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Example The solution set of the above equation can be written as

- $\{-2, 5\}$ *(listing)*
- $\{x : x = -2 \text{ or } x = 5\}$ *(description)*

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Definition *(Idea)*

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Listing List all the elements, enclosed by braces. Eg.

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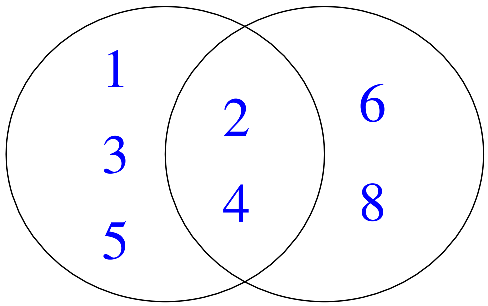

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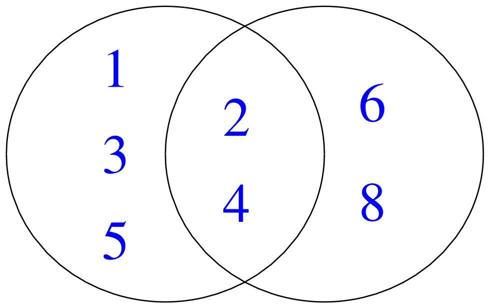

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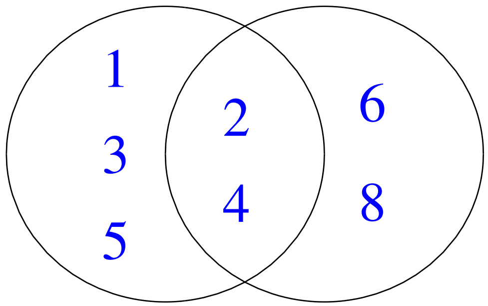

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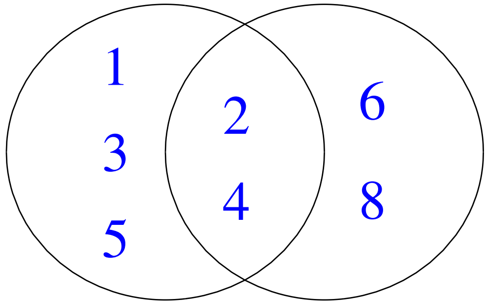

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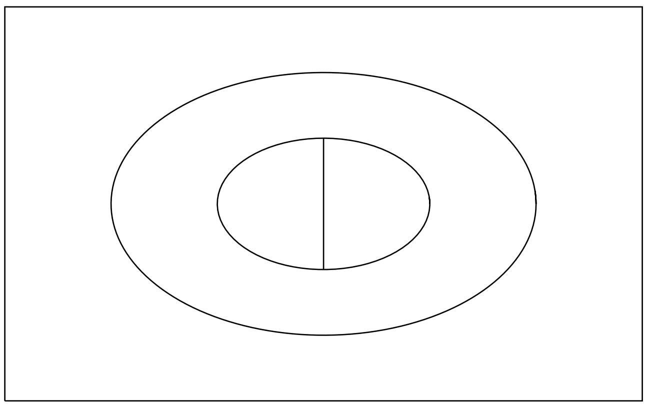

Interval Notation The following subsets of $\mathbb{R}$ are called *intervals*.

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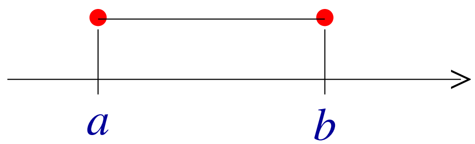

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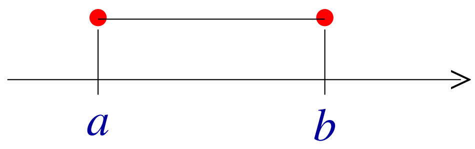

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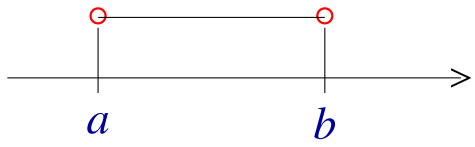

