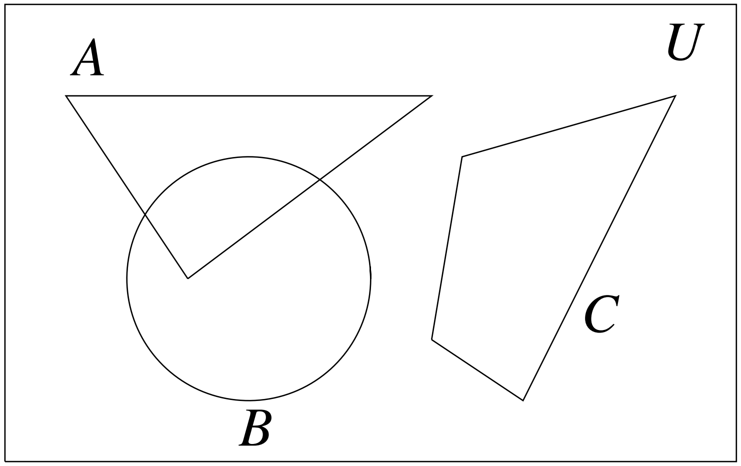
Venn diagram



- The universal set U is usually represented by a rectangle.
- Inside this rectangle, subsets of the universal set are represented by geometrical figures.

Venn diagrams help us *identify* some useful formulas in set operations.

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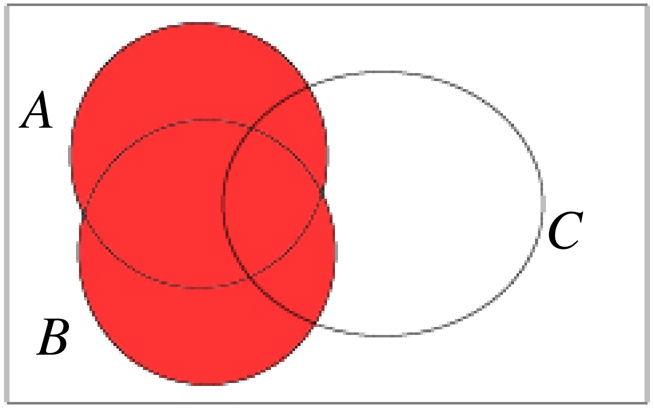

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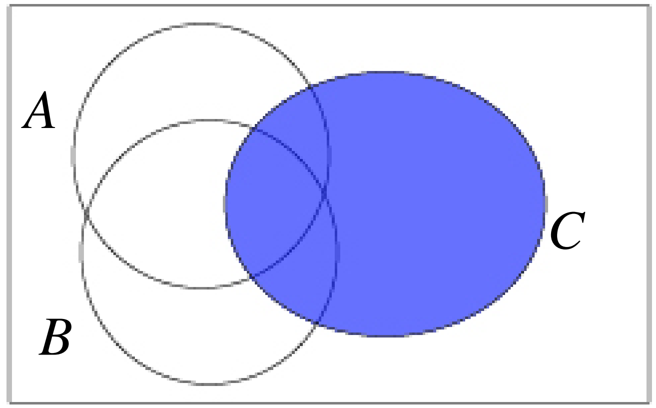

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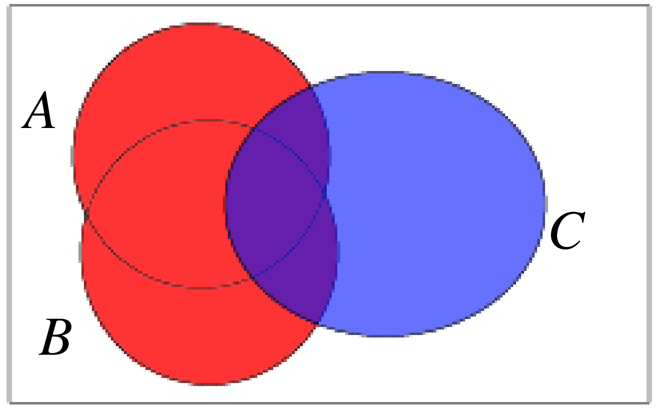

