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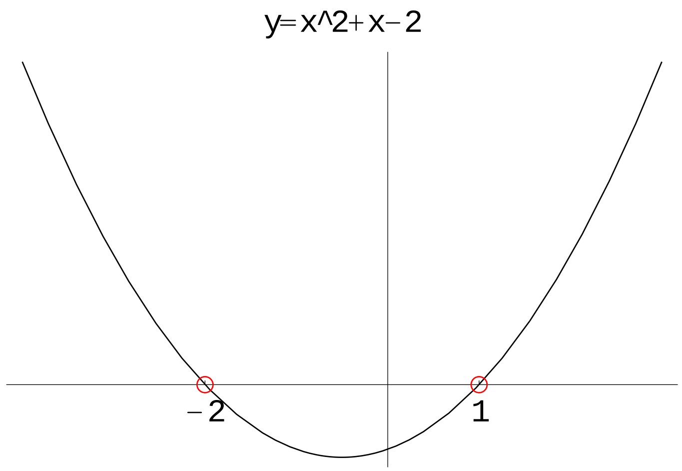

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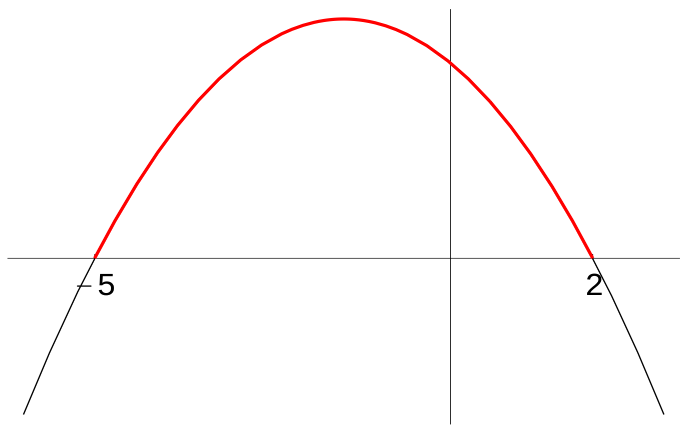

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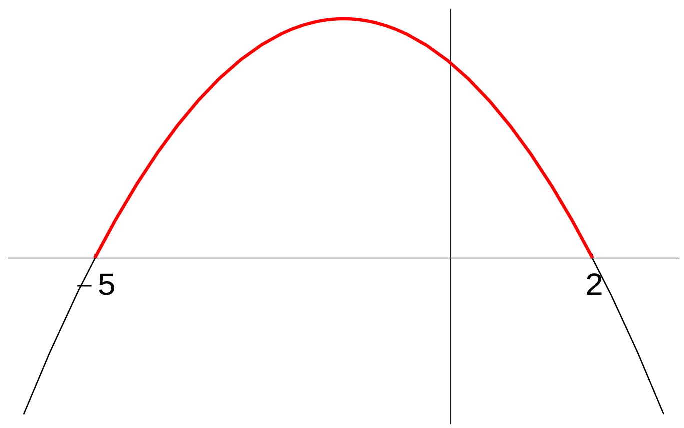

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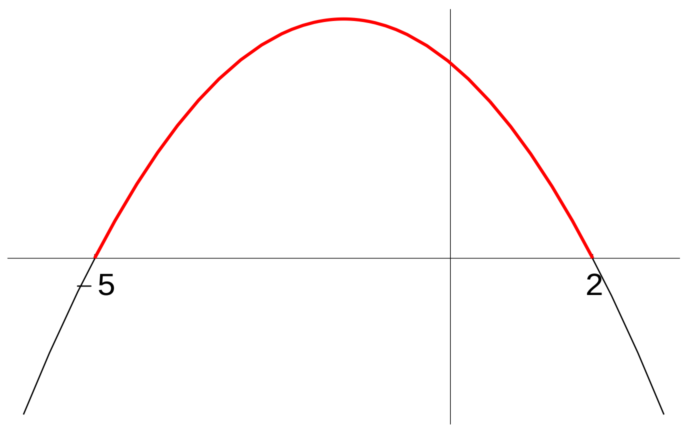

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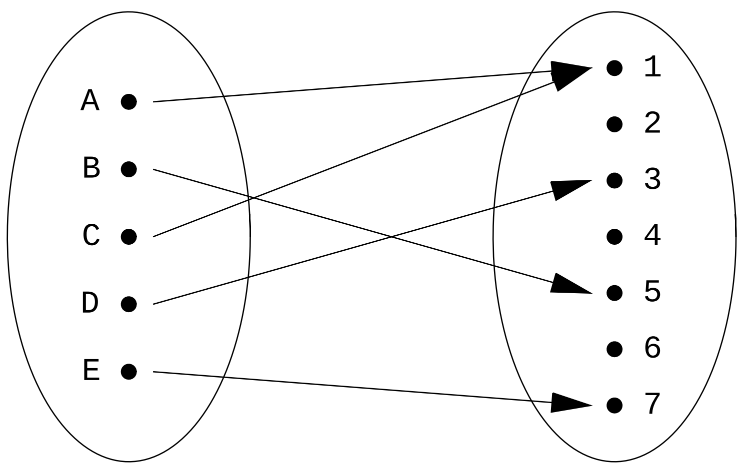