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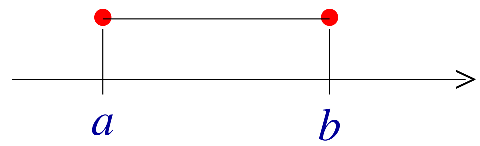

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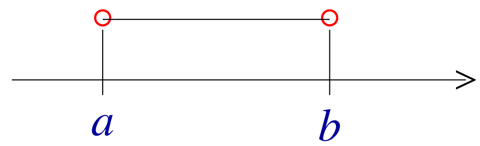

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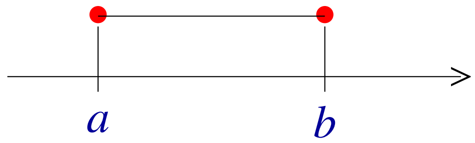

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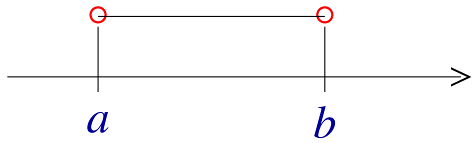

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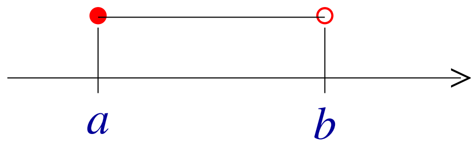

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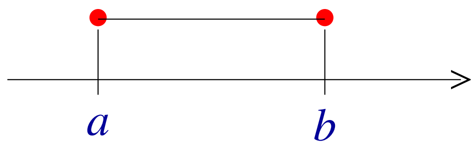

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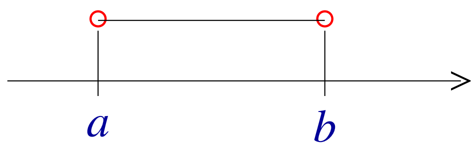

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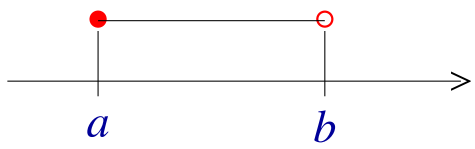

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Terminology (a, b) is called an *open interval* and $[a, b]$ a *closed interval*.

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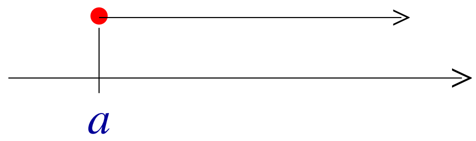

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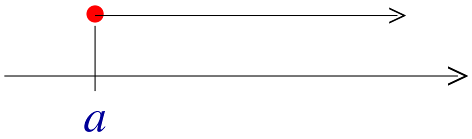

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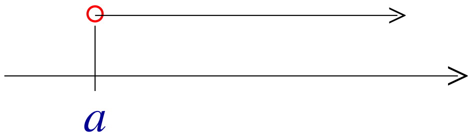

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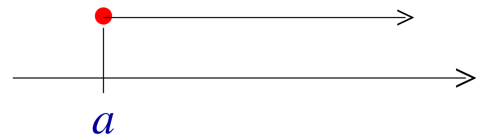

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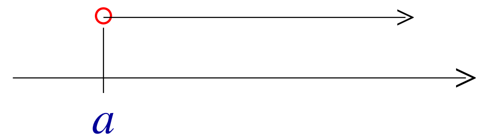

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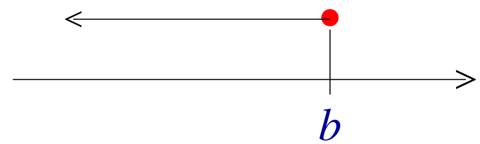

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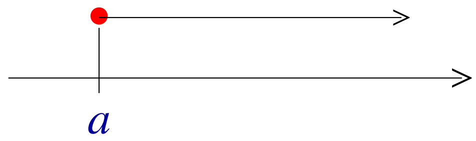

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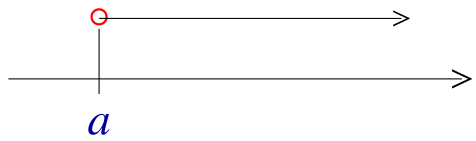