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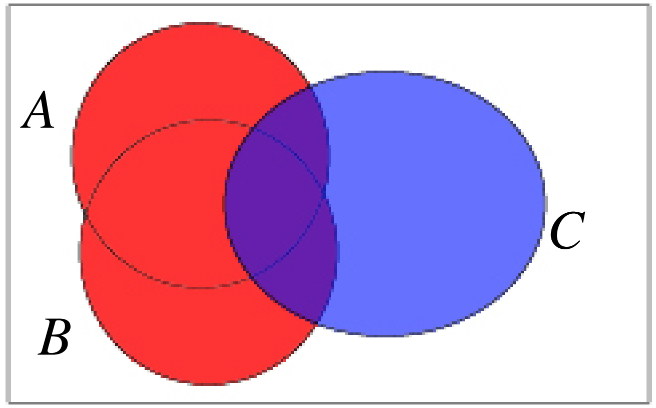

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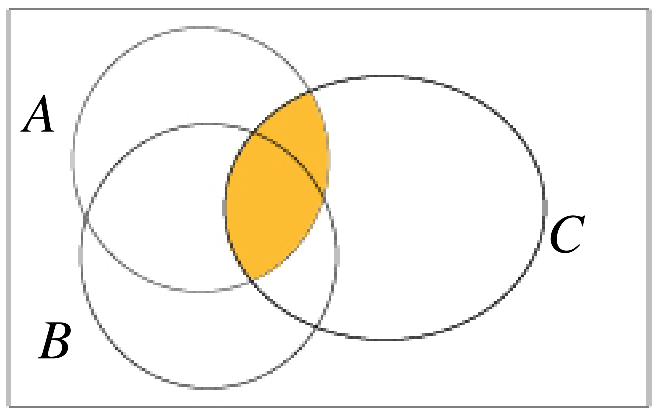

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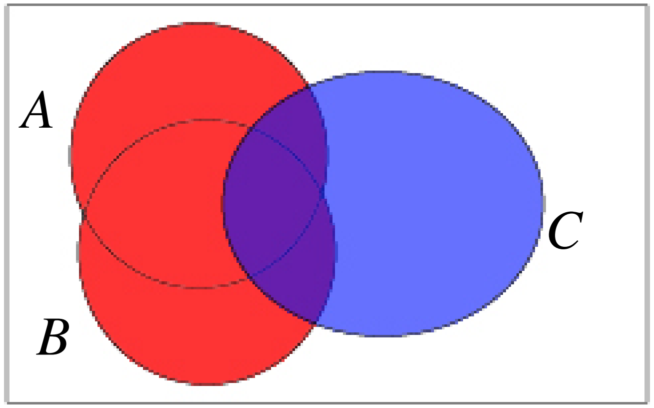

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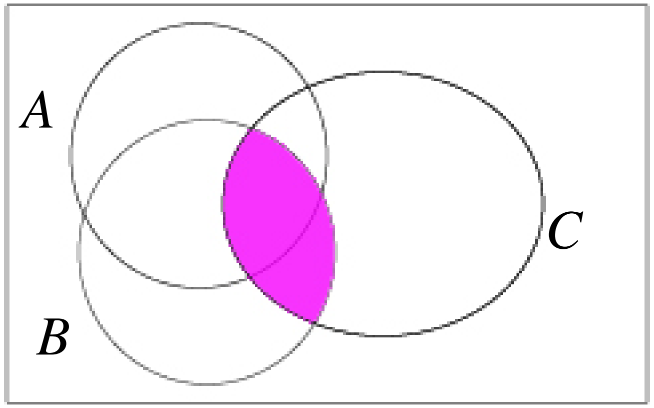