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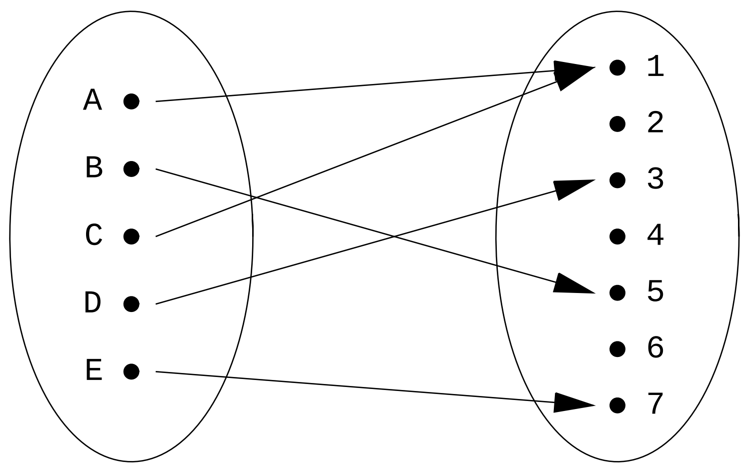

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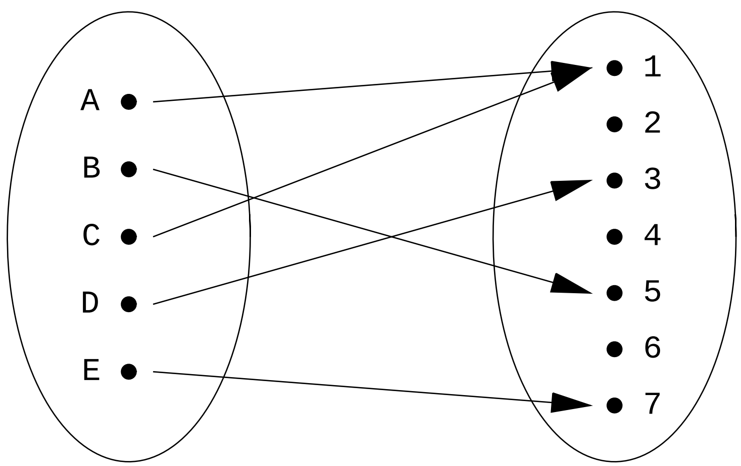

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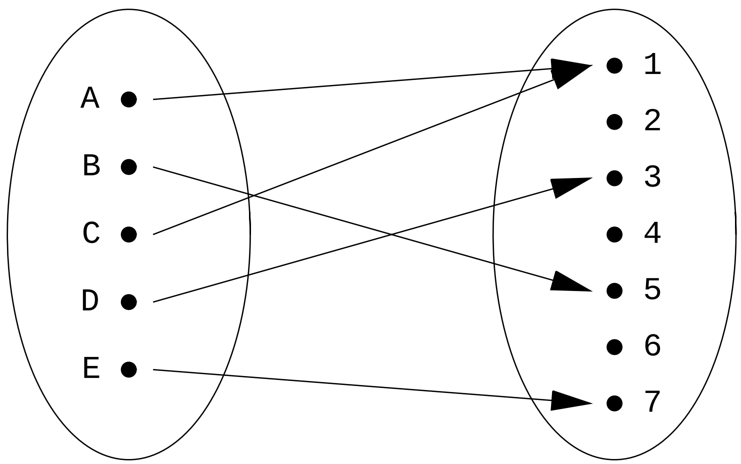

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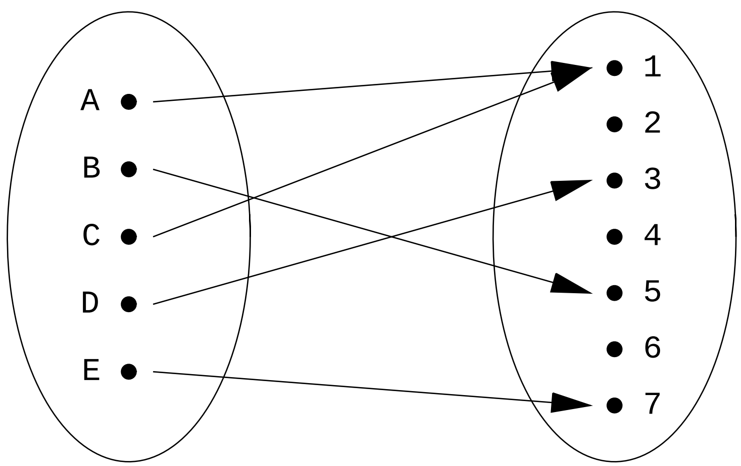