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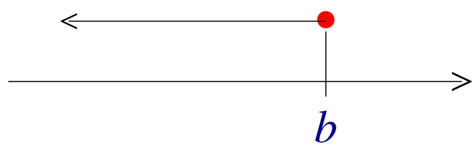

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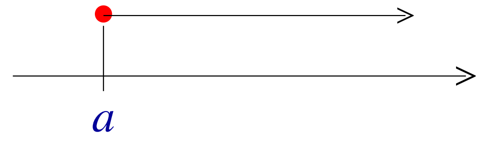

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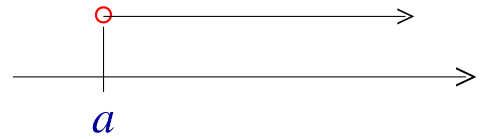

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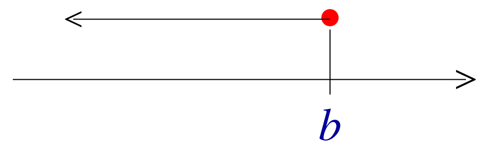

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Warning Because ∞ is not a real number, *don't write* $[a, \infty]$ etc.

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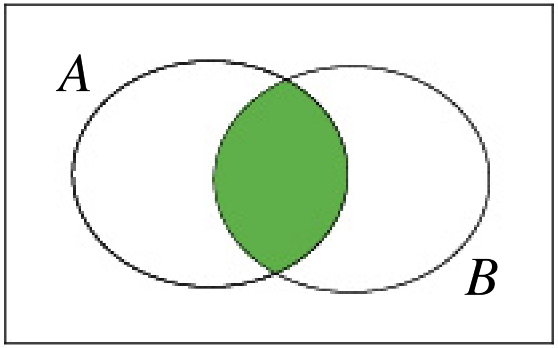

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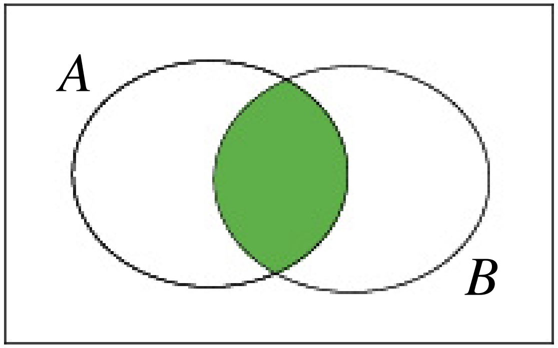

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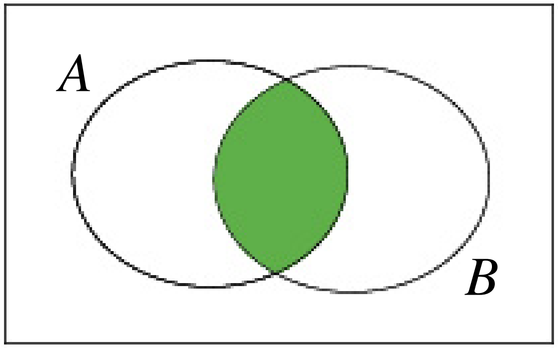

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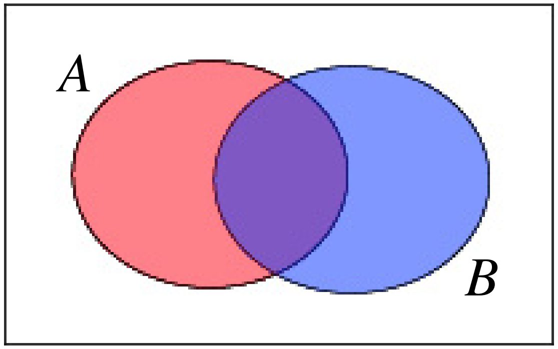