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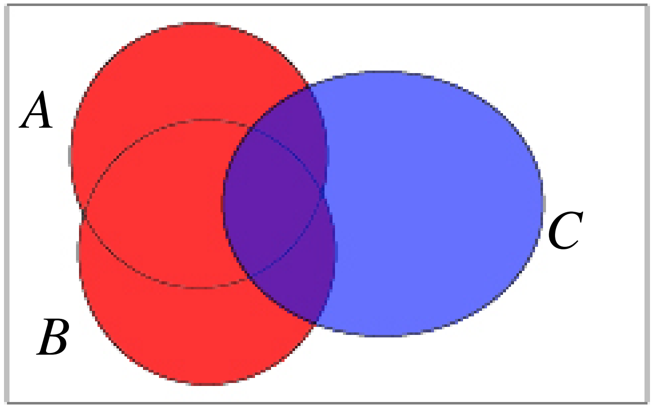

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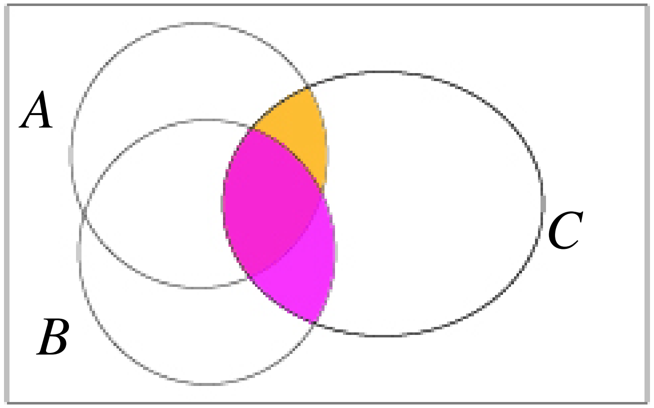

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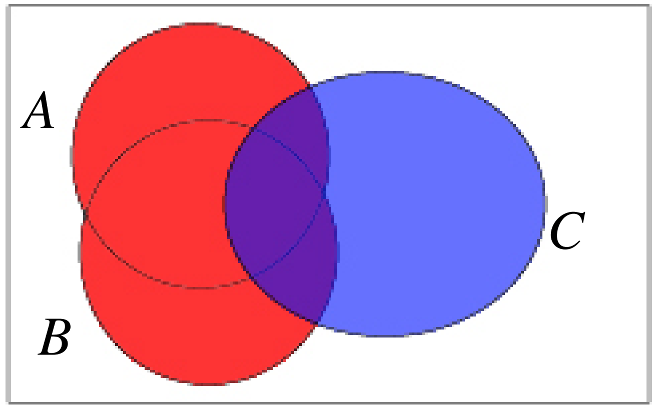

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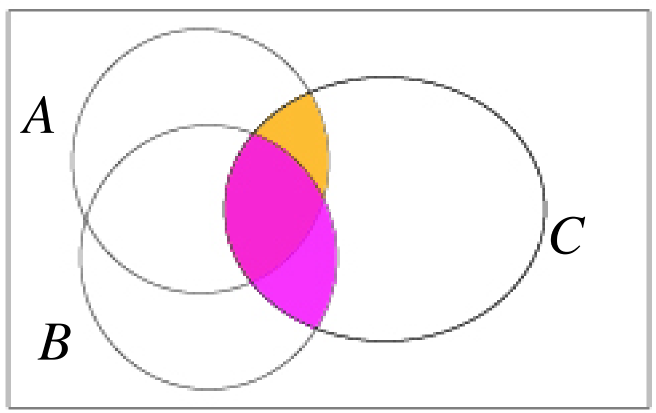

To prove $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ *in a* rigorous *manner, should use formal* mathematical logic*.*

A few remarks about set notations

- If a set has *finitely many elements*, use the listing method to express the set
  - write down all the elements
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  For example, $10 \in [7, 11]$

  $$9.123 \in [7, 11]$$

  $$\sqrt{50} \in [7, 11]$$

Inequalities

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Polynomial inequalities

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 < 0 \quad (\text{ or } > 0, \text{ or } \leq 0, \text{ or } \geq 0)$$