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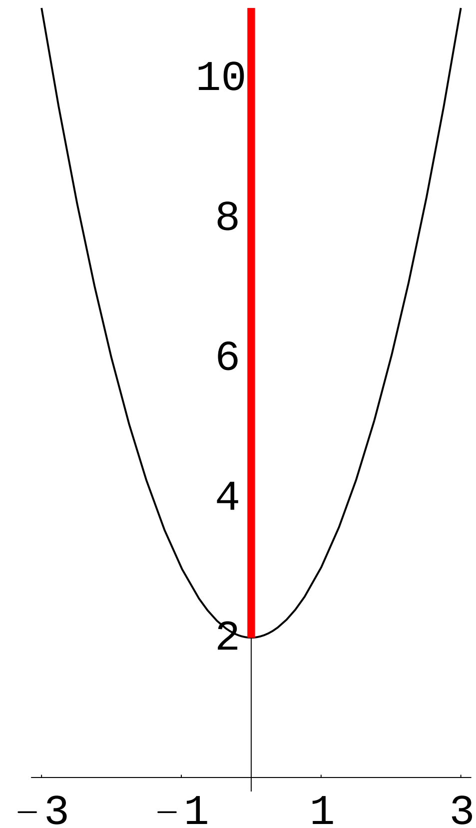

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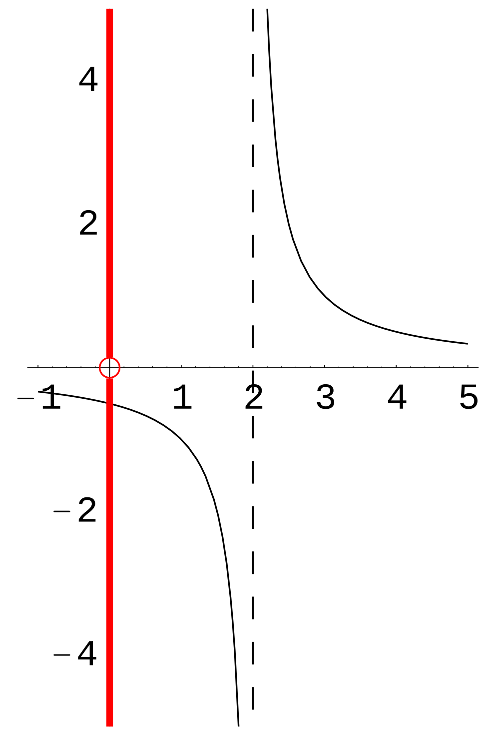
- Put $y = \dfrac{1}{x-2}$
- Solve for x :

$$\begin{aligned} y &= \frac{1}{x-2} \\ \frac{1}{y} &= x-2 \\ x &= \frac{1}{y}+2 \end{aligned}$$

- x can be solved *if and only if* $y \neq 0$.
- Range of g is $\{y \in \mathbb{R} : y \neq 0\}$, that is, $\mathbb{R} - \{0\}$.



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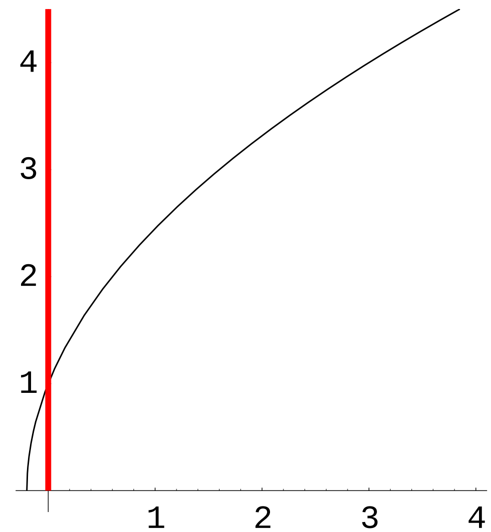

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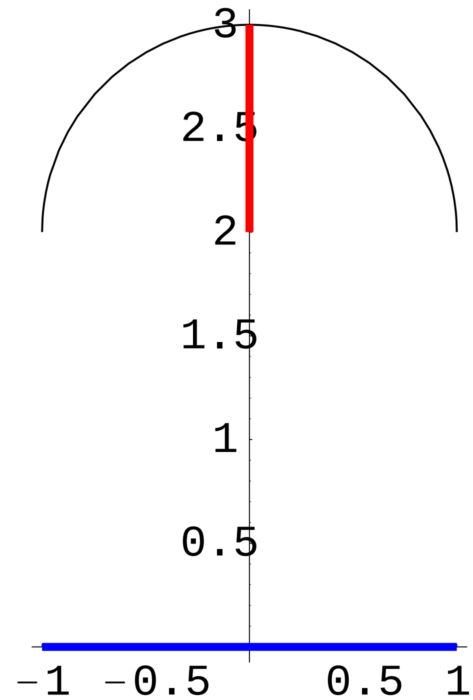