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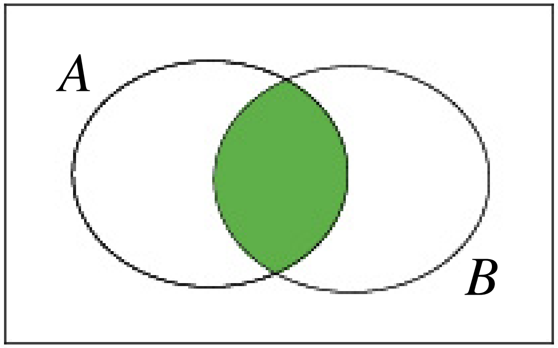

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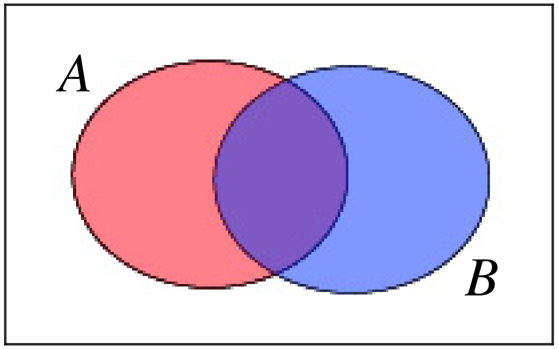

Note In mathematics, "P or Q" means "*either P or Q or both P and Q*".

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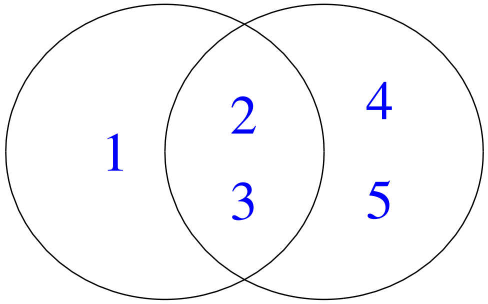

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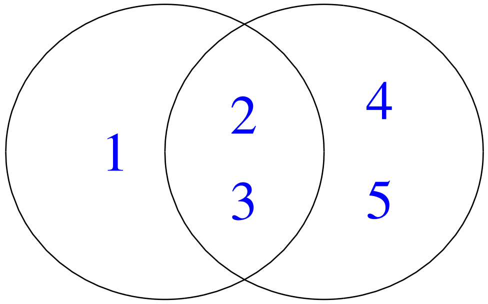

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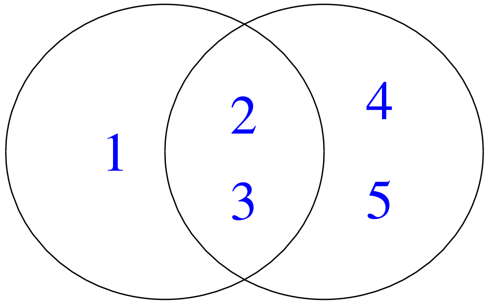

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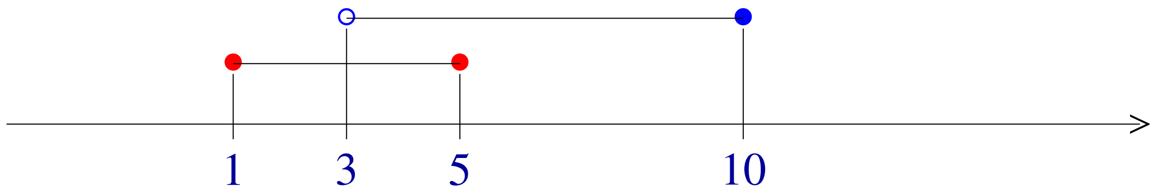

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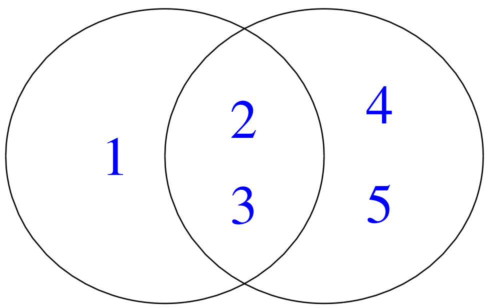

