Example Solve the following inequality

$$x(3 + x)(1 - x) < 0$$

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Solution

		-3		0		1	
x							
$3+x$							
$1-x$							
$p(x)$							

Example Solve the following inequality

$$x(3 + x)(1 - x) < 0$$

Solution

	$x < -3$	-3	$-3 < x < 0$	0	$0 < x < 1$	1	$1 < x$
x							
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Solution

	$x < -3$	-3	$-3 < x < 0$	0	$0 < x < 1$	1	$1 < x$
x				0			
$3+x$		0					
$1-x$						0	
$p(x)$		0		0		0	

Example Solve the following inequality

$$x(3+x)(1-x) < 0$$

Solution

	$x < -3$	-3	$-3 < x < 0$	0	$0 < x < 1$	1	$1 < x$
x				0	$+$	$+$	$+$
$3 + x$		0					
$1 - x$						0	
$p(x)$		0		0		0	

Example Solve the following inequality

$$x(3+x)(1-x) < 0$$

Solution

	$x < -3$	-3	$-3 < x < 0$	0	$0 < x < 1$	1	$1 < x$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$3+x$		0					
$1-x$						0	
$p(x)$		0		0		0	

Example Solve the following inequality

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Solution

	$x < -3$	-3	$-3 < x < 0$	0	$0 < x < 1$	1	$1 < x$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$3+x$		0	$+$	$+$	$+$	$+$	$+$
$1-x$						0	
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$3 + x$	$-$	0	$+$	$+$	$+$	$+$	$+$
$1 - x$						0	
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	$x < -3$	-3	$-3 < x < 0$	0	$0 < x < 1$	1	$1 < x$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$3+x$	$-$	0	$+$	$+$	$+$	$+$	$+$
$1-x$						0	$-$
$p(x)$		0		0		0	

Example Solve the following inequality

$$x(3+x)(1-x)<0$$

Solution

	$x<-3$	-3	$-3<x<0$	0	$0<x<1$	1	$1<x$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$3+x$	$-$	0	$+$	$+$	$+$	$+$	$+$
$1-x$	$+$	$+$	$+$	$+$	$+$	0	$-$
$p(x)$		0		0		0	

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Solution

	$x < -3$	-3	$-3 < x < 0$	0	$0 < x < 1$	1	$1 < x$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$3+x$	$-$	0	$+$	$+$	$+$	$+$	$+$
$1-x$	$+$	$+$	$+$	$+$	$+$	0	$-$
$p(x)$	$+$	0		0		0	

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	$x < -3$	-3	$-3 < x < 0$	0	$0 < x < 1$	1	$1 < x$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$3+x$	$-$	0	$+$	$+$	$+$	$+$	$+$
$1-x$	$+$	$+$	$+$	$+$	$+$	0	$-$
$p(x)$	$+$	0	$-$	0		0	

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	$x < -3$	-3	$-3 < x < 0$	0	$0 < x < 1$	1	$1 < x$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$3 + x$	$-$	0	$+$	$+$	$+$	$+$	$+$
$1 - x$	$+$	$+$	$+$	$+$	$+$	0	$-$
$p(x)$	$+$	0	$-$	0	$+$	0	

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	$x < -3$	-3	$-3 < x < 0$	0	$0 < x < 1$	1	$1 < x$
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$3 + x$	$-$	0	$+$	$+$	$+$	$+$	$+$
$1 - x$	$+$	$+$	$+$	$+$	$+$	0	$-$
$p(x)$	$+$	0	$-$	0	$+$	0	$-$

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	$x < -3$	-3	$-3 < x < 0$	0	$0 < x < 1$	1	$1 < x$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$3+x$	$-$	0	$+$	$+$	$+$	$+$	$+$
$1-x$	$+$	$+$	$+$	$+$	$+$	0	$-$
$p(x)$	$+$	0	$-$	0	$+$	0	$-$

Solution: $-3 < x < 0$ or $x > 1$

Example Solve the following inequality

$$-5(1-x)^2(1+x)^3(2-x) \geq 0$$

Solution

Example Solve the following inequality

$$-5(1-x)^2(1+x)^3(2-x) \geq 0$$

Solution

		-1		1		2	
-5							
$(1-x)^2$							
$(1+x)^3$							
$2-x$							
$p(x)$							

Example Solve the following inequality

$$-5(1-x)^2(1+x)^3(2-x) \geq 0$$

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	$x < -1$	-1	$-1 < x < 1$	1	$1 < x < 2$	2	$2 < x$
-5							
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Solution

	$x < -1$	-1	$-1 < x < 1$	1	$1 < x < 2$	2	$2 < x$
-5	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$(1-x)^2$				0			
$(1+x)^3$		0					
$2-x$						0	
$p(x)$		0		0		0	