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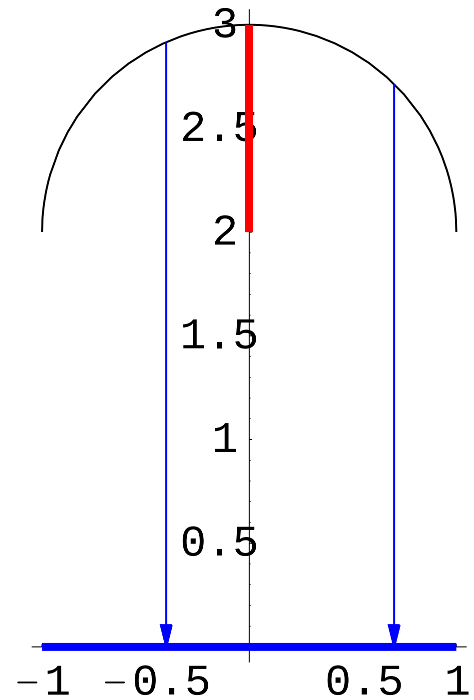

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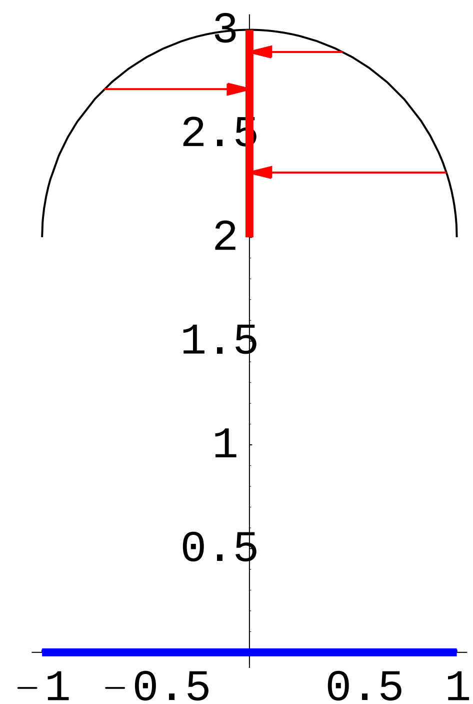

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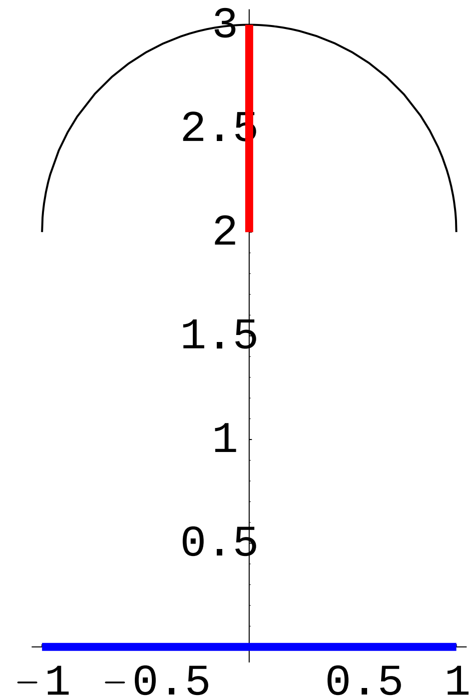

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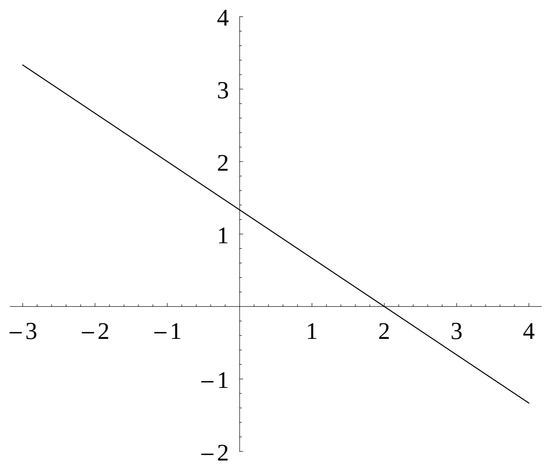

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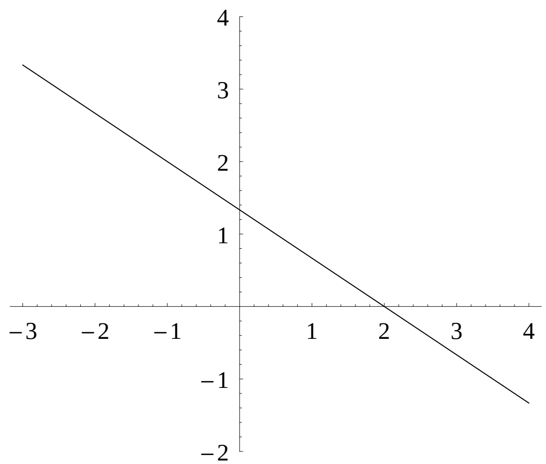