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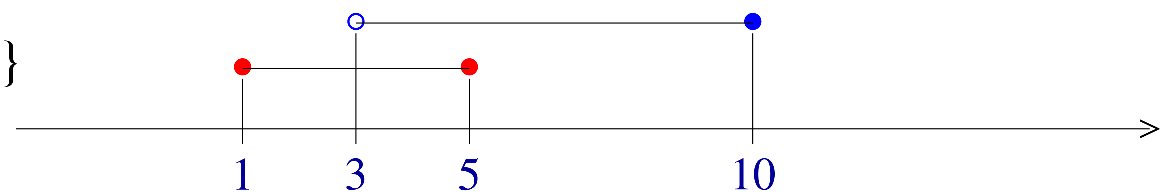

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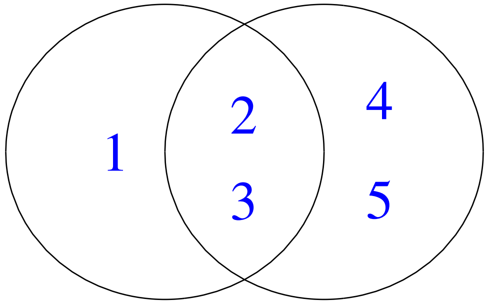

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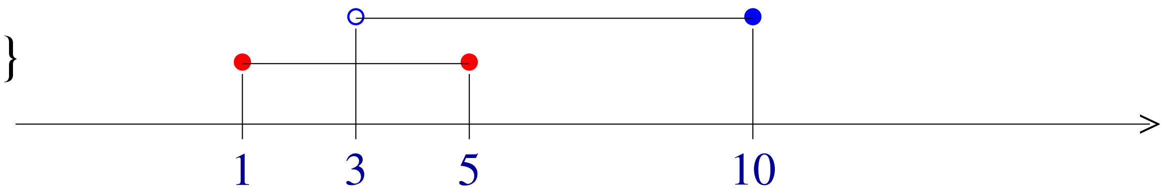

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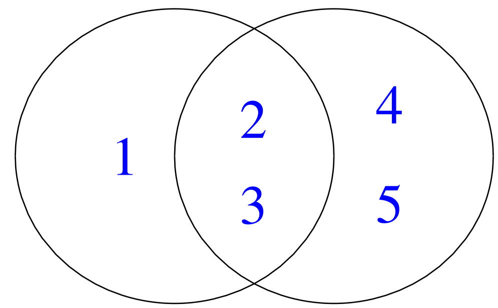

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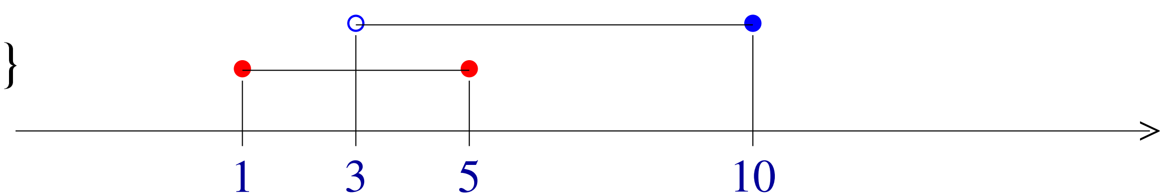

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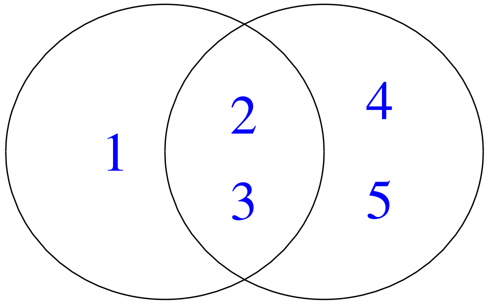

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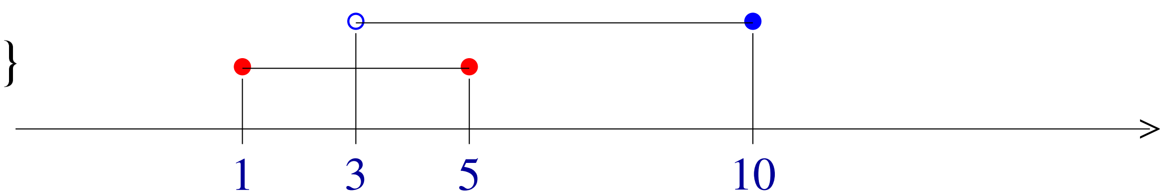