Example Solve the following inequality

$$-5(1-x)^2(1+x)^3(2-x) \geq 0$$

Solution

	$x < -1$	-1	$-1 < x < 1$	1	$1 < x < 2$	2	$2 < x$
-5	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$(1-x)^2$	$+$	$+$	$+$	0	$+$	$+$	$+$
$(1+x)^3$		0					
$2-x$						0	
$p(x)$		0		0		0	

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Solution

	$x < -1$	-1	$-1 < x < 1$	1	$1 < x < 2$	2	$2 < x$
-5	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$(1-x)^2$	$+$	$+$	$+$	0	$+$	$+$	$+$
$(1+x)^3$		0	$+$	$+$	$+$	$+$	$+$
$2-x$						0	
$p(x)$		0		0		0	

Example Solve the following inequality

$$-5(1-x)^2(1+x)^3(2-x) \geq 0$$

Solution

	$x < -1$	-1	$-1 < x < 1$	1	$1 < x < 2$	2	$2 < x$
-5	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$(1-x)^2$	$+$	$+$	$+$	0	$+$	$+$	$+$
$(1+x)^3$	$-$	0	$+$	$+$	$+$	$+$	$+$
$2-x$						0	
$p(x)$		0		0		0	

Example Solve the following inequality

$$-5(1-x)^2(1+x)^3(2-x) \geq 0$$

Solution

	$x<-1$	-1	$-1<x<1$	1	$1<x<2$	2	$2<x$
-5	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$(1-x)^2$	$+$	$+$	$+$	0	$+$	$+$	$+$
$(1+x)^3$	$-$	0	$+$	$+$	$+$	$+$	$+$
$2-x$						0	$-$
$p(x)$		0		0		0	

Example Solve the following inequality

$$-5(1-x)^2(1+x)^3(2-x) \geq 0$$

Solution

	$x < -1$	-1	$-1 < x < 1$	1	$1 < x < 2$	2	$2 < x$
-5	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$(1-x)^2$	$+$	$+$	$+$	0	$+$	$+$	$+$
$(1+x)^3$	$-$	0	$+$	$+$	$+$	$+$	$+$
$2-x$	$+$	$+$	$+$	$+$	$+$	0	$-$
$p(x)$		0		0		0	

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	$x < -1$	-1	$-1 < x < 1$	1	$1 < x < 2$	2	$2 < x$
-5	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$(1-x)^2$	$+$	$+$	$+$	0	$+$	$+$	$+$
$(1+x)^3$	$-$	0	$+$	$+$	$+$	$+$	$+$
$2-x$	$+$	$+$	$+$	$+$	$+$	0	$-$
$p(x)$	$+$	0		0		0	

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Solution

	$x < -1$	-1	$-1 < x < 1$	1	$1 < x < 2$	2	$2 < x$
-5	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$(1-x)^2$	$+$	$+$	$+$	0	$+$	$+$	$+$
$(1+x)^3$	$-$	0	$+$	$+$	$+$	$+$	$+$
$2-x$	$+$	$+$	$+$	$+$	$+$	0	$-$
$p(x)$	$+$	0	$-$	0		0	

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Solution

	$x < -1$	-1	$-1 < x < 1$	1	$1 < x < 2$	2	$2 < x$
-5	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$(1-x)^2$	$+$	$+$	$+$	0	$+$	$+$	$+$
$(1+x)^3$	$-$	0	$+$	$+$	$+$	$+$	$+$
$2-x$	$+$	$+$	$+$	$+$	$+$	0	$-$
$p(x)$	$+$	0	$-$	0	$-$	0	

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	$x < -1$	-1	$-1 < x < 1$	1	$1 < x < 2$	2	$2 < x$
-5	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$(1-x)^2$	$+$	$+$	$+$	0	$+$	$+$	$+$
$(1+x)^3$	$-$	0	$+$	$+$	$+$	$+$	$+$
$2-x$	$+$	$+$	$+$	$+$	$+$	0	$-$
$p(x)$	$+$	0	$-$	0	$-$	0	$+$

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	$x < -1$	-1	$-1 < x < 1$	1	$1 < x < 2$	2	$2 < x$
-5	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$(1-x)^2$	$+$	$+$	$+$	0	$+$	$+$	$+$
$(1+x)^3$	$-$	0	$+$	$+$	$+$	$+$	$+$
$2-x$	$+$	$+$	$+$	$+$	$+$	0	$-$
$p(x)$	$+$	0	$-$	0	$-$	0	$+$