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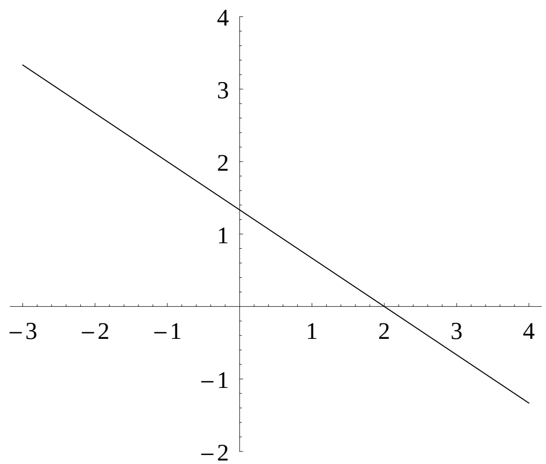

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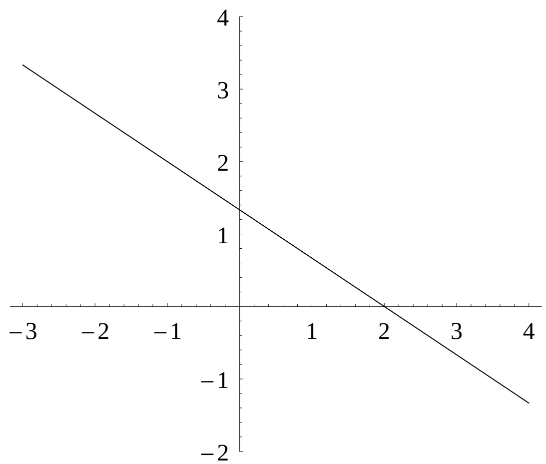

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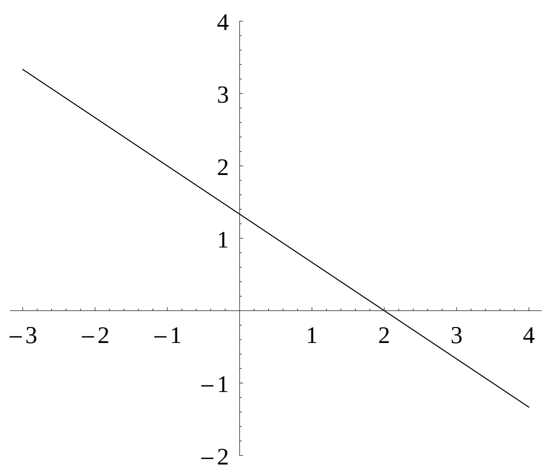

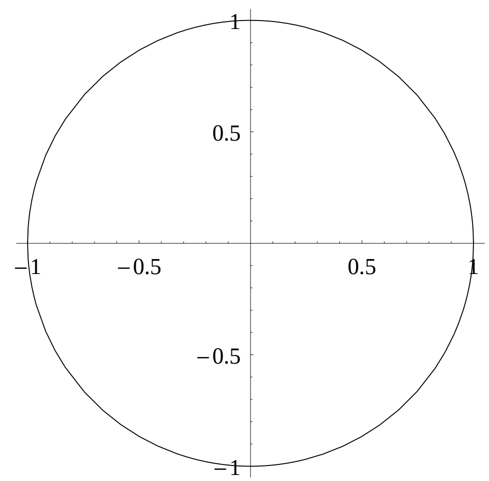

Graphs of Equations

Consider an equation in two variables x and y.

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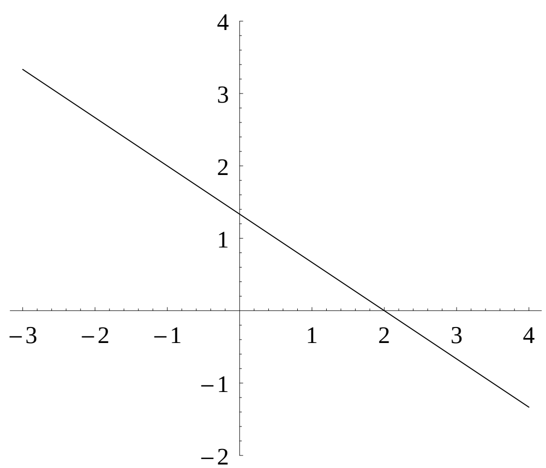