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Exercise Let $U = \{1, 2, 3, \ldots, 12\}$. Let

$$\begin{aligned} A &= \{x \in U : x \text{ is a prime number}\} \\ B &= \{x \in U : x \text{ is an even number}\} \\ C &= \{x \in U : x \text{ is divisible by } 3\} \end{aligned}$$

Find $(A \cup B) \cap C$ and $(A \cap C) \cup (B \cap C)$.

Note 1 *is not prime.*

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Answer $(A \cup B) \cap C = (A \cap C) \cup (B \cap C) = \{3, 6, 12\}$

Definition The *set difference* of two sets A and B, written $A - B$ (or $A \setminus B$), is the set consisting of *all the elements of A that are not elements of B*

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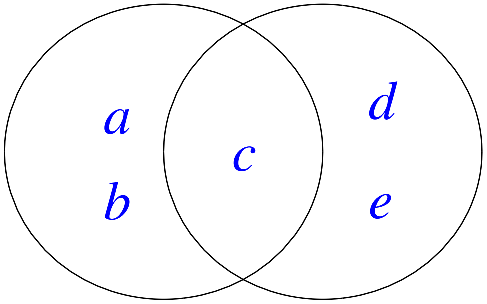

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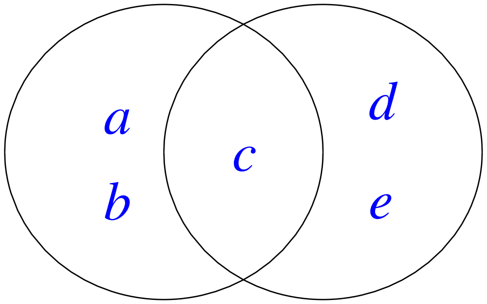

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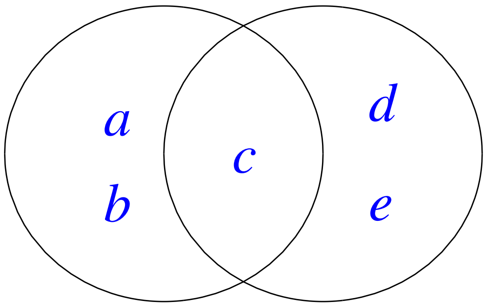

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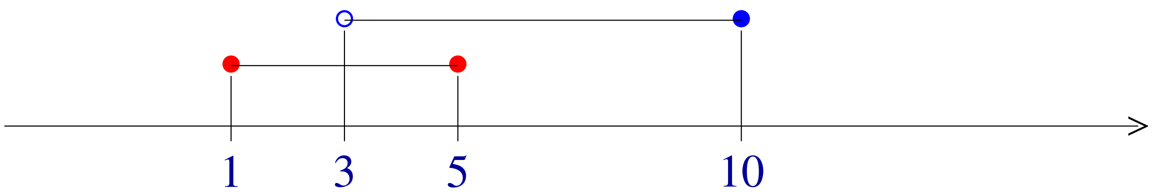

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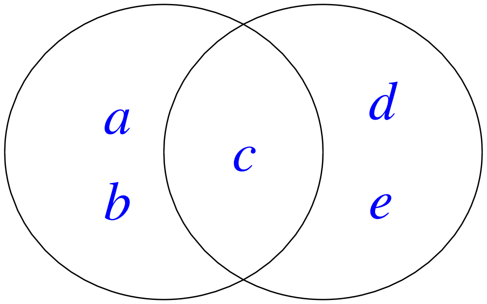