Solution: $x \leq -1$ or $x = 1$ or $x \geq 2$

Example Find the solution set to the inequality

$$\frac{x+1}{x-2} \geq 0$$

Solution

Example Find the solution set to the inequality

$$\frac{x+1}{x-2} \geq 0$$

Solution

		$x = -1$		$x = 2$	
$x + 1$					
$x - 2$					
$\frac{x+1}{x-2}$					

Example Find the solution set to the inequality

$$\frac{x+1}{x-2} \geq 0$$

Solution

	$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
$x + 1$					
$x - 2$					
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Solution

	$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
$x+1$		0			
$x-2$					
$\frac{x+1}{x-2}$					

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	$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
$x+1$		0	$+$	$+$	$+$
$x-2$					
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Solution

	$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
$x+1$	$-$	0	$+$	$+$	$+$
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	$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
$x+1$	$-$	0	$+$	$+$	$+$
$x-2$				0	
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$x+1$	$-$	0	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	0	$+$
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	$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
$x+1$	$-$	0	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	0	$+$
$\frac{x+1}{x-2}$	$+$				

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	$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
$x+1$	$-$	0	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	0	$+$
$\frac{x+1}{x-2}$	$+$	0			

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	$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
$x+1$	$-$	0	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	0	$+$
$\frac{x+1}{x-2}$	$+$	0	$-$		

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	$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
$x+1$	$-$	0	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	0	$+$
$\frac{x+1}{x-2}$	$+$	0	$-$	undefined	

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$x-2$	$-$	$-$	$-$	0	$+$
$\frac{x+1}{x-2}$	$+$	0	$-$	undefined	$+$

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	$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
$x+1$	$-$	0	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	0	$+$
$\frac{x+1}{x-2}$	$+$	0	$-$	undefined	$+$

Solution set: $(-\infty, -1] \cup (2, \infty)$

Chapter 2: Functions and Graphs

- *What are functions ?*
- *Why study functions ?*

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Example

(1) Represented by tables:

Total Midyear Population for the World	
Year	Population
1990	5,282,765,827
1991	5,366,815,901
1992	5,450,861,723
1993	5,532,578,016
1994	5,613,424,524
1995	5,694,418,460
1996	5,773,464,448
1997	5,852,360,768
1998	5,929,735,977
1999	6,006,163,019
2000	6,081,527,896
2001	6,155,942,526
2002	6,229,629,168
2003	6,303,112,453
2004	6,376,863,118
2005	6,451,058,790

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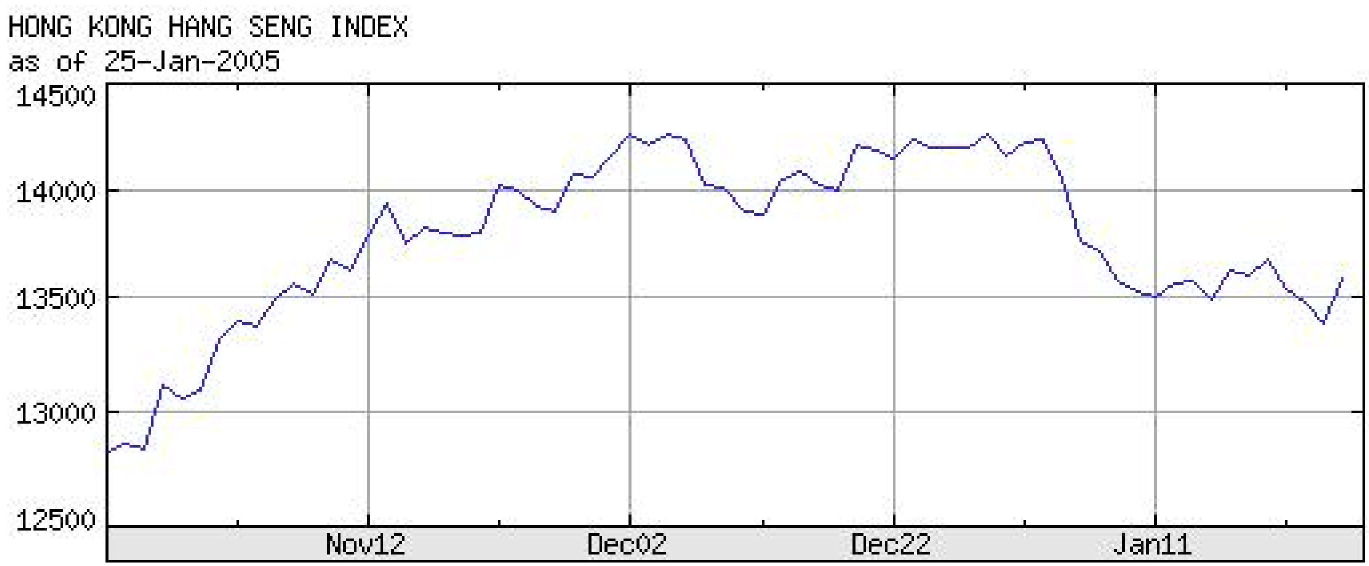
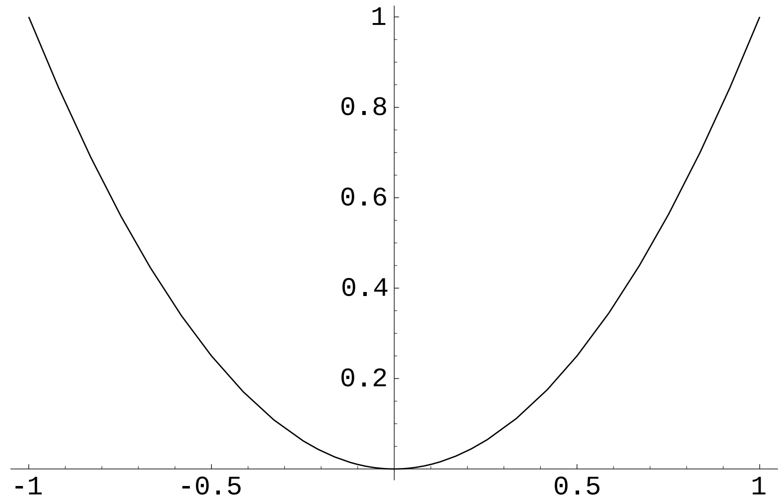
(2) Represented by graphs:

(a)



(b) *Graph of* $y = x^2$ *for* $-1 \leq x \leq 1$



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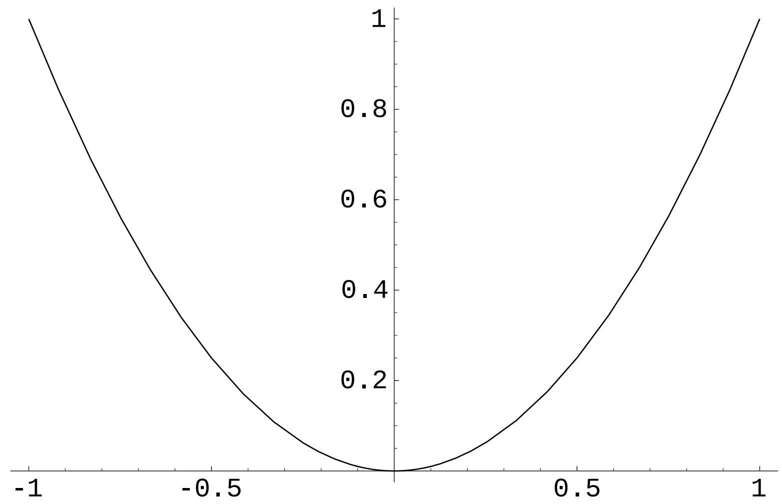
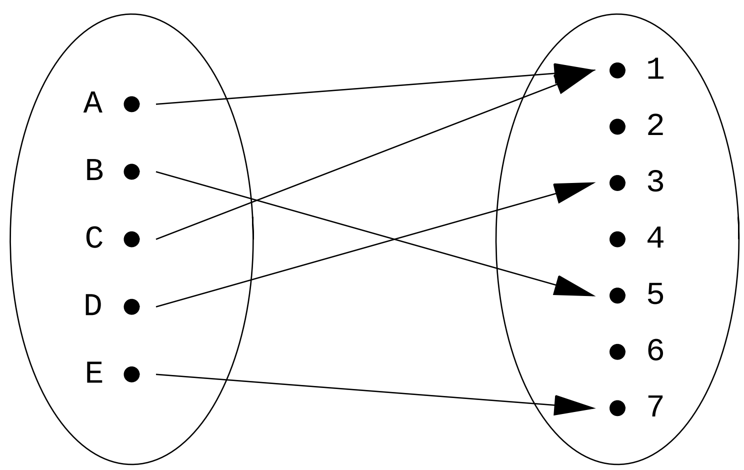


(3) Represented by mappings:



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Definition *(Idea)*

A *function* f from a set A to a set B, denoted by $f : A \longrightarrow B$, is a *"rule"* that *assigns to each element of A* **exactly one** *element of B*.

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For the above *“functions”*,

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Definition *(Idea)*

A *function* f from a set A to a set B, denoted by $f : A \longrightarrow B$, is a *“rule”* that *assigns to each element of* A **exactly one** *element of* B.

Terminology A is called the *domain* of the function f, denoted by dom(f).

B is called the *codomain* of f.

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- The domain is $[0, \infty)$.
- The codomain is $\mathbb{R}$.

For each element x in A (input), the corresponding element in B (output) assigned by the function f is denoted by $f(x)$.

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