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(1) $n = 1$ Linear inequalities

(2) $n = 2$ Quadratic inequalities

(3) $n \geq 3$ Higher degree inequalities

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Quadratic Inequalities

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From these we get

(4) $m \cdot n > 0 \iff (m > 0 \text{ and } n > 0)$ or $(m < 0 \text{ and } n < 0)$

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$$x^2 + 2x - 15 > 0$$

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Apply Rule (4)

$(x + 5 > 0 \text{ and } x - 3 > 0)$ or $(x + 5 < 0 \text{ and } x - 3 < 0)$

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$$(x > -5 \text{ and } x > 3) \quad \text{or} \quad (x < -5 \text{ and } x < 3)$$

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Solution set $= \{x \in \mathbb{R} : x < -5 \text{ or } x > 3\}$

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Method 2 **Graphical method**

Example Find the solution set to the inequality

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- Graph of $y = x^2 + 2x - 15$

Example Find the solution set to the inequality

$$x^2 + 2x - 15 > 0$$

Method 2 **Graphical method**

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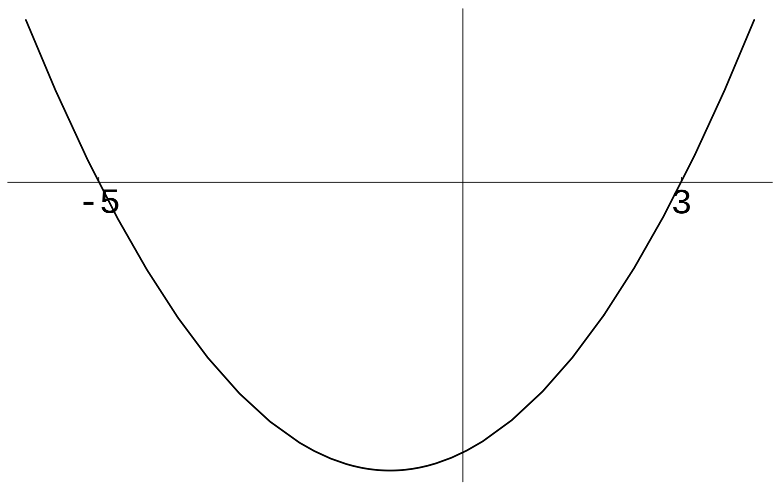

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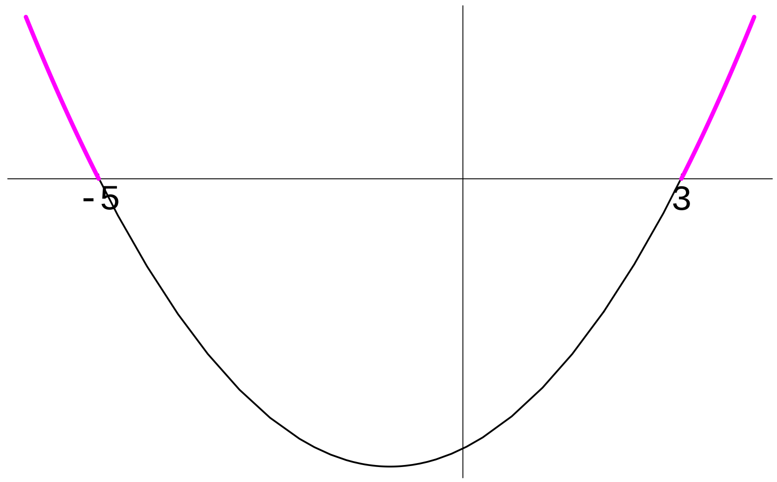