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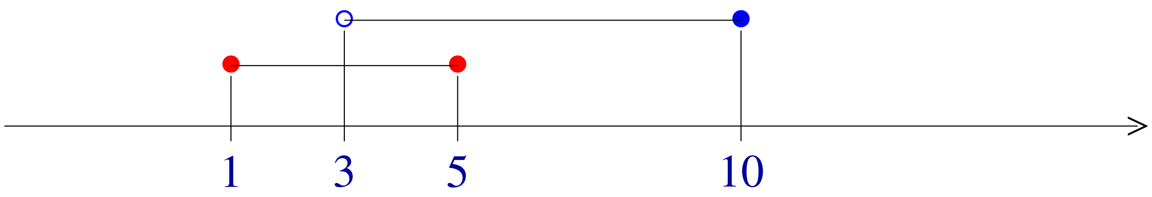

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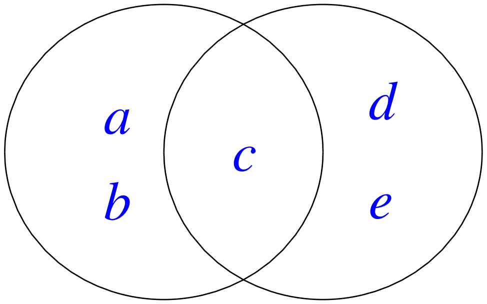

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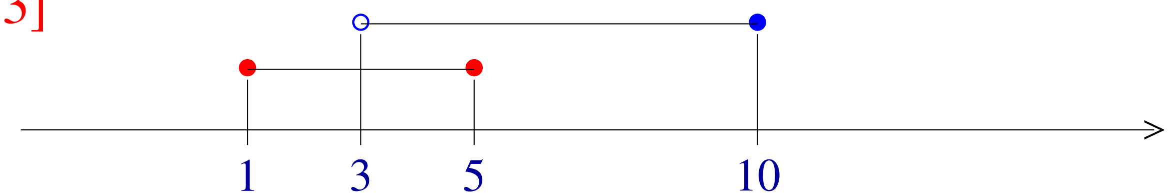

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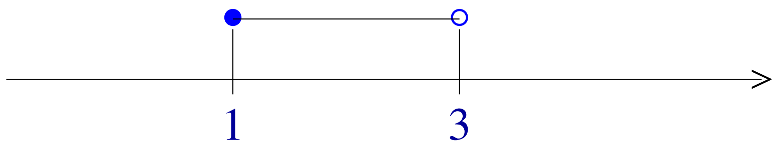

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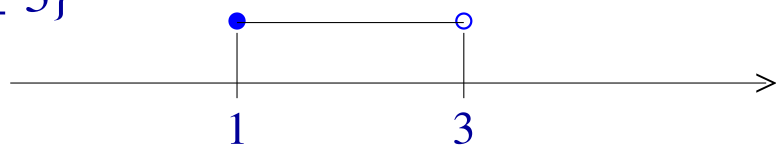

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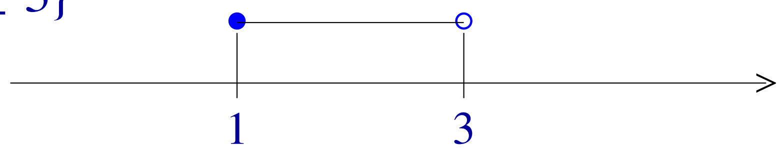

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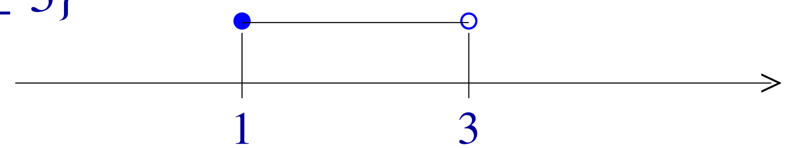

Exercise Let $U = \{1, 2, 3, \ldots, 12\}$. Let

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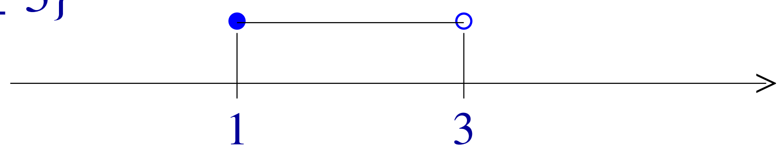

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Answer $(A \cup B)' = (A' \cap B') = \{1, 9\}$