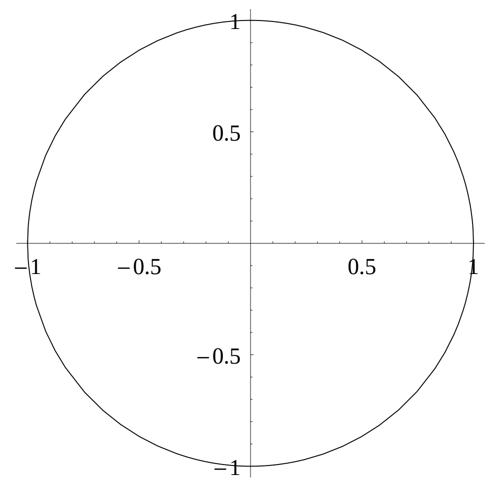

Remark Graph of the equation $F(x, y) = 0$ is the following subset of $\mathbb{R}^2$

$$\{(x, y) \in \mathbb{R}^2 : F(x, y) = 0\}$$

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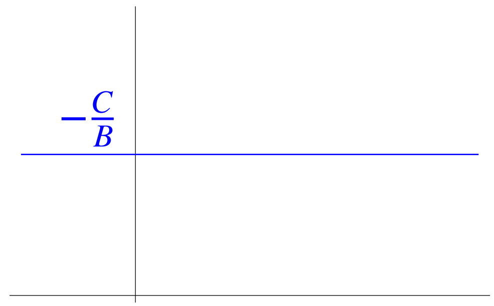

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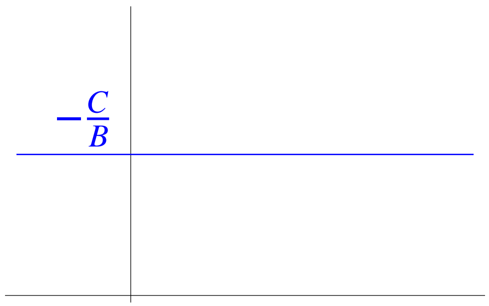

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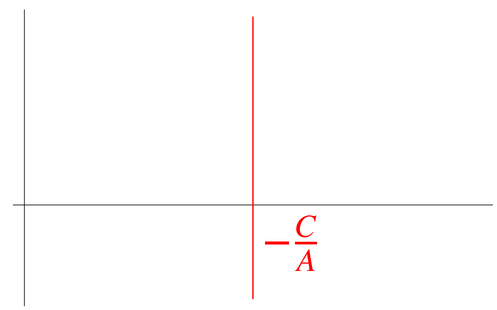

Example Consider the line L_1 given by

$$2x + 3y - 4 = 0 \tag{2}$$

For each of the following points, determine whether it lies on L_1 or not.

(1) $A = (4, -1)$

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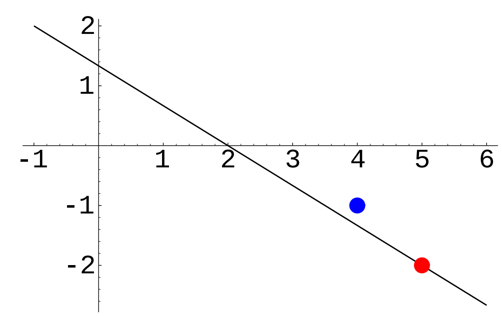

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Find the point of intersection of L_2 with the x-axis and the y-axis.

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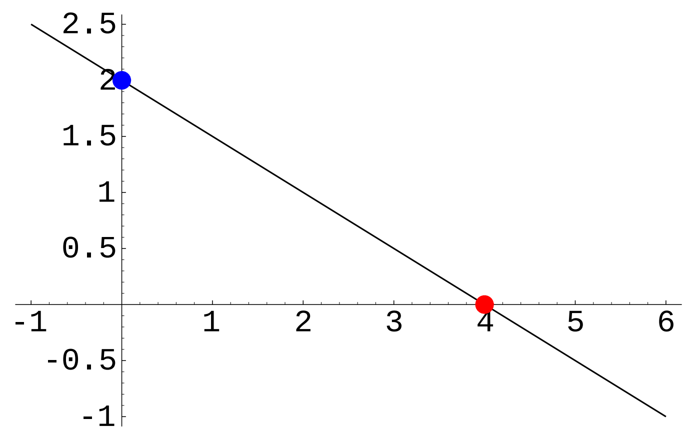

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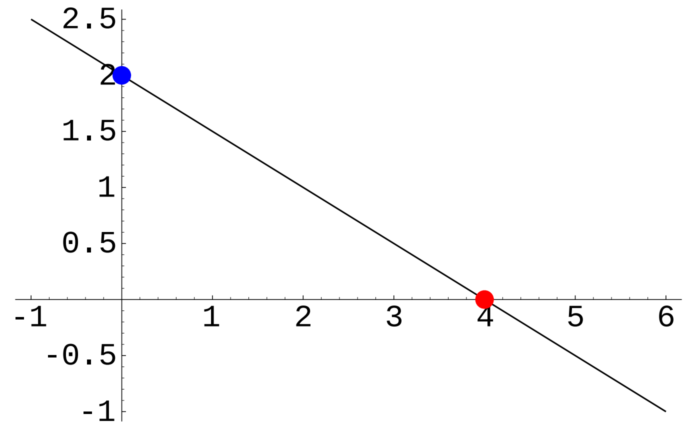

Remark $(4, 0)$ is called the x-*intercept* of the line L_2 and $(0, 2)$ the y-*intercept*.