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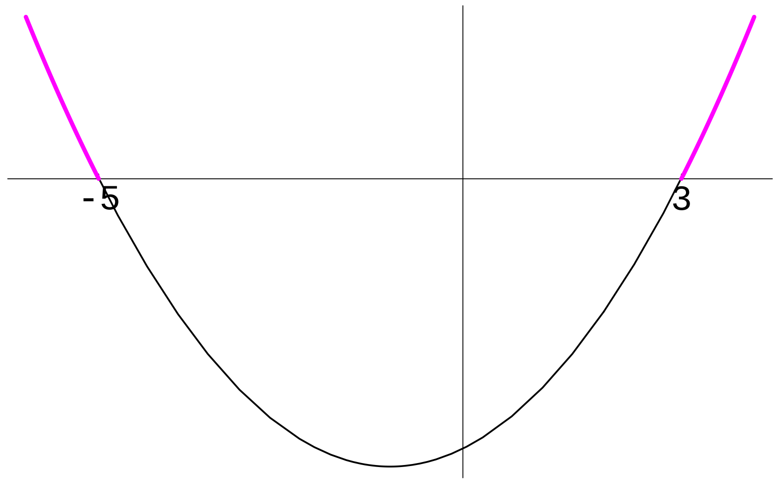

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$$x^2 + 2x - 15 > 0$$

Method 3

	$x < -5$	$x = -5$	$-5 < x < 3$	$x = 3$	$x > 3$
$(x+5)(x-3)$	$+$	0	$-$	0	$+$

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$$x^2 + 2x - 15 > 0$$

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	$x < -5$	$x = -5$	$-5 < x < 3$	$x = 3$	$x > 3$
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$x - 3$	–	–	–	0	+
$(x+5)(x-3)$	+	0	–		

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	$x < -5$	$x = -5$	$-5 < x < 3$	$x = 3$	$x > 3$
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$x - 3$	−	−	−	0	+
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Steps

Example Find the solution set to the inequality

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	$x < -5$	$x = -5$	$-5 < x < 3$	$x = 3$	$x > 3$
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Steps

- Factorize left-side $x^2 + 2x - 15 = (x+5)(x-3)$

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Steps

- Factorize left-side $x^2 + 2x - 15 = (x+5)(x-3)$
- Zeros of left-side -5 and 3

Example Find the solution set to the inequality

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$(x+5)(x-3)$	$+$	0	$-$	0	$+$

Steps

- Factorize left-side $x^2 + 2x - 15 = (x+5)(x-3)$
- Zeros of left-side -5 and 3
- Divide real number line into three parts: $(-\infty, -5)$, $(-5, 3)$, $(3, \infty)$

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Method 3

$$x^2 + 2x - 15 > 0$$

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- Zeros of left-side -5 and 3
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- On each of these intervals, determine the *sign* of $(x + 5)$ and $(x - 3)$,

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$x + 5$	$-$	0	$+$	$+$	$+$
$x - 3$	$-$	$-$	$-$	0	$+$
$(x + 5)(x - 3)$	$+$	0	$-$	0	$+$

Steps

- Factorize left-side $x^2 + 2x - 15 = (x + 5)(x - 3)$
- Zeros of left-side -5 and 3
- Divide real number line into three parts: $(-\infty, -5)$, $(-5, 3)$, $(3, \infty)$
- On each of these intervals, determine the *sign* of $(x + 5)$ and $(x - 3)$, hence the *sign* of $(x + 5)(x - 3)$

Polynomial inequalities (degree ≥ 3)

$$a_n x^n + a_{n-1} x^{n-1} + \cdots a_1 x + a_0 \quad > \ 0$$

Polynomial inequalities (degree ≥ 3)

$$
\begin{aligned}
a_n x^n + a_{n-1} x^{n-1} + \cdots a_1 x + a_0 \quad &> \ 0 \\
&\geq \ 0 \\
&< \ 0 \\
&\leq \ 0
\end{aligned}
$$

