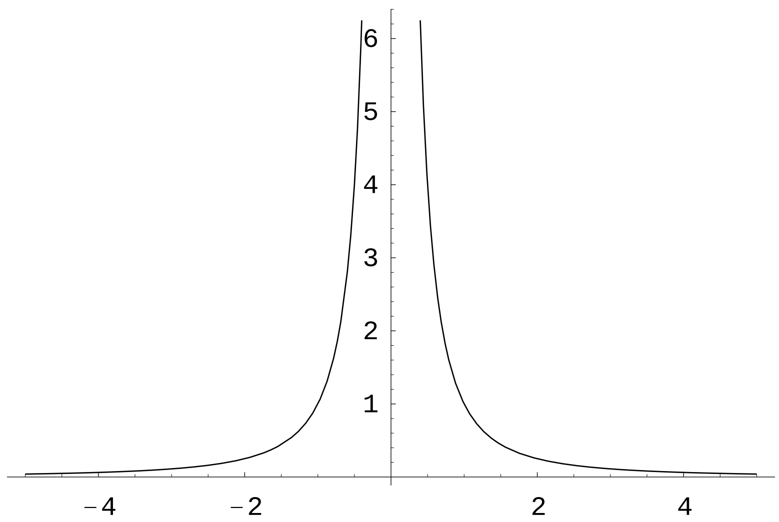

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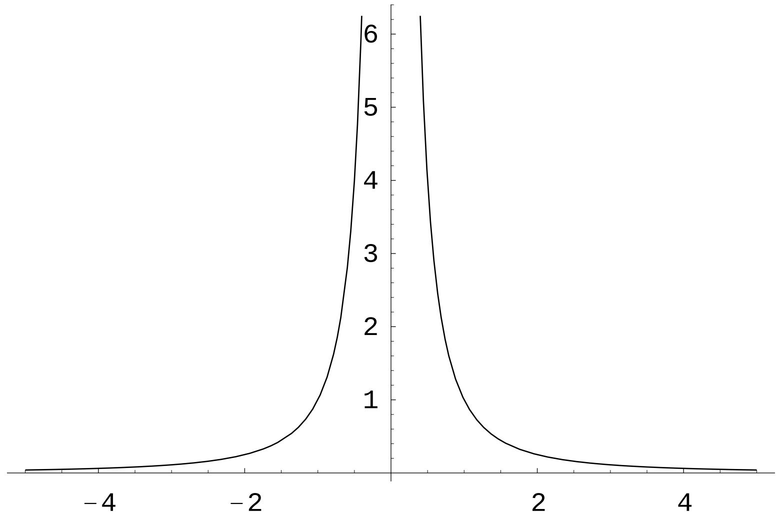

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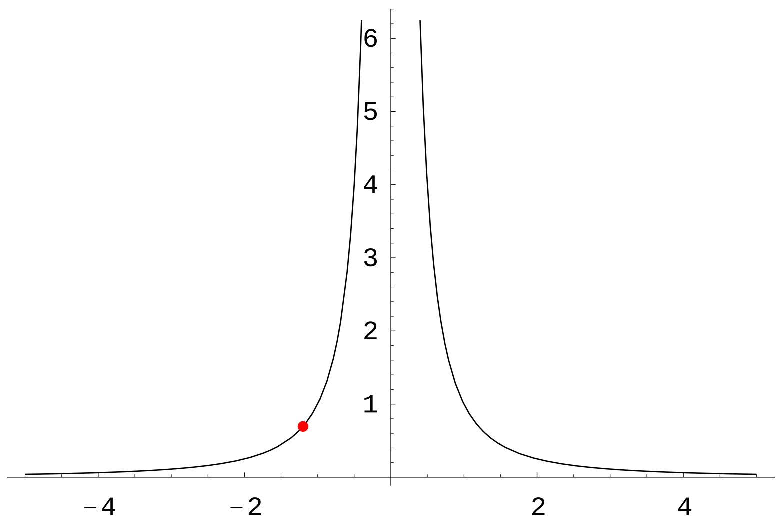

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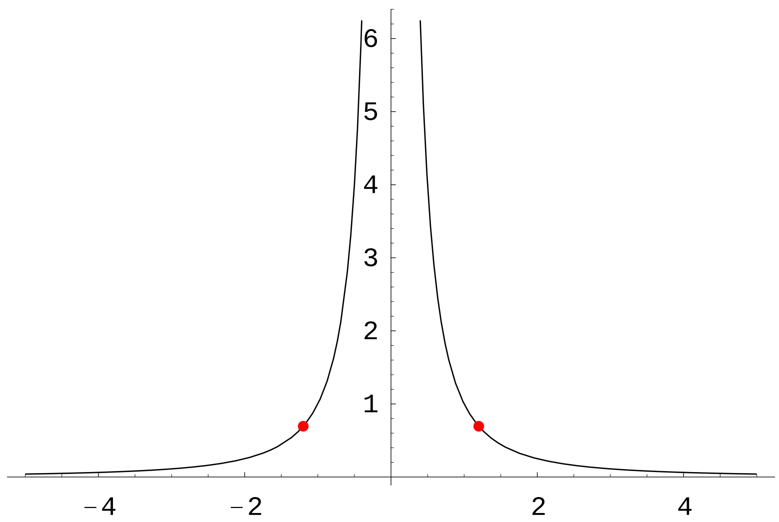