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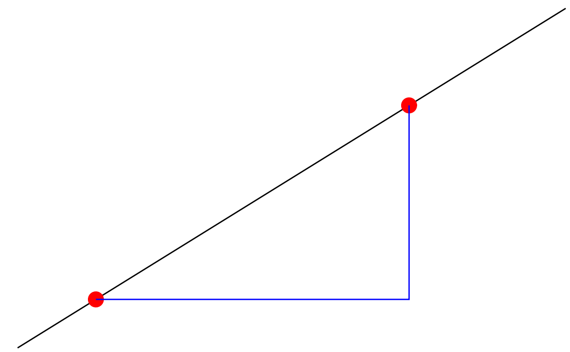

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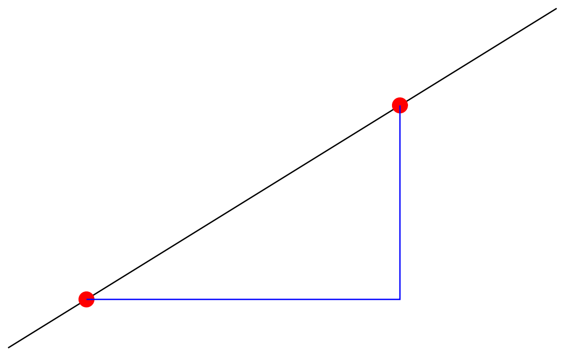

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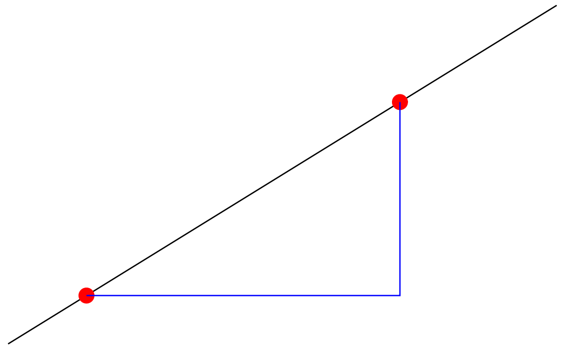

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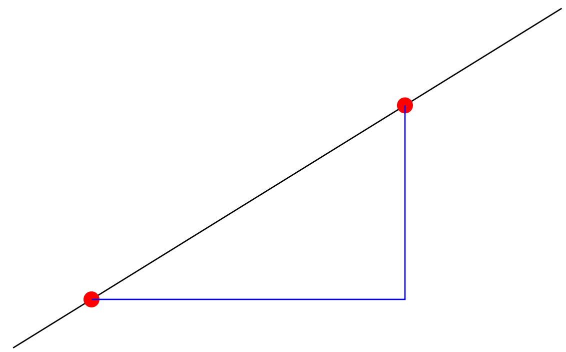

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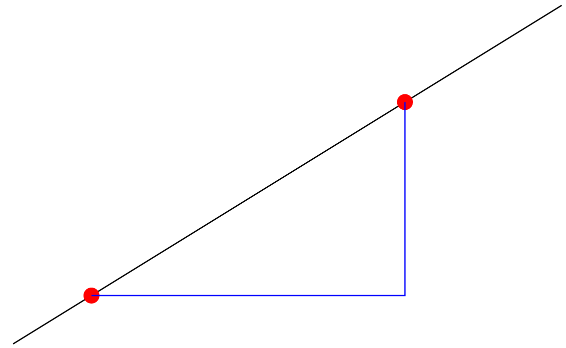

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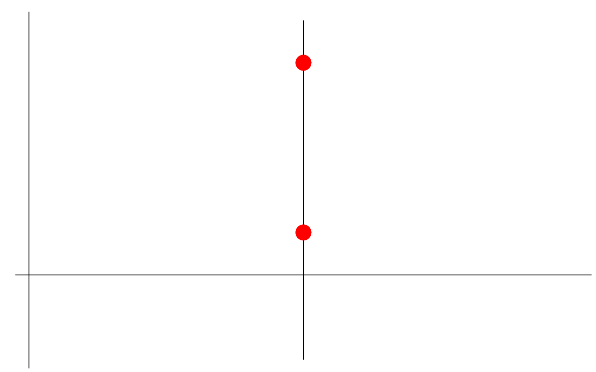

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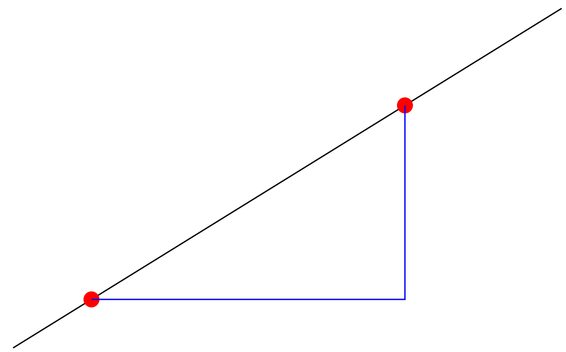