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Method 1 2^{n-1} cases

For example, $n = 3$: $a \cdot b \cdot c > 0$

4 cases: $+++$ $+--$ $-+-$ $--+$

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$n = 4$: $a \cdot b \cdot c \cdot d > 0$

8 cases: $++++$ $\quad ++--$ $\quad -++-$ $\quad --++$

$+-+-$ $\quad -+-+$ $\quad +--+$ $\quad ----$

Polynomial inequalities (degree ≥ 3)

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Method 2 Need graphs polynomials of *degrees* ≥ 3

Polynomial inequalities (degree ≥ 3)

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Method 2 Need graphs polynomials of *degrees* ≥ 3

Method 3 By table

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$$\leq \ 0$$

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Method 2 Need graphs polynomials of *degrees* ≥ 3

Method 3 By table

ALL three methods need to *factorize L.S.*

Example Solve the following inequality $x^3 + 3x^2 - 4x - 12 \le 0$

Example Solve the following inequality $x^3 + 3x^2 - 4x - 12 \leq 0$

Solution Denote $p(x) = x^3 + 3x^2 - 4x - 12$.

- Factorize $p(x)$

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  - ◇ $p(2) = 2^3 + 3(2^2) - 4(2) - 12 = 0$

    By Factor Theorem, $(x - 2)$ *is a factor of* $p(x)$.

Example Solve the following inequality $x^3 + 3x^2 - 4x - 12 \leq 0$

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$$\begin{aligned} p(x) &= (x-2)(x^2+5x+6) \\ &= (x-2)(x+2)(x+3) \end{aligned}$$

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    $= (x-2)(x+2)(x+3)$
-

		-3		-2		2	
$x - 2$							
$x + 2$							
$x + 3$							
$p(x)$							

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$x - 2$						0	
$x + 2$				0			
$x + 3$		0					
$p(x)$		0		0		0	

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$x - 2$						0	+
$x + 2$				0			
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$p(x)$		0		0		0	

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$x - 2$	$-$	$-$	$-$	$-$	$-$	0	$+$
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$x - 2$	$-$	$-$	$-$	$-$	$-$	0	$+$
$x + 2$	$-$	$-$	$-$	0	$+$	$+$	$+$
$x + 3$		0					
$p(x)$		0		0		0	

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$x-2$	$-$	$-$	$-$	$-$	$-$	0	$+$
$x+2$	$-$	$-$	$-$	0	$+$	$+$	$+$
$x+3$		0	$+$	$+$	$+$	$+$	$+$
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$x - 2$	$-$	$-$	$-$	$-$	$-$	0	$+$
$x + 2$	$-$	$-$	$-$	0	$+$	$+$	$+$
$x + 3$	$-$	0	$+$	$+$	$+$	$+$	$+$
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  - ◇ Long division or compare coeffficients

$$p(x) = (x-2)(x^2+5x+6)$$
$$= (x-2)(x+2)(x+3)$$

-

	$x < -3$	-3	$-3 < x < -2$	-2	$-2 < x < 2$	2	$2 < x$
$x - 2$	$-$	$-$	$-$	$-$	$-$	0	$+$
$x + 2$	$-$	$-$	$-$	0	$+$	$+$	$+$
$x + 3$	$-$	0	$+$	$+$	$+$	$+$	$+$
$p(x)$	$-$	0		0		0	

Example Solve the following inequality $x^3 + 3x^2 - 4x - 12 \leq 0$

Solution Denote $p(x) = x^3 + 3x^2 - 4x - 12$.

- Factorize $p(x)$
  - ◇ $p(2) = 2^3 + 3(2^2) - 4(2) - 12 = 0$

    By Factor Theorem, $(x - 2)$ *is a factor of* $p(x)$.
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$p(x)$	$-$	0	$+$	0	$-$	0	$+$

- Solution: $x < -3$ or $x = -3$ or $x = -2$ or $-2 < x < 2$ or $x = 2$, that is,

$x \le -3$ or $-2 \le x \le 2$.