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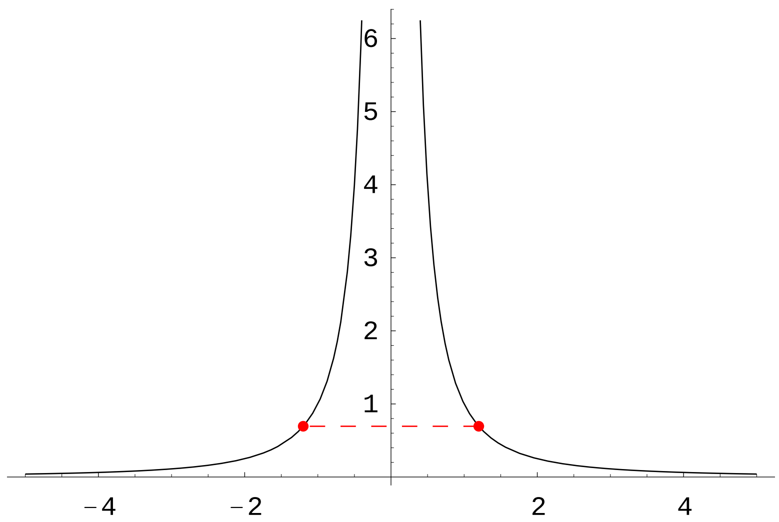

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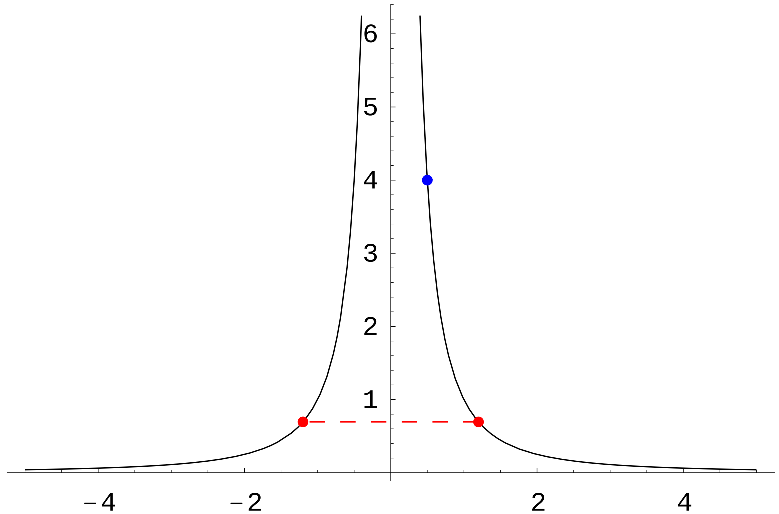

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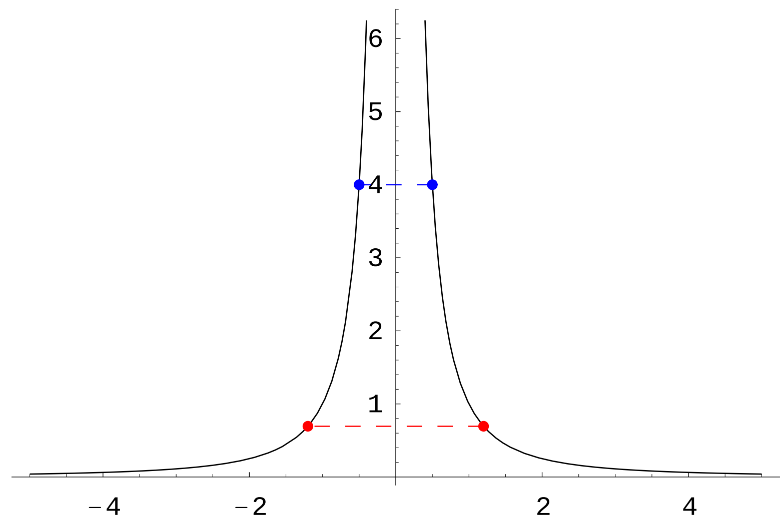

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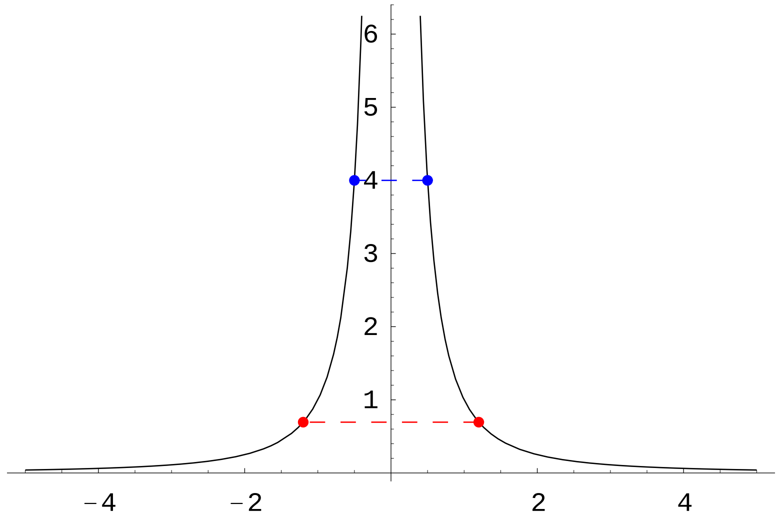

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The y-axis is called the *axis of symmetry*.