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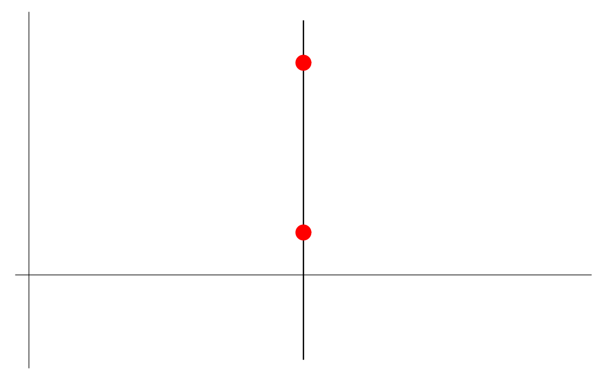

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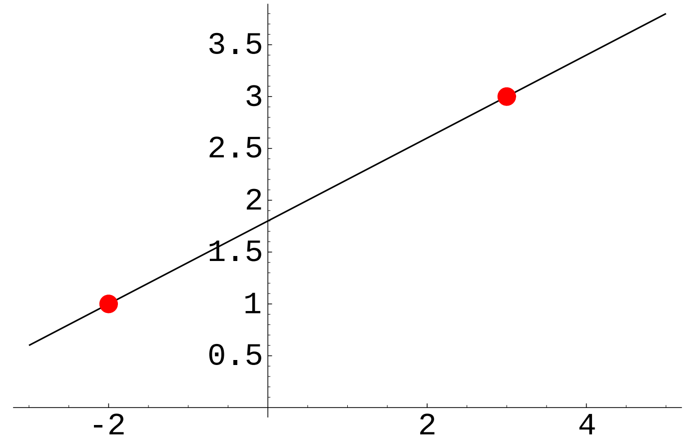

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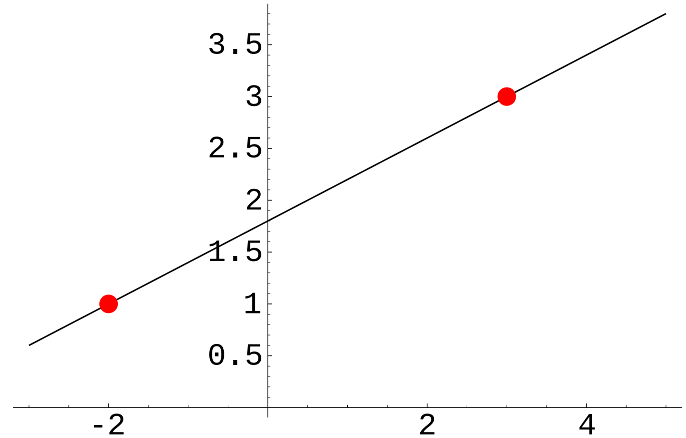

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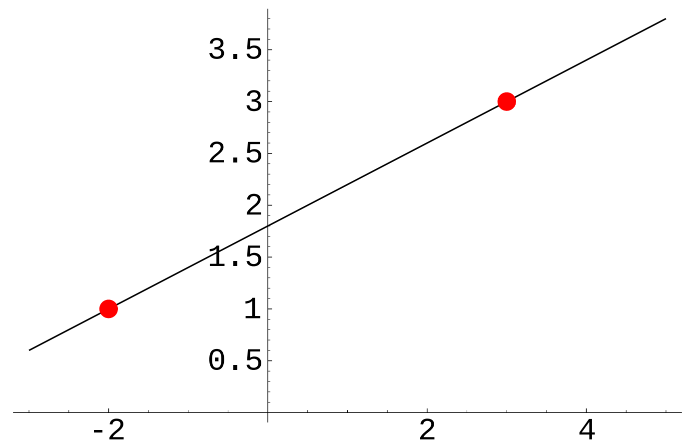

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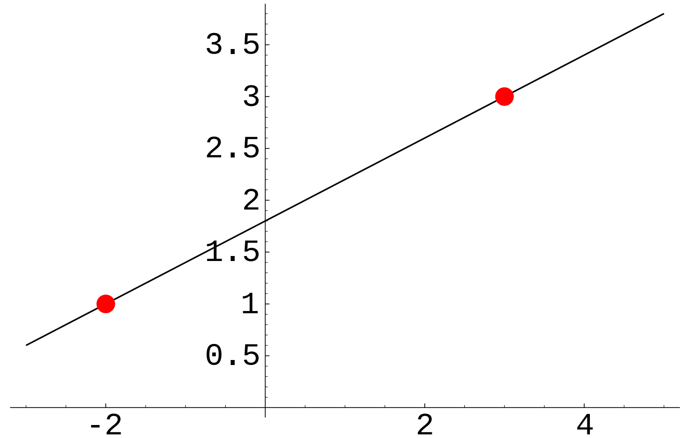

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Remark Can take other pair of points on the line.